Lecture 5

P.S. 2 ... PSPICE.

- Last time:
  - Bode plots for first-order transfer functions (low-pass and high-pass filters)
- Today:
  - Rapid sketching techniques for more complicated transfer functions.

LECTURE 6: 2ND ORDER

$|Z_1/Z_2| = |Z_1| - |Z_2|$

Magnitude Bode Plot

V_r/V_s

$$|H|_{dB} = \left|\frac{j\omega\tau}{1+j\omega\tau}\right|_{dB} = |j\omega\tau|_{dB} + \underbrace{\left|\frac{1}{1+j\omega\tau}\right|_{dB}}_{L.P.F.}$$

First term (numerator):

$$|j\omega\tau|_{dB} = 20\log_{10}|0 + j\omega\tau|$$

$$= 20\log_{10}(\omega\tau)$$

$\omega\tau$	$\lvert j\omega\tau\rvert_{dB}$
$\frac{1}{10}$	-20
1	$0 \leftarrow dB$
10	20
100	40

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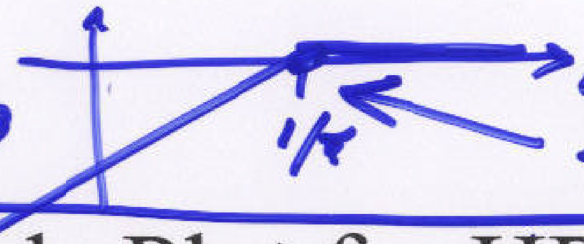
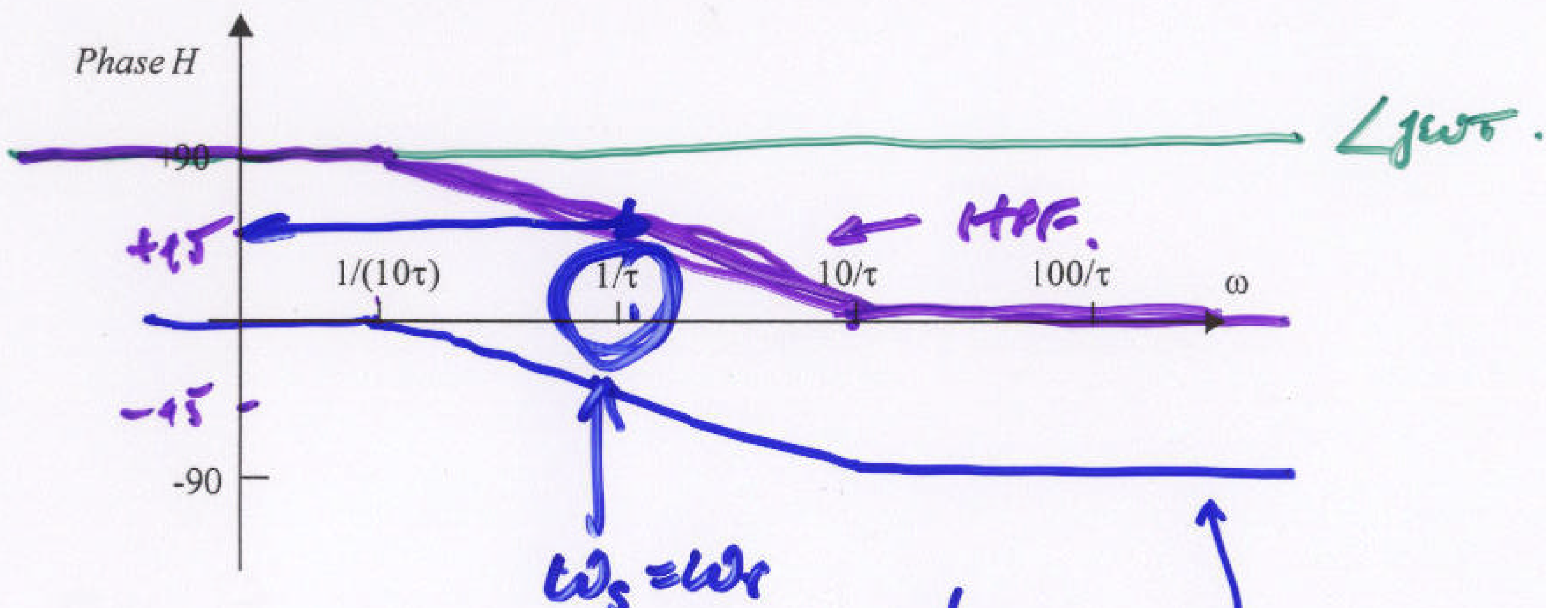


