a) Find $I_D$ given $V_{GS} = 0.9\,\text{V}$, $V_{DS} = 4\,\text{V}$

$$I_D = \frac{W}{L_0(1-K_{VDS})} \left(4.95 \times 10^{-9} \text{ cm}^2\right) e^{\frac{V_{GS}-V_{TH}}{V_{TH}}}$$

$$= (500 \text{ cm}^2/\text{Vs})(0.026\,\text{V}) \left(\frac{4.5\,\text{mm}}{1.5\,\text{mm}(1.01))}\right) \left(4.95 \times 10^{-9} \text{ cm}^2\right) e^{(0.9-1)/0.026}$$

$$I_D = 41.7\,\text{nA}$$

b) Transconductance

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{W}{L_0(1-K_{VDS})} \left(4.95 \times 10^{-9} \text{ cm}^2\right)$$

$$= \left(4.17 \times 10^{-9} \text{A}\right) \frac{0.026\,\text{V}}{0.026\,\text{V}} = 1.6\,\mu\text{S}$$

c) Output Resistance

$$r_o = \left(\frac{\Delta I_D}{\Delta V_{DS}}\right)^{-1} = r_o^{-1} = \frac{2}{V_{TH}} \left[ (1-K_{VDS})^{-1} \left(\frac{W}{L_0} \left(4.95 \times 10^{-9} \text{ cm}^2\right) e^{V_{GS}-V_{TH}}\right) \right]$$

$$= - (1-K_{VDS})^{-2}(1-K_{VDS})\left(\frac{W}{L_0(1-K_{VDS})} \left(4.95 \times 10^{-9} \text{ cm}^2\right) e^{V_{GS}-V_{TH}}\right)$$

$$= \frac{K_{VDS}}{(1-K_{VDS})} \left(\frac{W}{L_0(1-K_{VDS})} \left(4.95 \times 10^{-9} \text{ cm}^2\right) e^{V_{GS}-V_{TH}}\right)$$

$$= \frac{K_{VDS}}{1-K_{VDS}}$$

$$r_o = \frac{1-K_{VDS}}{K_{VDS}} = \frac{1-0.01}{(0.01)(41.7\,\text{nA})} = 2.37 \,\text{G}\Omega$$

d) $g_m r_o$ product:

$$g_m = \frac{I_D}{V_{TH}} \quad \text{(from part b)}$$

$$r_o = \frac{1-K_{VDS}}{K_{VDS}} \quad \text{(from part c)}$$

$$\Rightarrow g_m r_o = \left(\frac{W}{L_0} \frac{1-K_{VDS}}{K_{VDS}} \right)$$

*NOTE THAT THIS PRODUCT IS INDEPENDENT OF $I_D$.

(stays constant so long as $V_{GS}$ stays)

\[\frac{1}{4}\]
A few things to note about this minority-carrier concentration plot:

1. At $x=0.5$ micron, the concentration appears to go to zero. The actual value of at the depletion-region edge is equal to $n_{pB0}$, or $10^4$ cm$^{-3}$.

2. **But this doesn't matter**: by the law of the junction, the concentration at $x=0$ is more than *ten orders of magnitude* greater. Keep in mind that what we really care about is the slope of this concentration plot; whether the value at $x=0.5$ is 0 or $10^4$ doesn’t make a difference, since it is much less than the value at the other junction edge. So in the end, you can assume it is zero and still be ok.
3. a) To get $V_{out} = 0V$ (large signal)

$I_{bias}$ must be $-I_{RD}$ (the current through the resistor)

$$I_{RD} = \frac{S1 - V_{out}}{10k\Omega} = \frac{S1}{10k\Omega} = 500\mu A; \quad I_{Bias} = -500\mu A$$

b) To remain in saturation:

$$V_{DS} \geq V_{GS} - V_{th}; \quad V_{DS} = V_{out} - V_{S}, \text{ but we also know } V_{S} = V_{g} - V_{th}$$

so $V_{DS} = V_{out} - (V_{g} - V_{th}) = V_{out} + V_{th}$ (since $V_{g}$ is 0)

$$\Rightarrow \frac{(V_{out} + V_{th})}{V_{DS} - V_{th}} \geq \frac{V_{GS} - V_{th}}{V_{th}} \Rightarrow V_{out} \geq -V_{th}$$

or: $V_{out, min} = -1V$

c) To find $R_{in}$:

first, we recognize that this is just a common-gate amplifier;

$$R_{in} = \frac{1}{g_{m}}$$

$$g_{m} = \frac{2\mu_{n}C_{ox}}{W} \frac{I_D}{D} = \sqrt{2(50\mu A/\mu^{2})\left(\frac{50\mu A}{500\mu A}\right)} \approx 1.12 \text{ mS}$$

so $R_{in} = \frac{1}{1.12 \text{ mS}} = 894 \Omega$

d) To find transresistance: start with the CG small-signal model:

By Thevenin / Norton Equivalence: $-A_{i} \frac{R_{out}}{R_{in}} = R_{min}$

$$\Rightarrow R_{m} = -A_{i} R_{out}$$

But since this is a CG amp., we know $A_{i} = -1$

$$\Rightarrow R_{m} = R_{out}$$

So for a CG amp:

$$R_{out} = R_{up} || R_{down} = R_{D} || R_{o}(1 + g_{m}R_{S})$$

$$R_{o} = \frac{1}{\frac{I_{D}}{D}} \frac{1}{(0.5\mu A)(500\mu A)} = 40k\Omega; \quad R_{out} = 10k\Omega // 48k\Omega \approx 10k\Omega = R_{m}$$
3. e) Find $V_{out}$.

Plug in this "new" transresistance model, and hook up the sum of the input $i_s/R_s$.

\[
\begin{align*}
\text{By the current-divider equation, } i_m &= i_s \left(\frac{R_s}{R_{in}+R_s}\right) \\
V_{out} &= R_{min} \cdot R_m \frac{i_s}{R_{in}+R_s} \\
&= \frac{V_{out}}{i_s} = R_m \left(\frac{R_s}{R_{in}+R_s}\right) = (10 \text{ k}\Omega) \left(\frac{10 \text{ k}\Omega}{8948\Omega+10 \text{ k}\Omega}\right) = 9.18 \text{ k}\Omega
\end{align*}
\]