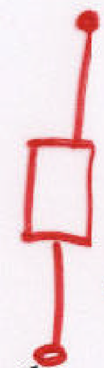
Lecture 4

I_c , $\tilde{V}_c$



$\frac{1}{j\omega C} = Z$

- Last time:
  - Circuit analysis with phasors: impedances
  - Bode plot of low-pass filter (start)
- Today : MAG, PHASE → VS. (ω or f)
  - Bode plot sketching for first-order transfer functions
  - Low-pass and high-pass filters

MONDAY 1-2 DISCUSSION

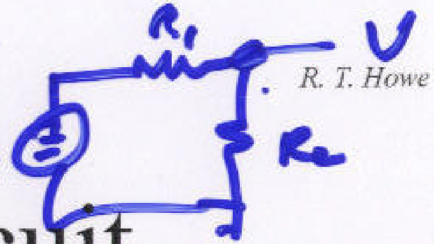
$\left\{ \begin{array}{l} \text{2/4 : 476 CORY HALL} \\ \text{2/11 AND LATER : 9 EVANS HALL} \end{array} \right\}$

R. T. Howe

KCL, KVL
Thev. Norton

WORKS?

DIVIDER



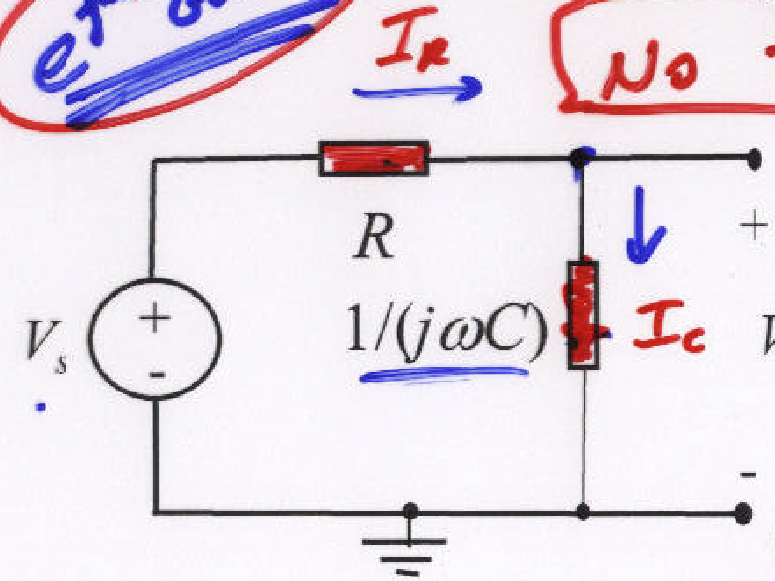
Redrawing the Circuit with Impedances

$V = V_s \left(\frac{R_2}{R_1 + R_2} \right)$

$e^{j\omega t}$ GONE

NO TIME

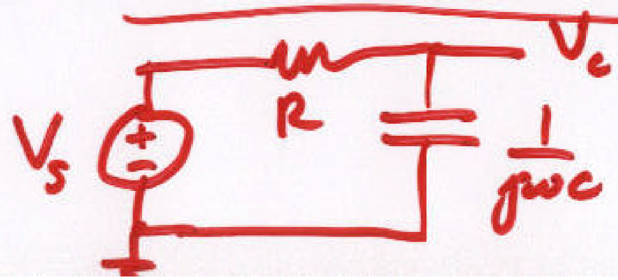
IMPEDANCE DIVIDER!



$$V_c = V_s \left[\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \right] \frac{j\omega C}{j\omega C}$$

$$V_c = V_s \left[\frac{1}{1 + j\omega RC} \right]$$

Note: this is not a "real" circuit that could be built and tested!



