

Lecture 40

- Last time:
 - Bias and output swing for BiCMOS voltage amp
- Start open-circuit time constant analysis
(back to Chapter 10)
- Today :
 - Applications of open-circuit time constant analysis: CE amplifier and cascode amplifier

Frequency Response of Multistage Amplifiers

We need a *systematic technique* rather than a bunch of qualitative results (e.g., CS suffers from Miller effect, CD and CG are wideband stages ...)

Disappointing news: our analytical technique is capable of estimating only the dominant (lowest) pole ... for a restricted class of amplifiers.

ω_1

DETAILED f RESPONSE
SPICE

The Special Case:

The transfer function can have no zeroes and must have a dominant pole $\omega_1 \ll \omega_2, \omega_3, \dots, \omega_n$ (common)

RELEVANT $\omega_2 \ll \omega_1$

Very/Min

$$H(j\omega) = \frac{H_0}{(1 + j\omega b_1 + (j\omega)^2 b_2 + (j\omega)^3 b_3 + \dots)}$$

Factor denominator: (SYMBOLIC ONLY)

$$H(j\omega) = \frac{H_0}{(1 + j\omega / \omega_1)(1 + j\omega / \omega_2) \dots (1 + j\omega / \omega_n)}$$

Approximating the Transfer Function

Multiply out denominator:

$$H(j\omega) = \frac{H_o}{(1 + j\omega / \omega_1)(1 + j\omega / \omega_2) \dots (1 + j\omega / \omega_n)} \approx$$

$$\frac{H_o}{1 + j\omega \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} + \dots + \frac{1}{\omega_n} \right) + \dots}$$

Since $\omega_1 \ll \omega_2, \omega_3, \dots, \omega_n \rightarrow$

$$b_1 = \frac{1}{\omega_1} + \frac{1}{\omega_2} + \dots + \frac{1}{\omega_n} \approx \frac{1}{\omega_1}$$

WHAT THE
a PENINANT
POLE MEANS!

How to Find b_1 ?

See P. R. Gray and R. G. Meyer, *Analysis and Design of Analog Integrated Circuits* (EE 140) for derivation.

Result: b_1 is the sum of open-circuit time constants τ_i which can be found by considering each capacitor C_i in the amplifier separately and finding its Thévenin resistance $R_{Ti} \rightarrow$

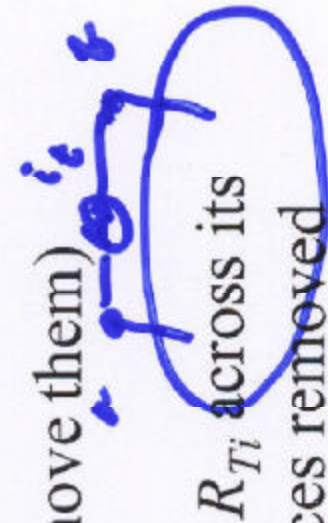
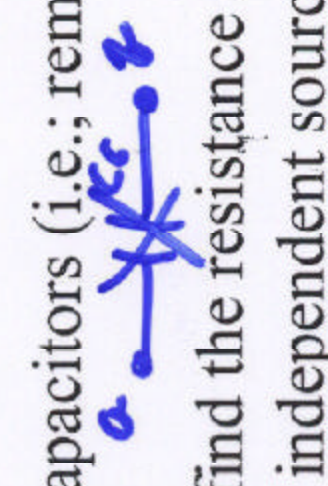
$$\tau_i = R_{Ti} C_i$$

$$\omega_1 = \frac{1}{\tau_i}$$

$$b_1 = \sum_{i=1}^n R_{Ti} C_i$$

$$\omega_1 \approx \frac{1}{\sum_{i=1}^n R_{Ti} C_i}$$

Finding the Thévenin Resistance

1. Open-circuit all capacitors (i.e.; remove them) 
2. For capacitor C_i , find the resistance R_{Ti} across its terminals with all independent sources removed (voltages shorted, currents opened) ... might need to apply a test voltage and find the current in some cases. 

$$\tau_i = \frac{1}{\omega_i} = \frac{1}{R_{Ti} C_i}$$

Insight for design: the bandwidth of the amplifier will be limited by the capacitor that contributes the largest

$\tau_i = R_{Ti} C_i$ \rightarrow not necessarily the largest C_i

$$\omega_1 = \frac{1}{\sum_{i=1}^n R_{Ti} C_i} \approx \frac{1}{(R_{Tj} C_j)_{\max}}$$