Phase Bode Plot for H.P.F.



$$\angle H = \underbrace{\angle j\omega\tau}_{\tan^{-1}\left(\frac{\omega\tau}{0}\right) = 90° = \pi/2} + \underbrace{\angle \frac{1}{1+j\omega\tau}}_{=}.$$

MAG PLOT FOR HPF.

"PART e)"

$$V_s = 1\,mV\,e^{j0°}$$

GIVEN $v_s(t) = \underbrace{1\,mV}_{\text{amplitude.}} \cos(\underbrace{2\pi f_s} \cdot t)$

$\omega_s = \frac{1}{\tau}$ ← PICK THIS.

- WHAT IS $v_r(t)$?

$$v_r(t) = \hat{v}_r \cos(\omega_r t + \phi_r)$$

$$\omega_r = \omega_s = \frac{1}{\tau}$$

$\hat{v}_r$, ϕ_r ... SEE BODE!

$$H(j\omega) = \frac{V_r}{V_s} \quad \cdots \quad \boxed{V_r = V_s H}$$

$\frac{1}{\sqrt{2}} = 0.707$ BIG DEAL!

$$V_r = H(1\,mV) = \{|H(j\omega_s)| \cdot 1\,mV\} e^{j\underbrace{\angle H(j\frac{1}{\tau})}_{\text{PHASE } +45°}}$$

MAG.

$$V_r = 0.7\,mV\,e^{j45°}$$

BIG POINT!

$$v_r(t) = \mathrm{Re}\left\{ V_r e^{j\underset{\uparrow\ 1/\tau}{\omega_r} t} \right\}$$

$$= (707\,\mu V) \cos\left[\left(\tfrac{1}{\tau}\right) t - 45^\circ\right]$$

Bode Plots

BREAK THEM UP!

Technique for plotting complicated transfer functions (e.g., several $(1+j\omega\tau_i)$ factors in numerator and denominator)

$$H(j\omega) = \frac{Aj\omega(1 + j\omega\tau_2)(1 + j\omega\tau_4)...(1 + j\omega\tau_n)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)...(1 + j\omega\tau_{n-1})}$$

"POLE"

