Announcements
- Midterm on 3/10! Next Wednesday! Get prepared!
- Today - Transistor Equations
  - DC applications
  - AC applications - What is Small Signal?
- Midterm Review Session, Monday OK?

Transistor Equations

**NMOS**
- Cutoff: \( I_D = 0, \ V_{DS} < V_T \)
- Linear: \( I_D = M_n \cdot \text{cox} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_T) \ V_{DS} - \frac{V_{DS}^2}{2} \right], \ V_{DS} > V_{DS} - V_T \)
  - If \( V_{DS} \) is small, \( I_D = M_n \cdot \text{cox} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_T) \ V_{DS} \right] \)
  - Resistance = \( \frac{V_{DS}}{I_D} = \frac{M_n \cdot \text{cox} \left( \frac{W}{L} \right)}{V_{DS}} \left( V_{GS} - V_T \right) \)
- Saturation: \( I_D = \frac{W}{2L} M_n \cdot \text{cox} (V_{GS} - V_T)^2 \left( 1 + R_D V_{DS} \right), \ V_{DS} > V_{DS} - V_T \ V_{GSS} > V_T \)
  - \( \Rightarrow 2\text{nd order effect}. \)

**PMOS Transistors**
- Cutoff: \( I_D = 0, \ V_{SG} > V_T \)
  - \( V_{SD} \leq V_{SG} + V_{TP} \)
- Linear: \( I_D = M_p \cdot \text{cox} \left( \frac{W}{L} \right) \left[ (V_{SG} + V_{TP}) \ V_{SD} - \frac{V_{SD}^2}{2} \right] \approx M_p \cdot \text{cox} \left( \frac{W}{L} \right) \left[ V_{SG} + V_{TP} \right] V_{SD} \)
- Saturation: \( I_D = M_p \cdot \text{cox} \left( \frac{W}{2L} \right) (V_{SG} + V_{TP})^2 \left( 1 + R_D V_{SD} \right), \ V_{SG} < V_T \)
  - \( V_{SD} > V_{SG} + V_{TP} \)

Some Properties: How do we know which terminal corresponds to source?

1. For NMOS, the lower potential is always the source.
2. For PMOS, the higher potential is always the source.
DC Application

\[ I = 5 \text{Volts} \]

\[ \frac{V_o}{R} = V_{in} \]

\[ V_T = 1 \text{V} \]

\[ M_n \alpha_n = 5 \times 10^{-6} \frac{A}{V^2} \]

\[ \frac{V_o}{V_T} = 1 \]

\[ R = 10 \text{M} \Omega = 10^7 \text{ohms} \]

Question: What's \( V_{out} \) when \( V_B = 0.5 \text{V}, 1.2 \text{V} \) and \( 3 \text{V} \)?

\[ V_R = 0.5 \text{V} \]

(1) Transistor is off because \( V_B < V_T = 1 \text{V} \)

\[ V_B = 1.2 \text{V} \]

\( V_B > V_T \), so transistor is on!

Let's guess that the transistor is in linear region. We write KCL at \( V_o \).

\[ I_D = \frac{M_n \alpha_n (\frac{V_o}{2})}{1} \left[ (1.2 - 1) V_{out} - \frac{V_{out}^2}{2} \right] = \frac{5 - V_{out}}{10^7} \]

\[ 5 \times 10^{-6} \left[ 0.2 V_{out} - \frac{V_{out}^2}{2} \right] = \frac{5 - V_{out}}{10^7} \]

\[ \rightarrow V_{out} \approx \begin{cases} 0.057764 \text{V} \\ 0.346236 \text{V} \end{cases} \]

Which is correct? We have 2 solutions! Check the boundary condition!

\[ V_{gs} - V_T = 0.2 \text{V} \]

In order for transistor to be in linear region, \( V_{out} = 0.05776 \text{V} \).