"TECHNIQUE"

Systematic Approach

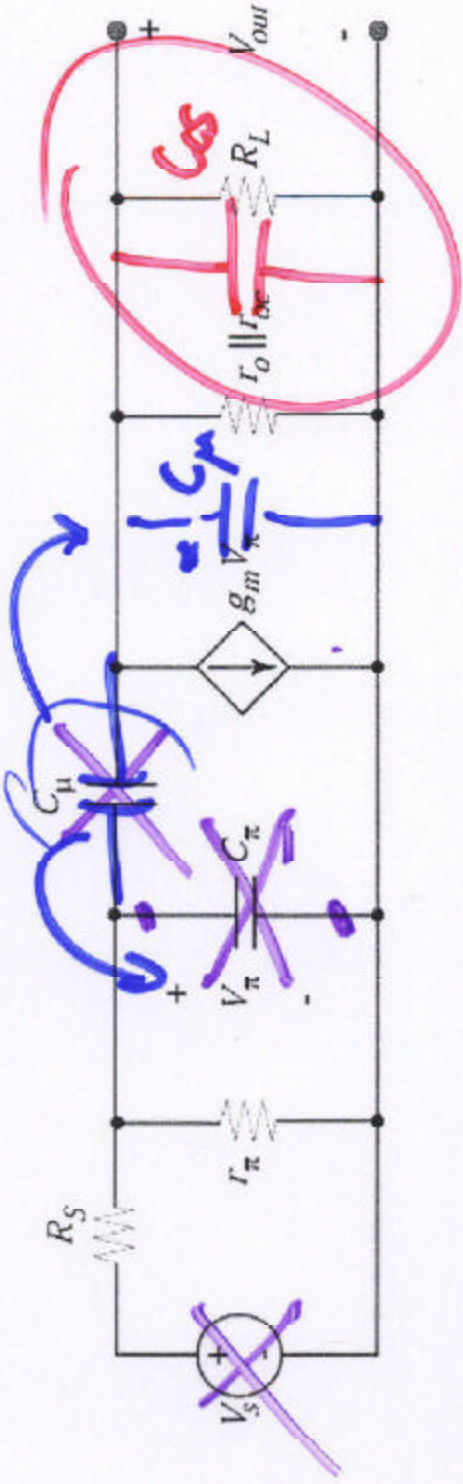
1. Construct two-port small-signal models for each stage
2. Add the capacitors for each device across the appropriate nodes in the two-port models. Make sure how the gate, drain, and source (or base, collector, and emitter) terminals of each device fit onto the two-port models!
3. (Optional) Use Miller's Theorem to transform capacitors across amplifiers into effective capacitances to ground (note that we ignore the "output Miller" in this course)

WATCH OUT!

A GOOD IDEA!

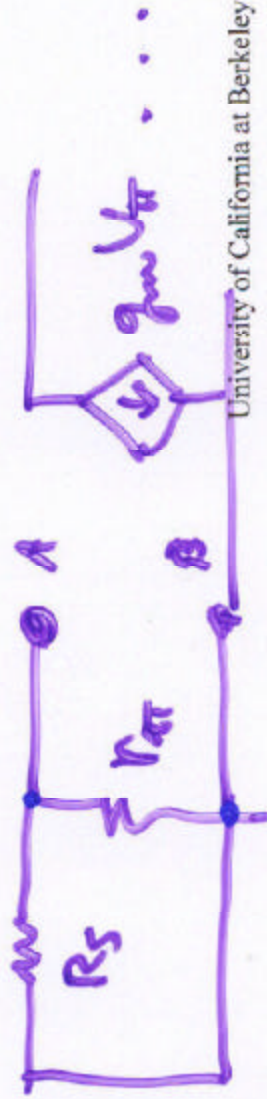
APPLYING O.C.T.C.