$\frac{I_{out}}{V_s} \ldots \frac{V_{out}}{I_s}$

$H = \frac{V_c}{V_s} \ldots$

$H(s);\ s = \sigma + j\omega.$

Transfer Function

Ratio of output to input phasor is called the transfer function of the circuit:

$$\boxed{H = \frac{V_c}{V_s} = \frac{1}{1 + j\omega RC}}$$

$H(j\omega)$. EE 120: EE 140:

$\omega^2 \rightarrow -(j\omega)^2$

$$|H|_{dB} = 20\log_{10}\left|\frac{1}{1 + j\omega RC}\right|$$

$$\angle H = \angle \frac{1}{1+j\omega RC} = \angle 1 - \tan^{-1}\left[\frac{\omega RC}{1}\right]$$

0, $1 + j0$

$\boxed{v_c(t) \text{ GIVEN } v_s(t)}$

Bode Plots for Low-Pass Filter

1. Plot magnitude $|H|$ in dB vs. ω (log scale)
2. Plot phase $\angle H$ in degrees vs. ω (log scale)

$$|H|_{dB} = \left|\frac{1}{1 + j\omega\tau}\right|_{dB} = \left[\frac{|1 + j0|}{|1 + j\omega\tau|}\right]_{dB} = 20\log_{10}\left\{\frac{\sqrt{1^2+0^2}}{\sqrt{1^2+(\omega\tau)^2}}\right\}$$

Why?

$$|z_1/z_2| = |z_1|/|z_2|$$

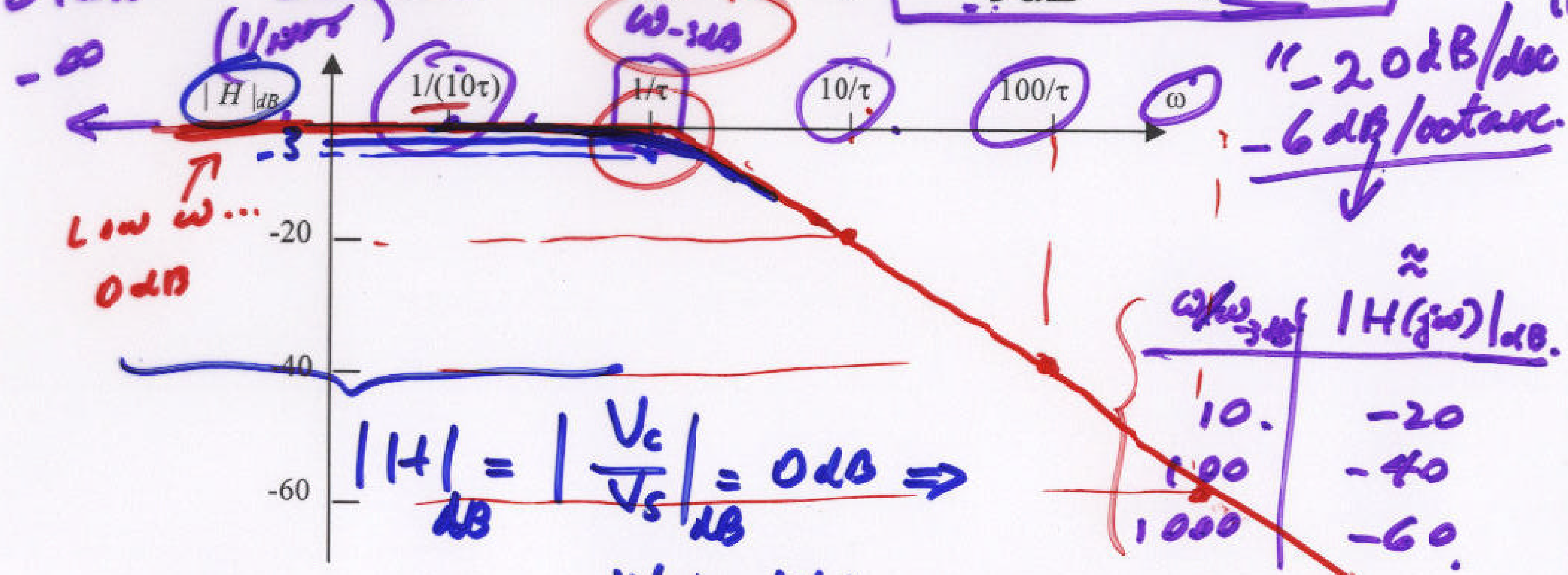
$$|H(j\omega)|_{dB} \equiv 20\log_{10}|H(j\omega)|$$