$\omega_i = (1/\tau_i)$ are the break frequencies

$(\sigma + j\omega)$

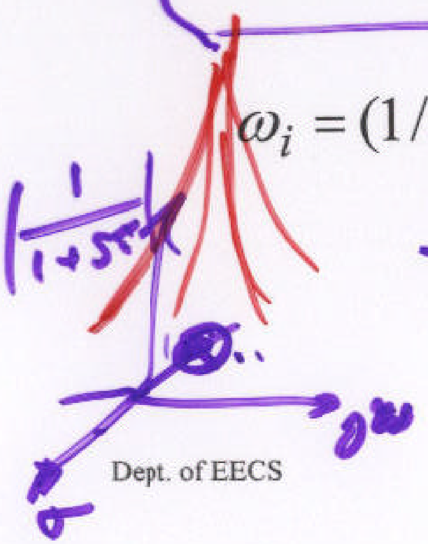
$(1 + s\tau)$

→ Denominator factors: *poles*

$s = -\frac{1}{\tau} \ldots$

Numerator factors: *zeroes*

$(1 + s\tau) \to 0$



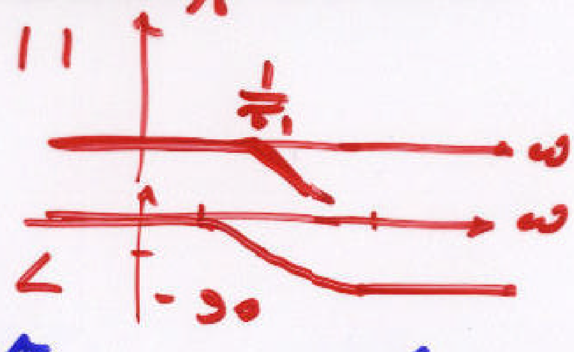
$\angle A = 0°$

$|A| = 60\text{ dB}$ STANDARD

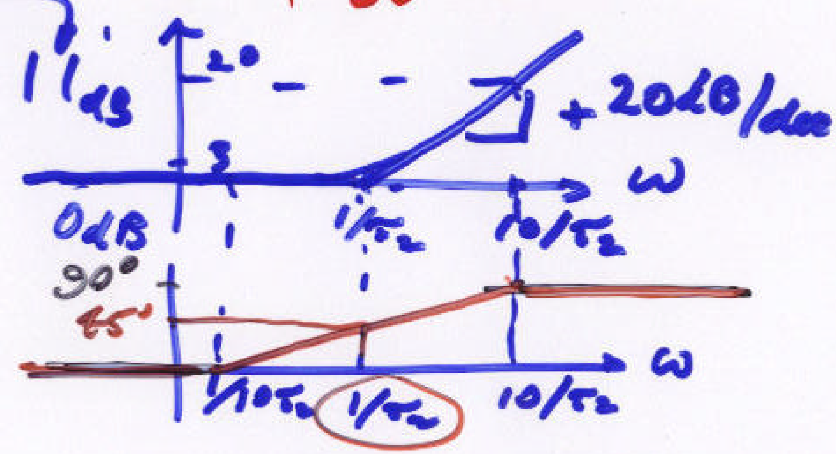
Summary of Individual Factors

CONST. $A = 1000.$

Poles: $\dfrac{1}{1 + j\omega\tau_1}$ ← LPF.



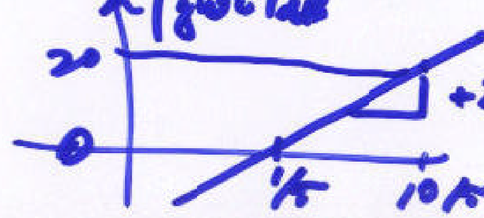
Zeroes: $1 + j\omega\tau_2$.



jωτ factors: $j\omega\tau$

$s\tau$

PICK $\omega = \frac{1}{\tau} \Rightarrow j\left(\frac{1}{\tau}\right)(\tau) = j \cdot 1$ 0dB.



$\angle j\omega\tau = 90°.$

?

$Z_c = \frac{1}{j\omega C} = \frac{1}{j(2\pi f)C}$.

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Example

"STANDARD FORM"

$$\boxed{H(j\omega) = \frac{10^{-5}\, j\omega (1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}}$$

$\times 10^{-9}$ s.

$\tau_1 = 200$ ns

$\tau_2 = 20$ ns

$\tau_3 = 200$ ps

10^{-12} s

Break frequencies: invert time constants

$\omega_1 = 1/\tau_1 = 5$ Mrad/s

$\omega_2 = 1/\tau_2 = 50$ Mrad/s.

$\omega_3 = 1/\tau_3 = 5$ Grad/s

< 900 MHz.

$$H(j\omega) = \frac{\boxed{(j\omega/10^5)}(1 + j\omega/\omega_2)}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_3)}$$

$f_1 = \frac{\omega_1}{2\pi} \approx \frac{5}{6}$ MHz.

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Breaking Down the Magnitude

$(j\omega/10^5)$ #1

$$|H(j\omega)|_{dB} = 20\log\left|\frac{\cancel{10^{-5}\, j\omega}(1+j\omega\tau_2)}{(1+j\omega\tau_1)(1+j\omega\tau_3)}\right|$$

#1 #2

$$= 20\log\left|j\omega/10^5\right| + 20\log\left|1+j\omega\tau_2\right|$$

#3 #4

$$\boxed{\underbrace{-20\log\left|1+j\omega\tau_1\right|}_{} \underbrace{-20\log\left|1+j\omega\tau_3\right|}_{}}$$

LPF OR PLOT THEM WITH MINUS SIGN...