Common-Emitter Voltage Amplifier



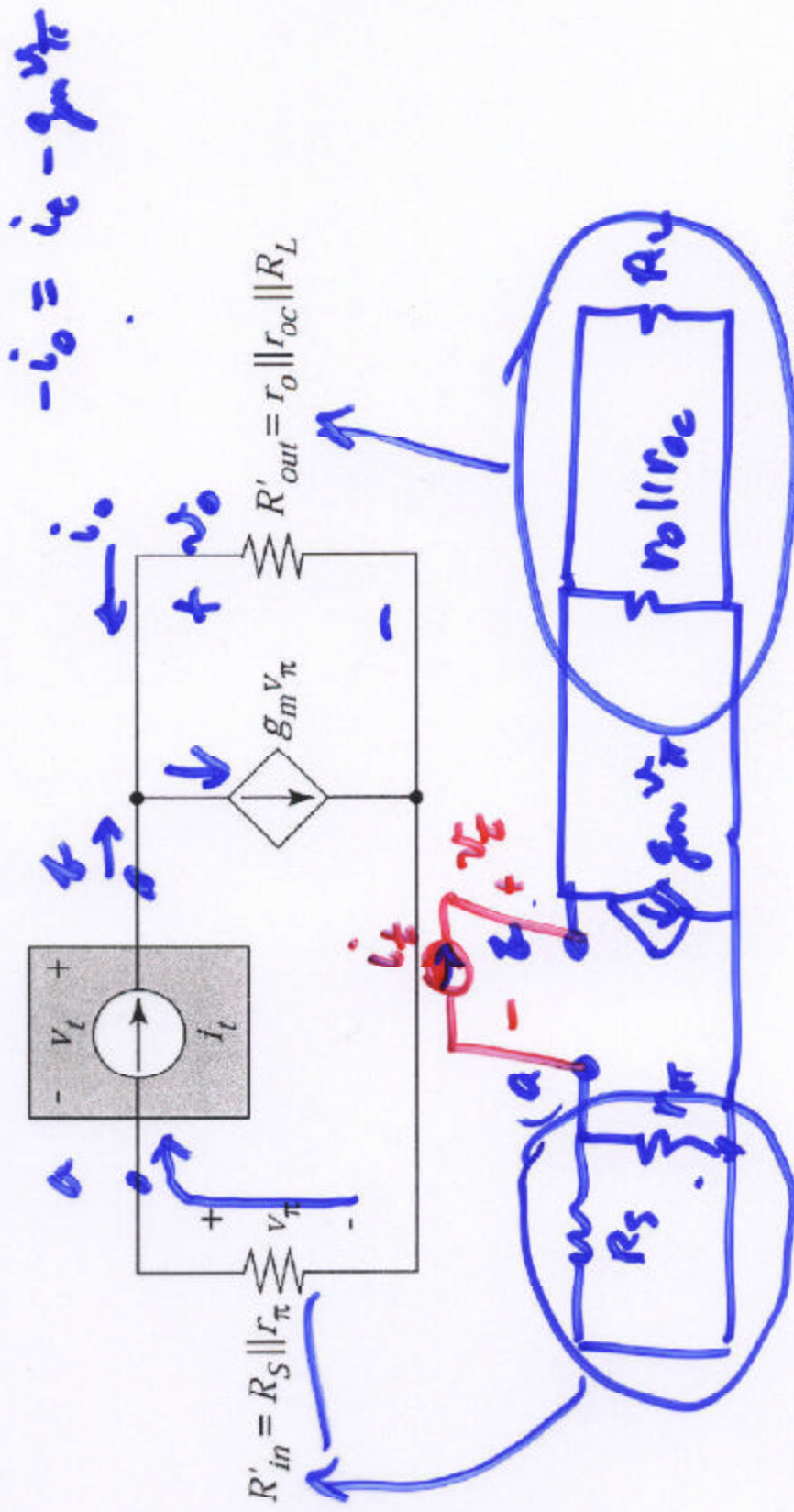
Time constant for base-emitter capacitance C_π :

$$\tau_\pi = R_{T\pi} C_\pi = (R_s || r_\pi) C_\pi$$



Base-Collector Time Constant

Must apply a test source (can't see $R_{T\mu}$ by inspection):



Solving for $R_{T\mu}$

• Find v_π :

$$v_\pi = -i_t (R_S \parallel r_\pi) = -i_t R'_{in}$$

• Find v_o :

$$v_o = -i_o R'_{out} = (i_t - g_m v_\pi) R'_{out} = i_t (1 + g_m R'_{in}) R'_{out}$$

• Find v_t :

$$v_t = v_o - v_\pi = i_t (1 + g_m R'_{in}) R'_{out} - (-i_t R'_{in})$$

Solve for Thévenin resistance:

$$v_t = i_t [R_{out}' + g_m R_{in}' R_{out}' + R_{in}']$$

$$R_{T\mu} = \frac{v_t}{i_t} = R_{in}' + R_{out}' + g_m R_{in}' R_{out}'$$

Dominant Pole of CE Amplifier

Estimate dominant pole as inverse of sum of OCTCs:

$$\omega_1 \approx \frac{1}{\tau_{C_\pi} + \tau_{C_\mu}} = \underbrace{\left(R'_{in} C_\pi + [R'_{out} + g_m R'_{in} R'_{out}] C_\mu \right)^{-1}}$$