$$= 20\log_{10}\left\{\left|\frac{1+j\cdot 0}{1+j\omega\tau}\right|\right\}$$

$$\frac{1}{1 + j(\omega/\omega_{-3dB})} = \frac{1}{1+j}$$

The Break Frequency $\boxed{\omega_{-3dB} = (1/\tau)}$

-3dB



ω/ω_{-3dB}	$\approx \lvert H(j\omega)\rvert_{dB}$
10.	-20
100	-40
1000	-60.

$$|H|_{dB} = \left|\frac{V_c}{V_s}\right|_{dB} = 0\,dB \Rightarrow |V_c| = |V_s|.$$

$\omega\tau \gg 1 \Rightarrow$

$$|H(j\omega)| = 20\log_{10}\left[\frac{1}{\sqrt{\cancel{1} + (\omega\tau)^2}}\right]$$

$$\cong -20\log_{10}(\omega\tau) = -20\log_{10}\left(\frac{\omega}{\omega_{-3dB}}\right)$$

$20\log\left(\frac{1}{x}\right) = 20\log_{10}\left((x)^{-1}\right) = -20\log_{10} x.$

Sketching the Magnitude Plot

$$|H|_{dB} = \left[\frac{|1|}{|1+j\omega\tau|}\right]_{dB} \doteq 20\log_{10}\left[\frac{1}{\sqrt{1+(\omega\tau)^2}}\right].$$

$\omega \to 0$

Low-frequency ($\omega\,\tau \ll 1$) asymtote : $\omega\tau = 0.1 \ldots \Rightarrow 0\,dB$

$|H(j0)| = 1;\ |H(j0)|_{dB} = 0\,dB$

High-frequency ($\omega\,\tau \gg 1$) asymtote

$$\frac{1}{\sqrt{1+(0.1)^2}} \approx \frac{1}{1} \qquad (0.1)^2 = 0.01$$

$$(1+0.01)^{1/2} \simeq 1 + 0.005 = 1.005$$

$\omega\tau = 10$

$$\frac{1}{\sqrt{1+\underbrace{(\omega\tau)^2}_{10^2}}} = \frac{1}{\sqrt{1+100}} \simeq \frac{1}{10} \Rightarrow |H(j\cdot(\tfrac{10}{\tau}))| = -20\,dB$$

$\angle z_1/z_2 = \angle z_1 - \angle z_2$.

Finding the Phase Plot

$\angle 1 + j0 = \tan^{-1}[0/1] = 0$

$$\angle(H) = \angle\left[\frac{1}{1+j\omega\tau}\right] = 0 - \arctan(\omega\tau)$$

Why? THEOREM!