This is because in linear region, \( V_{ds} = V_{out} \leq V_{gs} - V_T = V_B - V_T \)

\[ = 0.057764 \text{V} = 0.2 \text{V} \]

The trick here is that: Guess First and then make sure that your guess is self consistent.
\[ V_R = 5 \text{V} \]

Write KCL at output node again! Assume transistor is in saturation!

\[ \frac{W}{2L} V_{\text{in}} (V_{gss} - V_{T})^2 = \frac{5 - V_{\text{out}}}{10^7} \]

\[ \frac{1}{2} (5 \times 10^{-6}) (3 - 1)^2 = \frac{5 - V_{\text{out}}}{10^7} \]

\[ 10^{-4} = \frac{5 - V_{\text{out}}}{10^7} \rightarrow V_{\text{out}} = -4.95 \text{V} ! \text{ Doesn't make sense.} \]

Try again! Assume transistor is in linear region.

\[ \frac{W}{2L} V_{\text{in}} (V_{gss} - V_{T}) V_{\text{out}} - \frac{V_{\text{out}}}{2} = \frac{5 - V_{\text{out}}}{10^7} \rightarrow V_{\text{out}} = \frac{5}{3} = 1.667 \text{V} \]

\[ V_{\text{out}} = 0.005 \text{V} \text{ is correct! Because } V_{os} = V_{\text{out}} < V_{gss} - V_{T} = 2 \text{Vols} \]

If \[ V_{\text{out}} = 3.999 \text{V} \text{ were correct, then } V_{os} \geq V_{gss} - V_{T}, \text{ we would have used the WRONG equation!} \]

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**Flowchart:**

1. **Guess the Transistor Operation Region**
2. **Solve KCL, KVL Network**
3. **Check if consistent**
   - **Yeah?**
   - **No**
5. **How SPICE solves your circuit problems?**
AC Analysis — Intro to Small Signal Concept

\[ \frac{W}{L} = 1 \]
\[ M_{\text{lox}} = 5 \times 10^{-6} \frac{A}{V^2} \]
\[ R = 10^5 \Omega \]
\[ V_T = 0.7V_{\text{th}} \]
\[ V_B = V_{\text{gs}} = 1.5V_{\text{th}} \]

Start by Assume Saturation:

\[ \left( \frac{W}{L} \right) M_{\text{lox}} (V_{\text{gs}} - V_T)^2 = \frac{2.5 - V_{\text{vt}}}{R} \rightarrow V_{\text{vt}} = 0.9V \geq V_{\text{gs}} - V_T \]
\[ = 1.5V_{\text{th}} - 0.7V_{\text{th}} \]

So, yes! We guessed it right! NMOS is in saturation region.

What if we perturb the gate voltage a little? But not too much!

1. Since \( I_D = \left( \frac{W}{L} \right) M_{\text{lox}} (V_{\text{gs}} - V_T)^2 \)
   \[ \Delta I_D = \left( \frac{\partial I_D}{\partial V_{\text{gs}}} \right) \Delta V_{\text{gs}} \]
   \[ \frac{\partial I_D}{\partial V_{\text{gs}}} = J_m = \text{transconductance} = \sqrt{2 \left( \frac{W}{L} \right) M_{\text{lox}} I_D} \]

2. Current through NMOS would be \( I_D + \Delta I_D \)

3. Voltage at output = \( V_{\text{dd}} - R(I_D + \Delta I_D) \)
   \[ = (V_{\text{dd}} - RI_D) - R \Delta I_D \]

But \( (V_{\text{dd}} - RI_D) \) is the voltage at output when NO perturbation is present.

4. So the change at the output \( \Delta V \) caused by the perturbation is actually just \( -R \Delta I_D \)
(5) So \[ \Delta V_{out} = -R \Delta I_p = -R \left( g_m \Delta V_{gs} \right) \text{ where } g_m \equiv \frac{\Delta I_p}{\Delta V_{gs}} \]

\[ \frac{\Delta V_{out}}{\Delta V_{gs}} = -g_m R \]

Conclusion: 

\(-g_m R\) is known as the small-signal transfer function!

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Why care about AC analysis and DC analysis?

1. Analog Circuit design is 50% AC analysis, the other 50% is DC biasing.

2. When we introduce capacitors or inductors to the circuit, we would be able to analyze circuit's frequency response and build interesting applications — filters for instance.

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Why study analog if digital circuit is so powerful today?

1. We live in an analog world! \( \Leftarrow \) Most important reason!

2. RF Circuitry

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Voice signal

\[ \text{voice signal} \rightarrow \text{DSP} \rightarrow \text{filtered signal} \]

\[ \text{ADC} \quad \text{EE247} \quad \text{DAC} \quad \text{EE247} \]

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RF Circuit

\[ \text{Antenna} \quad \text{Oscillator} \quad \text{Mixer} \quad \text{ADC, DAC, etc.} \quad \text{DSP chip} \]