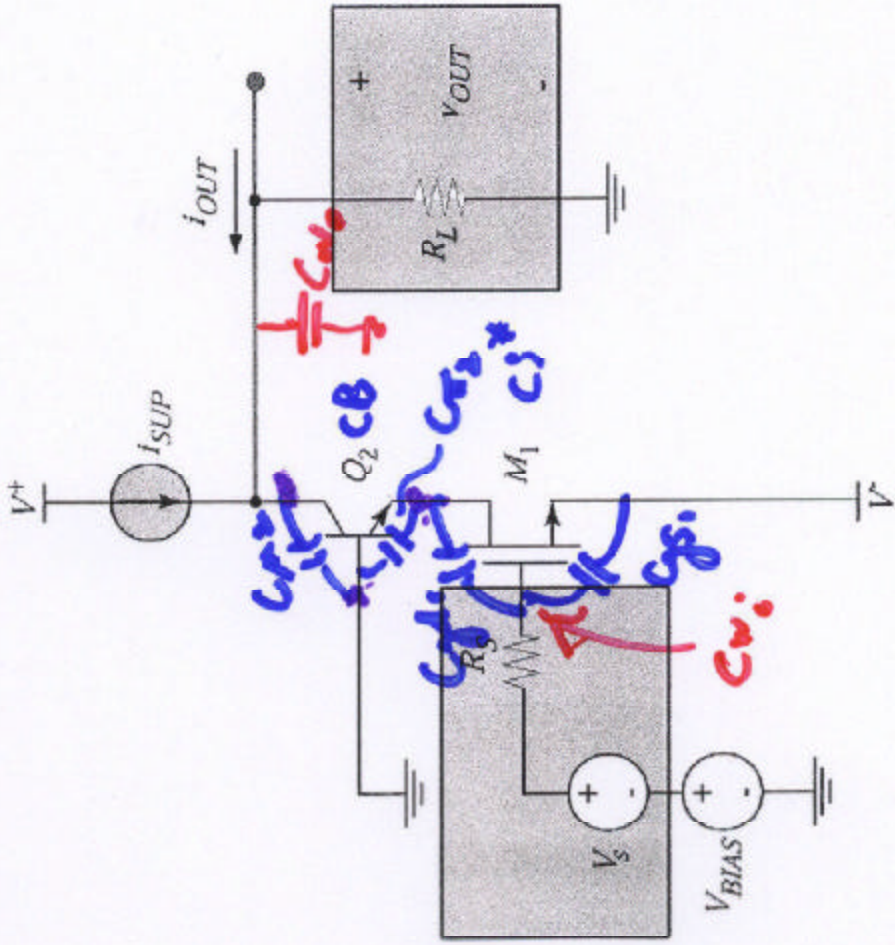
adder *no. in. || R_e*

Identical to the “exact” analysis in Chapter 10

Why bother with the OCTC technique ... add effect of C_{es}

$$\frac{-g_m \left(\frac{R_x}{R_x + r_\pi} \right) (r_o || R_e) (1 + g_m / \omega_c)}{\underbrace{\left(1 + g_m / \omega_1 \right) (1 + g_m / \omega_2)}} \quad \text{Root!!}$$