$\tau = 1/\omega_{-3dB}$

- Low-frequency asymtote $\omega\tau \ll 1$ $\arctan(\approx 0) = 0$

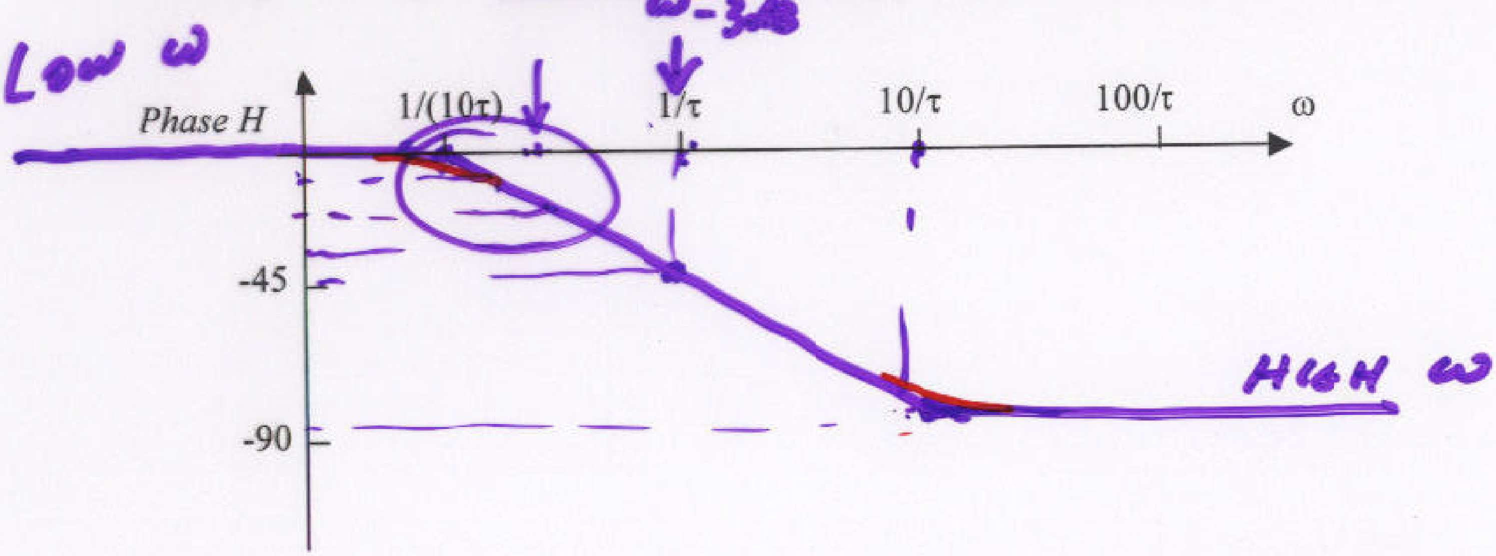
$\angle H \approx 0$.

High-frequency asymtote

$\omega\tau \gg 1$ $\angle H = 0 - \tan^{-1}\{\text{BIG!}\} = 0 - 90° = -90°$.

Approx. linear with ω for $1/(10\tau) < \omega < 10/\tau$

Rapidly Sketching the Phase Plot



$$\left[\begin{array}{l} \tan^{-1}\left(\frac{1}{10}\right) \cong 6^\circ \\ \tan^{-1}(10) \cong 84^\circ \end{array}\right.$$

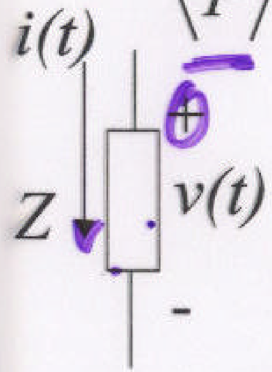
$\omega = \frac{1}{\tau}$

$\tan^{-1}[1] = 45^\circ$

$i \quad R \quad i^2 R = i \cdot v.$

Average Power and Phasors

Integrate $P(t)$ over one period: $P(t)$.



$$\langle P \rangle = \int_0^T \underbrace{i(t)v(t)}dt = \int_0^T |I|\cos(\omega t + \angle I)|V|\cos(\omega t + \angle V)dt$$

$\langle \ \rangle = 0.$

$$= |I||V|\left\langle \sum \{\cos(\omega t), \cos(2\omega t), \sin(\omega t), \sin(2\omega t)\}\right\rangle +$$

