EE105 Practice Midterm 2 Solutions

SPRING 2002

1a) Find I_D given $V_{GS}=0.9V$, $V_{DS}=1V$

$$I_D = \mu_n V_{th}\left(\frac{W}{L_0(1-\kappa V_{DS})}\right)(4.95\times10^{-9}\,C/cm^2)\,e^{V_{GS}-V_t/V_{th}}$$

$$=(500\,cm^2/V\cdot s)(0.026V)\left(\frac{45\mu m}{1.5\mu m(1-.01(1))}\right)(4.95\times10^{-9}\,C/cm^2)\,e^{(0.9-1)/0.026}$$

$$\boxed{I_D = 41.7\,nA}$$

b) Transconductance

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{V_{th}}\underbrace{\left(e^{(V_{GS}-V_t)/V_{th}}\right)\mu_n V_{th}\left(\frac{W}{L_0(1-\kappa V_{DS})}\right)(4.95\times10^{-9}\,C/cm^2)}_{\text{this is just } I_D}$$

$$\Rightarrow g_m = \frac{I_D}{V_{th}} = \frac{41.7\times10^{-9}A}{0.026V} = \boxed{1.60\mu S}$$

c) Output Resistance

$$r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} \Rightarrow r_o^{-1} = \frac{\partial}{\partial V_{DS}}\left[(1-\kappa V_{DS})^{-1}\left(\mu_n V_{th}\left(\frac{W}{L_0}\right)(4.95\times10^{-9}\,C/cm^2)\,e^{V_{GS}-V_t/V_{th}}\right)\right]$$

$$= -(1-\kappa V_{DS})^{-2}(-\kappa)\left(\mu_n V_{th}\left(\frac{W}{L_0}\right)(4.95\times10^{-9}\,C/cm^2)\,e^{V_{GS}-V_t/V_{th}}\right)$$

$$= \frac{\kappa}{(1-\kappa V_{DS})}\underbrace{\left(\mu_n V_{th}\left(\frac{W}{L_0(1-\kappa V_{DS})}\right)(4.95\times10^{-9}\,C/cm^2)\,e^{V_{GS}-V_t/V_{th}}\right)}_{\text{this is just } I_D}$$

$$\Rightarrow r_o^{-1} = \frac{\kappa I_D}{1-\kappa V_{DS}};$$

$$r_o = \frac{1-\kappa V_{DS}}{\kappa I_D} = \frac{1-0.01}{(0.01)(41.7nA)} = \boxed{2.37\,G\Omega}$$

d) $g_m r_o$ product:

$g_m = \frac{I_D}{V_{th}}$ (from part b); $r_o = \frac{1-\kappa V_{DS}}{\kappa I_D}$ (from part c)

$$\Rightarrow g_m r_o = \left(\frac{\cancel{I_D}}{V_{th}}\right)\left(\frac{1-\kappa V_{DS}}{\kappa \cancel{I_D}}\right) = \boxed{\frac{1-\kappa V_{DS}}{V_{th}}}$$

* NOTE that this product is independent of I_D.
(stays constant so long as V_{DS} does)

2a) C_μ: the depletion capacitance of the base-collector junction
- It's just a reverse-biased pn junction;

$$C_\mu = \frac{\varepsilon_s A_c}{X_{bc}}$$
A_c = area of base-collector junction interface
X_{bc} = depletion width of base-collector junction

From the cross-section: $X_{bc} = 0.6\,\mu m$
From the layout: $A_c = (1.5\,\mu m + 0.25\,\mu m)(2\,\mu m) = 3.5\times10^{-8}\,cm^2$

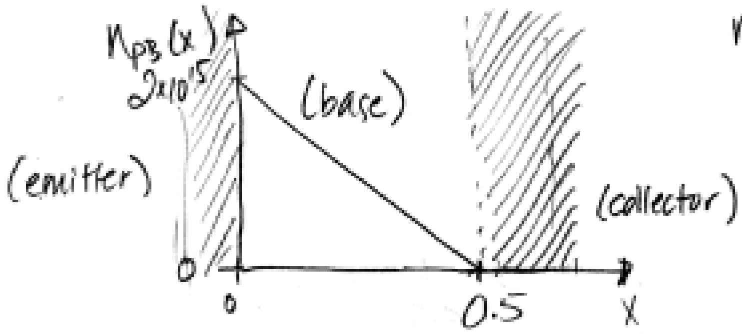
$$C_\mu = \frac{(11.7 \times 8.85\times10^{-14}\,F/cm)(3.5\times10^{-8}\,cm^2)}{0.6\times10^{-4}\,cm} = \boxed{0.604\,fF}$$

b) By the law of the junction:

$n_{pB}(x=0) = n_{pB0}\,e^{V_{BE}/V_{th}}$; n_{pB} = concentration of electrons in p-type base
$n_{pB0} = \frac{n_i^2}{N_{aB}}$ (equilibrium concentration of electrons in p-type base)

$n_{pB}(x=0.5) = 0$
(since this is at the edge of the reverse-biased base-collector junction)

And in between, the minority electron concentration falls linearly:



$n_{pB0} = \frac{10^{20}}{10^{16}} = 10^4\,cm^{-3}$

$n_{pB}(x=0) = 10^4 e^{650/26} = 2\times10^{15}\,cm^{-3}$

A few things to note about this minority-carrier concentration plot:

1. At x=0.5 micron, the concentration appears to go to zero. The actual value of at the depletion-region edge is equal to n_{pB0}, or 10^4 cm^{-3}.

2. **But this doesn't matter**: by the law of the junction, the concentration at x=0 is more than *ten orders of magnitude* greater. Keep in mind that what we really care about is the <u>slope</u> of this concentration plot; whether the value at x=0.5 is 0 or 10^4 doesn't make a difference, since it is much less than the value at the other junction edge. So in the end, you can assume it is zero and still be ok.

3. a) To get $V_{out} = 0V$ (large signal)

I_{BIAS} must be $-I_{R_D}$ (the current through the resistor)

$I_{R_D} = \frac{5V - V_{out}}{R_D} = \frac{5V}{10k\Omega} = 500\mu A$; $\boxed{I_{BIAS} = -500\mu A}$

b) To remain in saturation:

$V_{DS} \geq V_{GS} - V_{TN}$; $V_{DS} = V_{OUT} - V_S$, but we also know $V_S = V_G - V_{GS}$

so $V_{DS} = V_{OUT} - (V_G - V_{GS}) = V_{OUT} + V_{GS}$ (since V_G is 0)

$\Rightarrow (V_{OUT} + \cancel{V_{GS}}) \geq \cancel{V_{GS}} - V_{TN} \Rightarrow V_{OUT} \geq -V_{TN}$

or: $\boxed{V_{OUT,\,MIN} = -1V}$

c) To find R_{in}:

first, we recognise that this is just a common-gate amplifier;

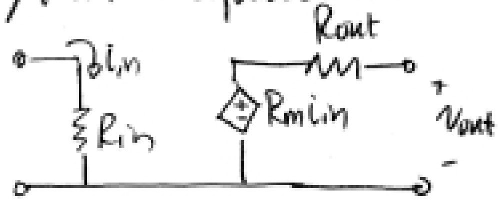
$R_{in} = \frac{1}{g_m}$

$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} = \sqrt{2(50\mu A/V^2)\left(\frac{50}{2}\right)(500\mu A)} = 1.12\,mS$

so $R_{in} = \frac{1}{1.12\,mS} = \boxed{894\Omega}$

d) To find transresistance: start w/ the CG small-signal model;

By Thevenin / Norton Equivalence: $-A_i i_{in}(R_{out}) = R_m i_{in}$



$\Rightarrow R_m = -A_i R_{out}$

But since this is a CG amp., we know $A_i = -1$

$\Rightarrow R_m = R_{out}$

So for a CG amp:

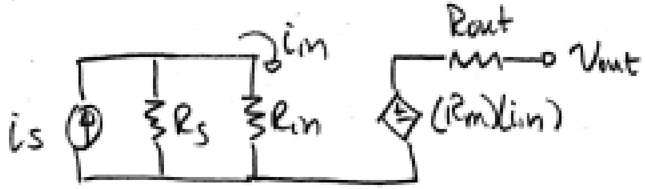
$R_{out} = R_{up} // R_{down} = R_D // r_o(1 + g_m R_S)$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.05)(500\times10^{-6})} = 40k\Omega$; $R_{out} = 10k\Omega // 488k\Omega \approx \boxed{10k\Omega = R_m}$

$\boxed{3/4}$

3 e) Find $\frac{v_{out}}{i_s}$:

Plug in this "new" transresistance model, and hook up the sm-sig input i_s / R_s



By the current-divider equation, $i_{in} = i_s \left(\frac{R_s}{R_{in}+R_s} \right)$

$$v_{out} = R_m i_{in} = R_m i_s \left(\frac{R_s}{R_{in}+R_s} \right)$$

$$\Rightarrow \frac{v_{out}}{i_s} = R_m \left(\frac{R_s}{R_{in}+R_s} \right) = (10\,k\Omega)\left(\frac{10k\Omega}{894\Omega + 10k\Omega} \right) = \boxed{9.18\,k\Omega}$$