Multistage Amplifier Frequency Response



CS*-CB cascode

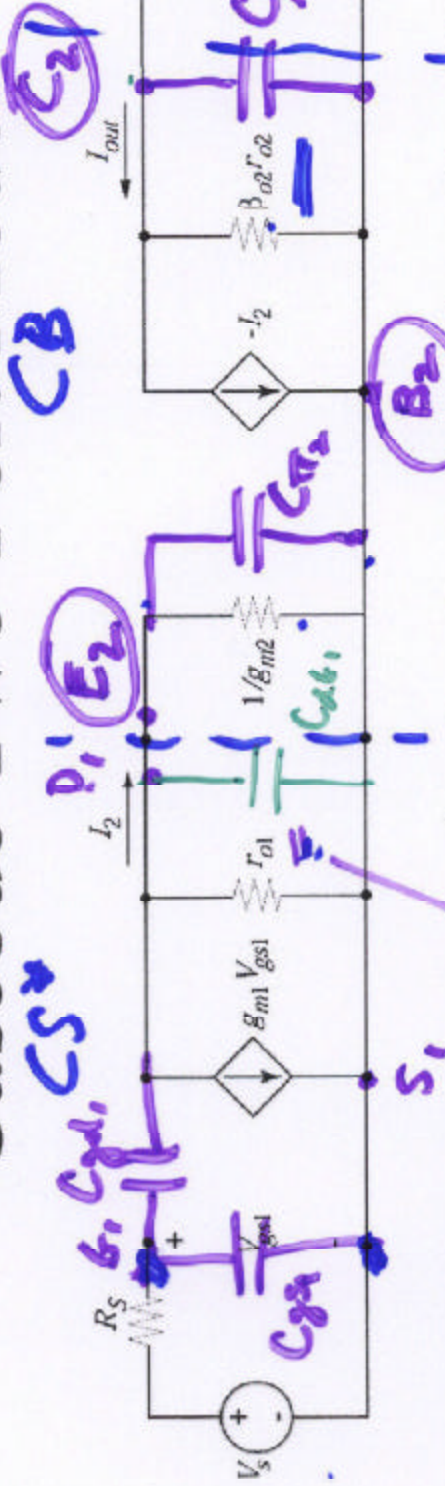
C_{gs1}
 $C_{gd1} = C_{gd2}$
 C_{db1}
 ~~C_{db2}~~ → shorted
 C_{gs2} ... omitted.

CURRENT SUPPLY...
 C_{gs2}
 C_{gd2}
 C_{pi2}

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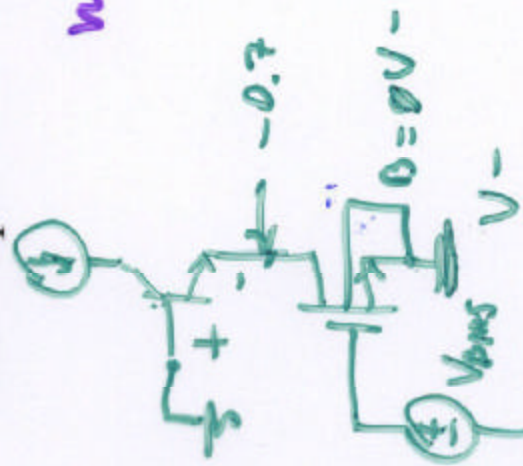
C_{wi}, C_{wo}

Cascode Two-Port Model



Add Capacitors: C_{gs1} , C_{gd1} , $C_{\mu 2}$, C_{gs2} ... what about C_{db1} , C_{cs2} ?

no roe!



Finding the Thévenin Resistances

o C_{gs1} : $R_{TC_{gs1}} = R_S$

$r_{o1} \parallel \frac{1}{g_{m2}}$

o C_{gd1} : $R_{TC_{gd1}} = R'_{in} + R'_{out} + g_{m1} R'_{in} R'_{out} = R_S + \frac{1}{g_{m2}} + g_{m1} (R_S \frac{1}{g_{m2}})$



$r_{o1} \rightarrow$

o $C_{\pi 2}$: $R_{TC_{\pi 2}} = r_{o1} \parallel \frac{1}{g_{m2}} \approx \frac{1}{g_{m2}}$

(+ C_{cb1})

o $C_{\mu 2}$: $R_{TC_{\mu 2}} = \beta_0 r_{o2} \parallel R_L$

(+ C_{cs2})

Dominant Pole

Applying the theorem:

$$\omega_1^{-1} \approx \underbrace{R_S C_{gs1}}_{\tau_{gs}} + \underbrace{R_S (1 + g_{m1} / g_{m2}) C_{gd1}}_{\tau_{gd}} + \underbrace{(1 / g_{m2}) C_{\pi 2}}_{\tau_{\pi 2}} + \underbrace{R_L C_{\mu 2}}_{\tau_{\mu 2}}$$