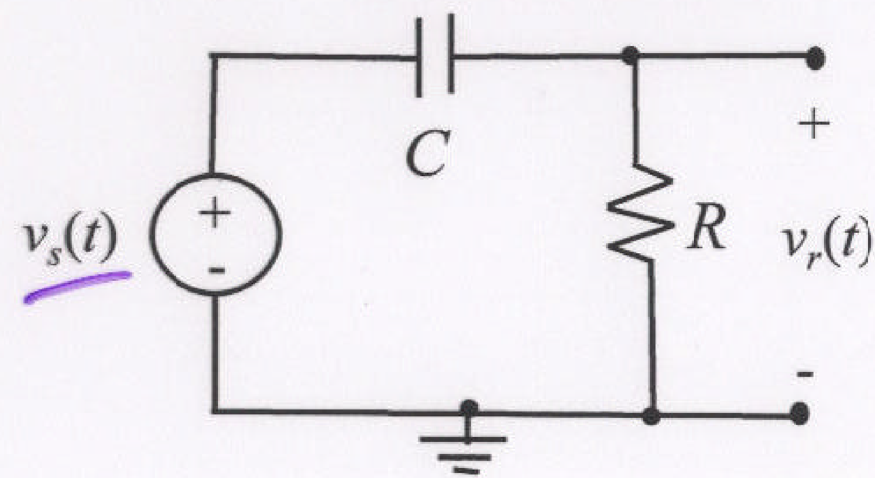
$$\cancel{2}\frac{|I||V|}{\cancel{2}}\underbrace{\left(\cos\angle I \cos\angle V + \sin\angle I \sin\angle V\right)}$$

Result: $\langle P \rangle = \dfrac{|I||V|}{\cancel{2}}\cos(\angle I - \angle V) = \mathrm{Re}\{I \cdot V^*\}$

$V = |V|e^{j\angle V} \qquad V^* = |V|e^{-j\angle V}$

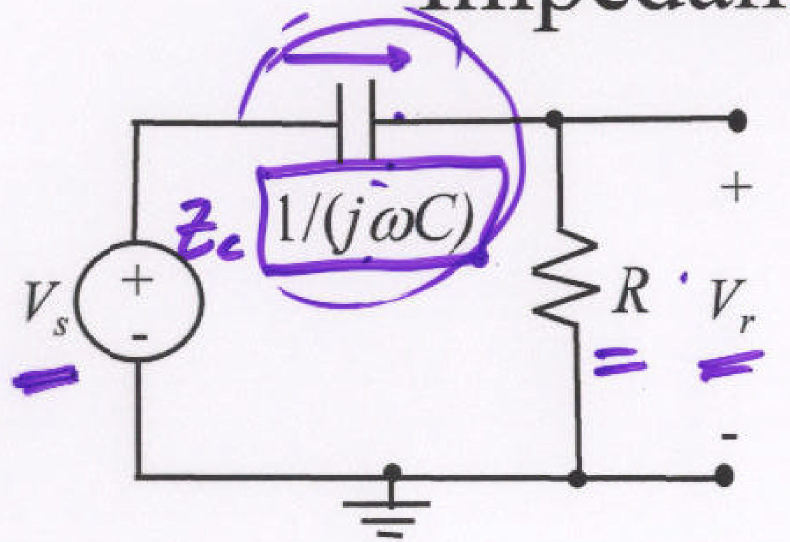
 R. T. Howe

The High-Pass Filter



$$\begin{cases} v_s(t) = v_s \cos \omega t \\ v_r(t) = v_r \cos[\omega t + \phi_r] \end{cases}$$

Impedance Divider



Insight:

$|Z_c|$ vs. ω.

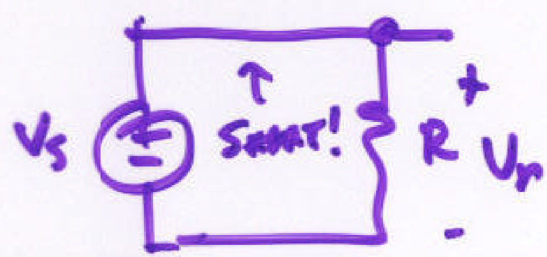
$\frac{1}{j\omega C}$

LOW ω ⇒ BLOWS UP!

$V_r \to 0$.

$$H = \frac{V_r}{V_s} =$$

HIGH ω ✓



LOW ω ✓

$$H = \frac{V_r}{V_s} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}.$$

$$= \frac{j\omega\tau}{1 + j\omega\tau}.$$

$\omega \to \infty$

$$H \to \frac{j\omega\tau}{j\omega\tau} \Rightarrow 1 \checkmark$$

$\omega \to 0$

$$H \to \frac{0}{1+0} \to 0 \checkmark$$

Graphical Addition of Magnitudes