Lecture 11: P-N Diode capacitors, intro to small signal models

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Context

In the last lecture, we looked at the PN diode at equilibrium, and under bias, and several applications for PN diodes.

We discussed a simple model for the PN diode, for an abrupt junction, and for a sharp edge depletion.

In this lecture, we will look at the reverse biased PN diode as a variable capacitor, and solve for the fields, voltages, and currents.

We will also use this device to introduce the concept of a small signal model.

Announcements

- Reading: Finish chapter 3 in the text
- Next week we will be starting on MOS transistors, chapter 4
- Don’t come to lecture on Monday, it’s a holiday!

MOS capacitor

- The PN diode as a variable capacitor (varactor) is useful in its own right:

- And we will also draw on this analysis to model field effect transistors, where the action of the gate on the channel is similar to this analysis.
**PN diode Under Reverse Bias**

- Under thermal equilibrium current is zero
- If we apply a reverse bias, we are increasing the barrier against diffusion current
- Drift current is low since the field only moves minority carriers across junction
- The small current under a reverse bias is due to minority carriers which diffuse into the depletion region from either side, and from generation (thermal generation, or light)

**Plot of Fields in Depletion Region**

- E-Field zero outside of depletion region
- Note the asymmetrical depletion widths
- Which region has higher doping?
- Slope of E-Field larger in n-region. Why?
- Peak E-Field at junction. Why continuous?

**PN Junction populations**

- Hole population
  - \( n_h = N_A \)
  - \( N_A \) = fixed negative charges

- Electron population
  - \( n_e = N_D \)
  - \( N_D \) = fixed positive charges

**Reverse bias**

- Under a reverse bias, a voltage is applied which increases the built in field, pulling the mobile carriers out of the depletion region. The drift current rises only slightly, because only the minority carriers and generated carriers get pulled across the region.

\[
E_p(x) = -\frac{qN_A}{\varepsilon} (x - x_p) \\
E_n(x) = -\frac{qN_D}{\varepsilon} (x - x_n)
\]
Forward or reverse bias (abrupt junction, full depletion model)

- **Forward bias**
  - $x_{p0}$
  - $\phi_N$
  - Depletion region narrows

- **Reverse Bias**
  - $x_{p0}$
  - $\phi_N$
  - Depletion region is larger

In any case, charge must balance so:

$qN_x x_p = qN_p x_p$

Where $x_{n0}$ and $x_{p0}$ are the widths of the depletion regions extending into their respective doped regions.

Electrostatics (1-D)

From Maxwell’s equations and the definition of the potential (voltage) we have a differential equation for the fields due to the charge distribution

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

Notice that this says that in a region of no charge, the potential will change linearly, which is a constant E field.

Abrupt junction, full depletion model

For the Abrupt junction, full depletion model of the PN diode, we can find the potential as a function of position by integrating over the charge distribution $\rho(x)$

Where $x_{n0}$ and $x_{p0}$ are the widths of the depletion regions extending into their respective doped regions.
Boundary conditions

- Since the dielectric constant is the same, and there are no sheet charges at x=0, we have:
  \[ \phi(0^+) = \phi(0^-) \]
  \[ \frac{qN_n}{2\varepsilon} x^2 + A_n x + B_n = -\frac{qN_p}{2\varepsilon} x^2 + A_p x + B_p \]
- And:
  \[ E(0^+) = E(0^-) \]
  \[ \frac{d}{dx}\phi(0^+) = \frac{d}{dx}\phi(0^-) \]
  \[ 2 \frac{qN_n}{2\varepsilon} x + A_n \bigg|_{x=0} = 2 \frac{qN_p}{2\varepsilon} x + A_p \bigg|_{x=0} \]

Notice that the E field at the junction is equal to A.

Calculating the depletion depth

- Depletion depth:
  \[ \frac{qN_n}{\varepsilon} x_p = -A \quad \Rightarrow \quad x_p = -\frac{A\varepsilon}{qN_n} \]
- Since \( E(x=0) = A \) we have:
  \[ -x_p = -\frac{E(x=0)\varepsilon}{qN_n} \]

This will be useful later, for calculating depletion caused by an E field penetrating into semiconductor regions in general.