$+20\log_{10}\left|\frac{1}{1+j\omega\tau_1}\right|$

$+20\log_{10}\left|\frac{1}{1+j\omega\tau_3}\right|$

→ Plot the terms separately and add them graphically!

$\log(xy) = \log x + \log y$

$\log(x/y) = \log x - \log y.$

Breaking Down the Phase

$$\angle H(j\omega) = \angle \frac{10^{-5}\, j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

But we know $\angle z_1 z_2 = \angle z_1 + \angle z_2$

POLAR FORM!

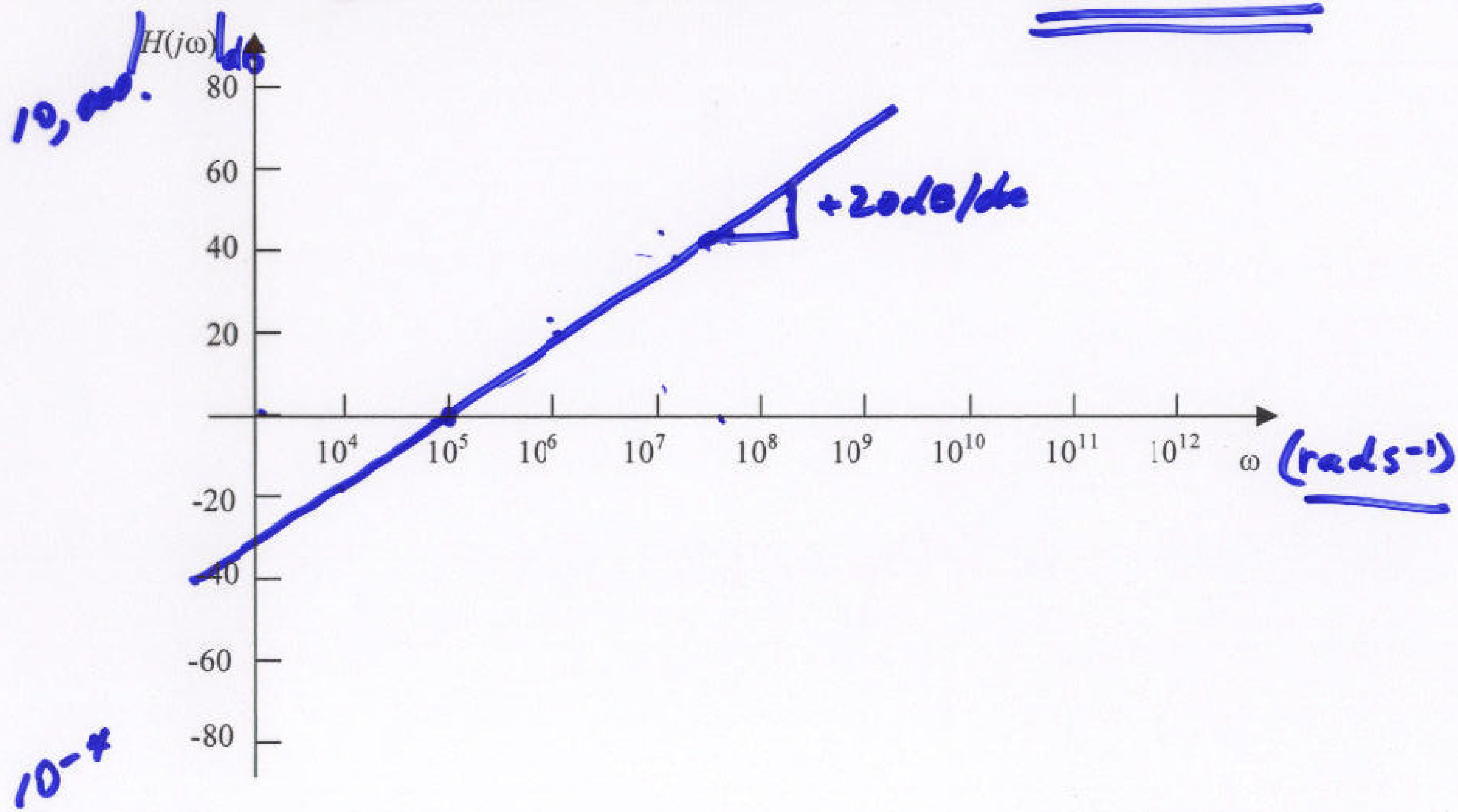
$\angle z_1/z_2 = \angle z_1 - \angle z_2.$

$$\angle H(j\omega) = \angle j\omega/10^5 + \angle(1 + j\omega\tau_2)$$
$$- \angle(1 + j\omega\tau_1) - \angle(1 + j\omega\tau_3)$$