$R_L \ll \beta_o r_{oc2} \parallel R_{\mu 2}$

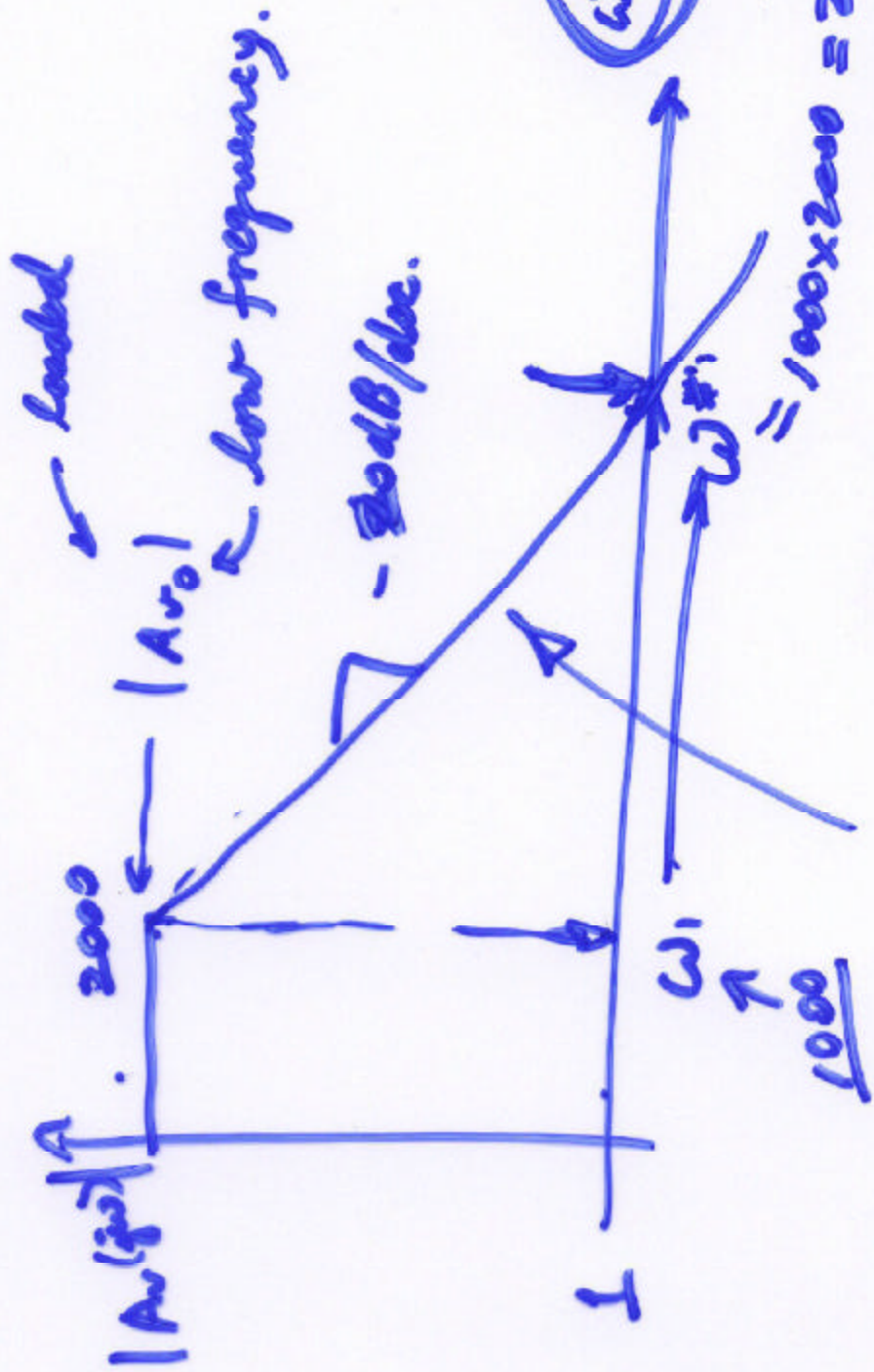
Find approximate voltage transfer function:

$$A_{vo} = \frac{V_{out}}{V_S} \Big|_{R_S, R_L} = -g_{m1} \left(\frac{r_{o1}}{r_{o1} + 1 / g_{m2}} \right) (\beta_o r_{oc2} \parallel r_{oc2} \parallel R_L) \approx -g_{m1} R_L$$

$R_L \ll r_{oc2}, \beta_o g_{m2}$

$$A_v(j\omega) \approx \frac{A_{vo}}{1 + j\omega(\tau_{gs} + \tau_{gd} + \tau_{\pi 2} + \tau_{\mu 2})}$$

(Avo) ← no capacitor



$$|A_v(j\omega)| \approx \frac{|A_{v0}| \cdot \omega_1}{\omega} = \frac{|A_{v0}|}{(\omega/\omega_1)} = 1$$

$$\left(\frac{|A_{v0}|}{\sqrt{1 + (\omega/\omega_1)^2}} \right) \propto \frac{|A_{v0}|}{(\omega/\omega_1)^2} \quad \omega \gg \omega_1$$

$\omega^\#$