More BC's

- So we now have
  \[ \phi(x) = \frac{qN_n}{2\varepsilon} x^2 + A_n x + B_n \quad \Rightarrow \quad x_p < x < 0 \]
  \[ \phi(x) = -\frac{qN_p}{2\varepsilon} x^2 + A_p x + B_p \quad 0 < x < x_s \]
- We also know that outside the depletion region, the E field will be small, so
  \[ E(-x_p) = \frac{d}{dx}\phi(x) \bigg|_{x=-x_p} = 0 \]
  \[ 2 \frac{qN_n}{2\varepsilon} x + A_n \bigg|_{x=-x_p} = 0 \quad \Rightarrow \quad \frac{qN_n}{\varepsilon} x_p = -A \]

Depletion depth, n region (N_d)

- The depletion depth into the n region is calculated the same way:
  \[ \frac{qN_d}{\varepsilon} x_n = A \quad \Rightarrow \quad x_n = \frac{A\varepsilon}{qN_d} \]

and:
  \[ x_s = \frac{E(x=0)\varepsilon}{qN_d} \]
Potential difference

- We can now plug these constants back in to find the potential difference from edge to edge of the depletion region:

$$\Delta \phi = \phi(-x_d) - \phi(x_p) = \frac{qN_x}{2\epsilon} x_p^2 - A x_p + B + \frac{qN_x}{2\epsilon} x_d^2 - A x_d - B$$

- And since:

$$x_p = \frac{Ae}{qN_x} \quad \text{and} \quad x_d = \frac{Ae}{qN_d}$$

$$\Delta \phi = \frac{qN_x}{2\epsilon} \left( \frac{Ae}{qN_x} \right)^2 - A \left( \frac{Ae}{qN_x} \right) qN_x + \frac{qN_d}{2\epsilon} \left( \frac{Ae}{qN_d} \right)^2 - A \left( \frac{Ae}{qN_d} \right) qN_d$$

$$\Delta \phi = \left( \frac{Ae}{2qN_x} \right) - A \left( \frac{Ae}{qN_x} \right) + \left( \frac{Ae}{2qN_d} \right) - A \left( \frac{Ae}{qN_d} \right)$$

Simplifying further:

$$\Delta \phi = \frac{Ae}{2qN_x} \left( \frac{1}{N_d} + \frac{1}{N_x} \right)$$

- Of course what we would really do is to set the potential (voltage) across the junction, and then get the depletion depths, etc., so solve for A:

$$A = E(x = 0) = \left( -\Delta \phi \right) \left( \frac{\epsilon}{2q} \right) \left( \frac{1}{N_d} + \frac{1}{N_x} \right)$$

Notice that this will be dominated by whichever region has the lighter doping.

bias

- The externally applied voltage will not be $\Delta \phi$ however, but the difference between the Fermi levels; (one contact is to a P type region, the other is to an N type region)

$$V_{ext}$$

- So the total potential for the electrons will be the sum of the external voltage plus the built in voltage!

$$\phi = V_{ext} + \Delta \phi$$

So we have:

$$A = E(x = 0) = \left( \phi_h - V_{ext} \right) \left( \frac{2q}{\epsilon} \right) \left( \frac{1}{N_d + N_x} \right)^{\frac{1}{2}}$$

$V_{ext}$ is taken to be positive for a forward bias.

- And we can find the penetration of the field into each region:

$$x_p = \frac{Ae}{qN_x} \quad x_d = \frac{Ae}{qN_d}$$
Voltage Dependence of Depletion Width

- Rewriting and simplifying:
  \[ x_\alpha(V_D) = \sqrt{\frac{2\varepsilon}{qN_D} \left( \frac{N_x}{N_x+N_d} \right)} = x_\alpha \sqrt{1 - \frac{V_D}{\phi_b}} \]
  \[ x_p(V_D) = \sqrt{\frac{2\varepsilon}{qN_p} \left( \frac{N_x}{N_x+N_p} \right)} = x_p \sqrt{1 - \frac{V_D}{\phi_b}} \]

- And then the total depletion width is:
  \[ X_d(V_D) = x_\alpha(V_D) + x_p(V_D) = \sqrt{\frac{2\varepsilon}{q} \left( \frac{\phi_b - V_D}{N_x} + \frac{1}{N_x+N_d} \right)} \]
  \[ X_d(V_D) = x_\alpha \sqrt{1 - \frac{V_D}{\phi_b}} \]

Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge
  \[ Q_j(V_D) = -qN_x x_j(V_D) = -qN_x \sqrt{1 - \frac{V_D}{\phi_b}} \]

- Charge is **not** a linear function of voltage
- This is a non-linear capacitor
- We can define a small signal capacitance for small signals by breaking up the charge into two terms
  \[ Q_j(V_D + \Delta V) = Q_j(V_D) + q(V_D) \]

Small signal models

- In a small signal model, we can take a nonlinear circuit or device and make approximations which linearize it about a DC operating point.
- To do this, we write all voltages and currents as the sum of a DC value for each, plus a small extra amount which is the small signal:
  \[ V(t) = V_0 + v(t) \]
  \[ I(t) = I_0 + i(t) \]