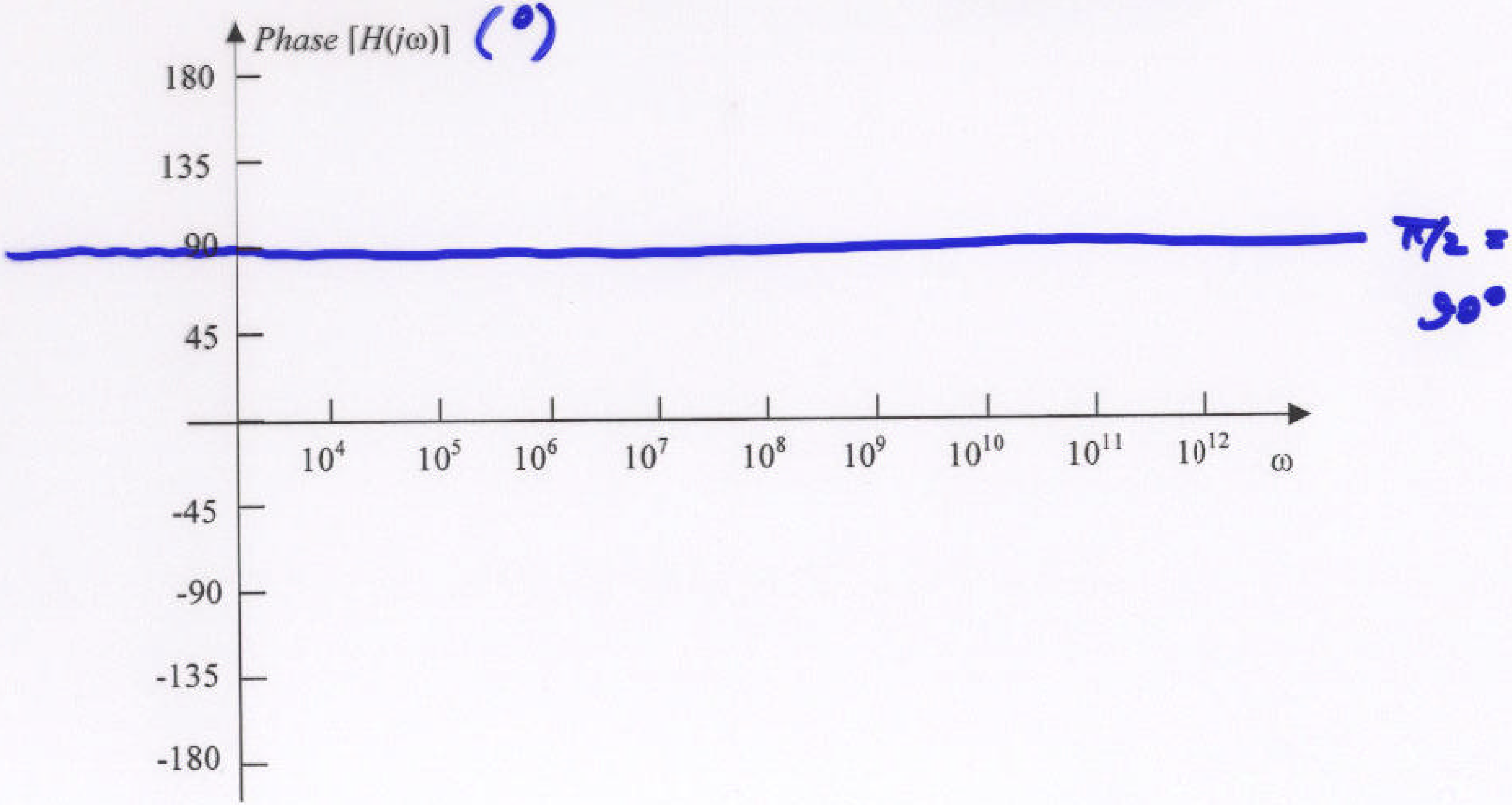
OR $+ \angle \frac{1}{1+j\omega\tau_1} + \angle \frac{1}{1+j\omega\tau_3}$

→ Plot each term separately and add them graphically!

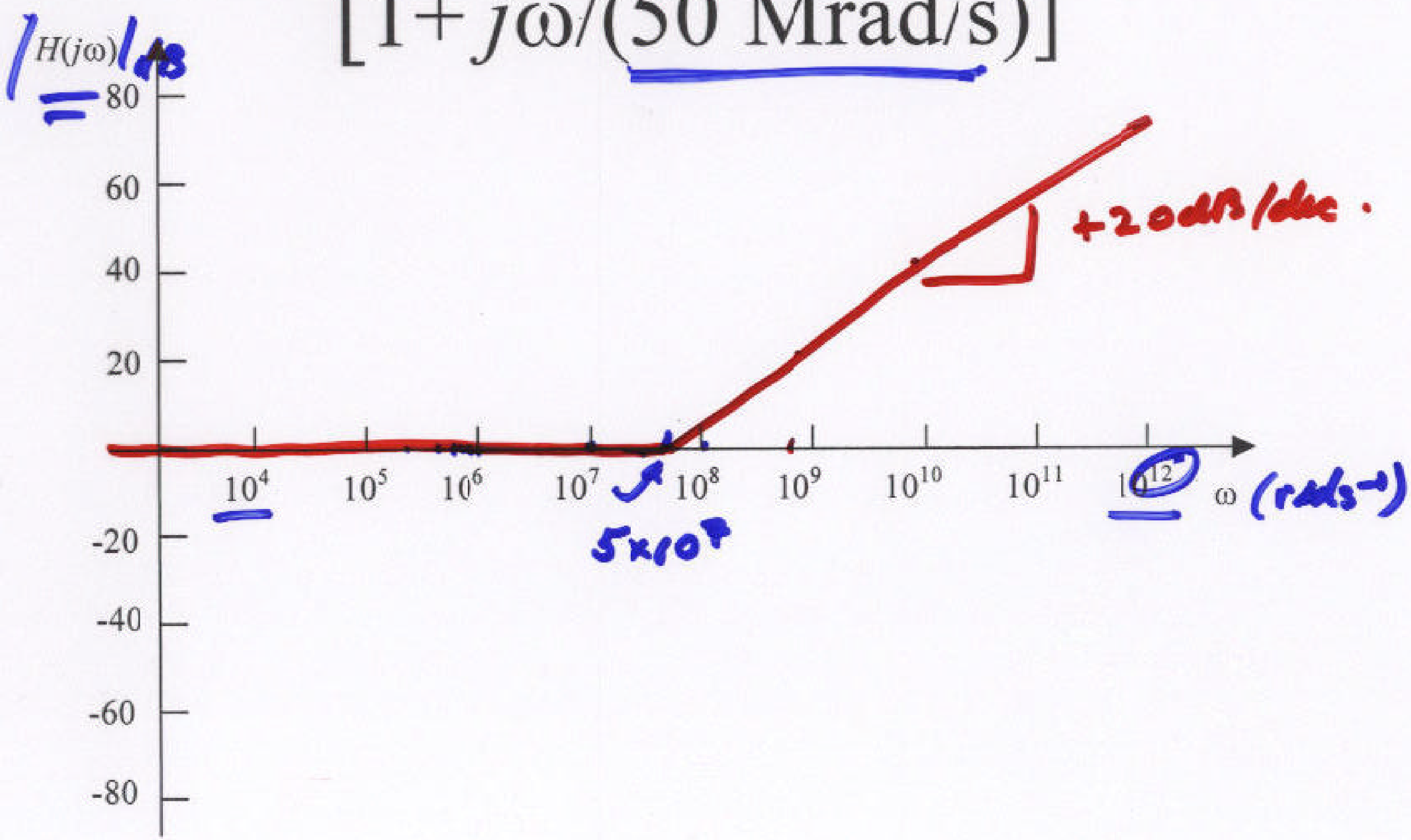
Magnitude Bode Plot: $j\omega/10^5$



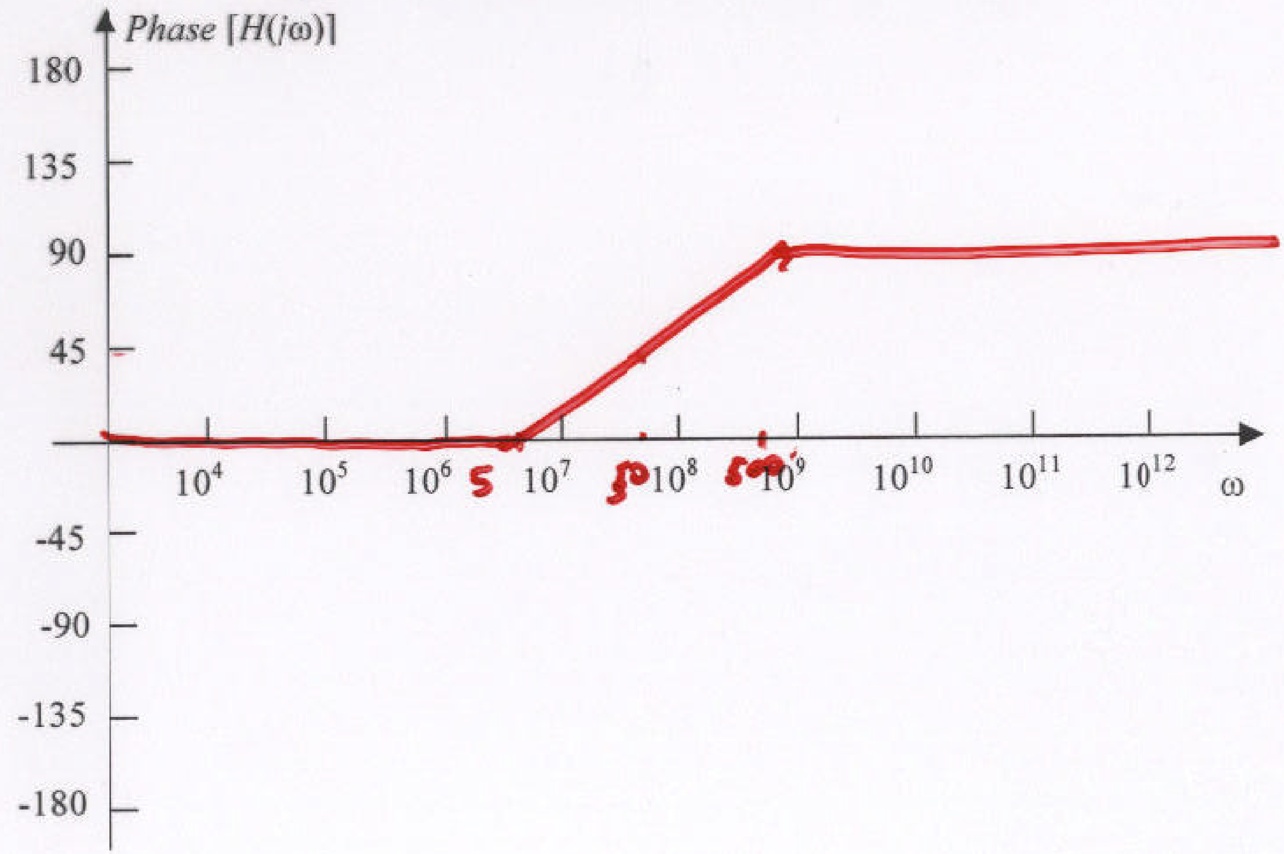
Phase Bode Plot: $j\omega/10^5$



Magnitude Bode Plot: $[1 + j\omega/(50\ \mathrm{Mrad/s})]$



Phase Bode Plot: $[1 + j\omega/(50 \text{ Mrad/s})]$



Magnitude Bode Plot: $|H(j\omega)|_{dB}$