- Since the circuit is nonlinear, changes in the operating point will change the response to the small signal, but if the small signal is small enough we will be able to find a linear model for it

Derivation of Small Signal Capacitance

- As an example, for our capacitor, we can do a Taylor expansion about the voltage \( V_D \), the operating point:
  \[ Q_j(V_D + \Delta V) = Q_j(V_D) + \frac{dQ_j}{dV} \bigg|_{V=V_D} V_D + \cdots \]
  \[ C_j = C_j(V_D) = \frac{dQ_j}{dV} \bigg|_{V=V_D} = \frac{d}{dV} \left( -qN_x \sqrt{1 - \frac{V}{\phi_b}} \right) \bigg|_{V=V_D} \]

Where \( C_j \) is what we call the small signal capacitance (it depends on \( V_D \))
Derivation of Small Signal Capacitance

- Taking the derivative:

\[ C_j = \frac{qN_x x_{j0}}{2\varepsilon_0 \sqrt{1 - \frac{V_d}{\Phi_0}}} - \frac{C_{j0}}{\sqrt{1 - \frac{V_d}{\Phi_0}}} \]

- Notice that

\[ C_j = \frac{qN_x X_{j0}}{2\varepsilon_0 \sqrt{1 + \frac{1}{N_x + N_y}}} = \frac{qN_x N_y}{2\varepsilon_0 \sqrt{N_x + N_y}} \]

A Variable Capacitor (Varactor)

- Capacitance varies versus bias:

\[ \frac{C_j}{C_{j0}} \]

Application: Radio Tuner

Physical Interpretation of Depletion Cap

- The expression on the right-hand-side is just the depletion width in thermal equilibrium

\[ C_{j0} = \frac{qN_x N_y}{2\varepsilon_0 \sqrt{N_x + N_y}} \]

- This looks like a parallel plate capacitor with a spacing \( X_{j0} \)

\[ C_j(V_d) = \frac{\varepsilon_s}{X_s(V_d)} \]

- This is because the differential charge is stored at that distance, which is all that counts!

MOS Capacitor

- MOS = Metal Oxide Silicon
- Sandwich of conductors separated by an insulator
- “Metal” is more commonly a heavily doped polysilicon layer n⁺ or p⁺ layer
- NMOS \( \rightarrow \) p-type substrate, PMOS \( \rightarrow \) n-type substrate

MOS: Oxide (SiO₂) \( \varepsilon_o = 3.9\varepsilon_s \)

Very Thin! \( t_o \sim 1\text{nm} \)

Gate (n⁺ poly)

Body (p-type substrate)

Oxide (SiO₂) \( \varepsilon_o = 3.9\varepsilon_s \)

Very Thin! \( t_o \sim 1\text{nm} \)

Gate (n⁺ poly)

Body (p-type substrate)
Under thermal equilibrium, the n-type poly gate is at a higher potential than the p-type substrate.

\[ \phi_n = -\frac{kT}{q} \ln \frac{N_d}{n_i} \quad \phi_p \approx 550 \text{mV} \]

No current can flow because of the insulator but this potential difference is accompanied with an electric field.

Fields terminate on charge!

At equilibrium there is an electric field from the gate to the body. The charges on the gate are positive. The negative charges in the body come from a depletion region.

If we apply a bias, we can compensate for this built-in potential.

\[ V_{FB} = -(\phi_p - \phi_n) \]

In this case the charge on the gate goes to zero and the depletion region disappears.

In solid-state physics lingo, the energy bands are “flat” under this condition.

If we further decrease the potential beyond the “flat-band” condition, we essentially have a parallel plate capacitor.

Plenty of holes and electrons are available to charge up the plates.

Negative bias attracts holes under gate.
Depletion

- Similar to equilibrium, the potential in the gate is higher than the body
- Body charge is made up of the depletion region ions
- Potential drop across the insulator and depletion region

\[ Q_b(V_{gs}) = -Q_d \]

\[ Q_b = -qN_d X_d(V_{gs}) \]

Inversion

- As we further increase the gate voltage, eventually the surface potential increases to a point where the electron density at the surface equals the background ion density

\[ n_e = n_i e^{\phi / kT} = N_d \quad \rightarrow \quad \phi_s = \phi_p \]

- At this point, the depletion region stops growing and the extra charge is provided by the inversion charge at surface

Next week

- Next week, we will see how we can use this control of the number of carriers under an oxide to make a switch, a field effect transistor