Lecture 26:
Single stage amps, CD, BJT

Prof. J. S. Smith

Context
Today, we will continue to discuss biasing and small signal models for single transistor amplifiers, including the common base amplifier, and the similar bipolar transistor amplifiers, common emitter, common base, and common collector.

Next, we will be discussing the frequency response of these amplifiers.

Transistor terminals

- Gate (Base)
  - Less current than other terminals
  - S→D current a sensitive function of G-S voltage
    \((E→C)\) \((B-E)\)
- Source (emitter)
  - Relatively large amount of current
  - S→D current a sensitive function of G-S voltage
    \((E→C)\) \((B-E)\)
- Drain (collector)
  - Relatively large amount of current (=Source)
  - S→D current not a sensitive function of voltage at this terminal

Lecture Outline
- Common Drain Amp
- Bipolar:
  - Common Emitter Amp
  - Common Base Amp
  - Common Collector Amp
Common-Source Amplifier

The input is at the gate, which doesn’t need any (much) current, and is a sensitive terminal (→S): large voltage and current gain.

Common Gate Amplifier

Notice that $I_{out}$ must equal $-I_s$.

Gain of transistor tends to hold this node at ss ground: low input impedance load for current input.

CG as a Current Amplifier: Find $A_i$

$$i_{out} = i_d = -i_s$$

$$A_i = -1$$

CG Input Resistance

At input:

$$i_i = \frac{v_o - v_{in}}{r_{in}} = i_r (r_m \parallel R_i)$$

Output voltage:

$$v_{out} = -i_r (r_m \parallel R_i) = i_r (r_m \parallel R_i)$$

$$i_i = g_m v_i + g_{mn} v_m + \left( \frac{v_i - v_{in}}{r_{in}} \right)$$
Approximations…

- We have this messy result

\[ \frac{1}{R_m} = \frac{i}{v} = \frac{g_m + g_{ab} + \frac{1}{r_o}}{1 + \frac{r_o}{R_m} r_s} \]

- But we don’t need that much precision. Let’s start approximating:

\[ g_m + g_{ab} \gg \frac{1}{r_o} \quad r_{oc} \parallel R_z \approx R_z \quad \frac{R_i}{r_o} \approx 0 \]

\[ R_m = \frac{1}{g_m + g_{ab}} \]

CG Output Resistance

Substituting \( v_s = iR_S \)

\[ iR_S \left( \frac{1}{R_S} + g_m + g_{ab} + \frac{1}{r_o} \right) = \frac{v}{r_c} \]

The output resistance is \((v_t/ i_t)|| r_{oc}\)

\[ R_{oc} = r_{oc} \parallel \left( R_z \left( \frac{R_i}{R_S} + g_m r_o + g_{ab} r_o + 1 \right) \right) \]

Approximating the CG \( R_{out} \)

\[ R_{out} = r_{oc} \parallel \left[ r_o + g_m r_o R_S + g_{ab} r_o R_S + R_S \right] \]

The exact result is complicated, so let’s try to make it simpler:

\[ g_m \approx 500 \mu \Omega \quad g_{ab} \approx 50 \mu \Omega \quad r_o \approx 200k \Omega \]

\[ R_{oc} \approx r_{oc} \parallel \left[ r_o + g_m r_o R_S + R_S \right] \]

Assuming the source resistance is less than \( r_{oc} \),

\[ R_{out} \approx r_{oc} \parallel \left[ r_o + g_m r_o R_S \right] = r_{oc} \parallel \left[ r_o \left( 1 + g_m R_S \right) \right] \]
**CG Two-Port Model**

Function: a current buffer
- Low Input Impedance
- High Output Impedance

**CD Voltage Gain**

Note \( v_{gs} = v_i - v_{out} \)

\[
\frac{v_{out}}{r_{ds}} = g_m v_{gs} - g_{ds} v_{out}
\]

\[
\frac{v_{out}}{r_{ds}} = g_m (v_i - v_{out}) - g_{ds} v_{out}
\]

In the common drain amp, the output is taken from a terminal of which the current is a sensitive function.

\[
I_{ds} = \mu C_w \frac{W}{L} \left( V_{gs} - V_T \right)^2
\]

\[
V_{gs} = V_T + \frac{2I_{ds}}{\mu C_w \frac{W}{L}}
\]

Weak \( I_{ds} \) dependence

**Common-Drain Amplifier**

**CD Voltage Gain (Cont.)**

KCL at source node:

\[
\frac{v_{out}}{r_{ds}} \parallel r_i = g_m \left( v_i - v_{out} \right) - g_{ds} v_{out}
\]

\[
\left( \frac{1}{r_{ds}} + g_{ds} + g_m \right) V_{out} = g_m V_i
\]

Voltage gain (for \( V_{gs} \) not zero):

\[
\frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + g_{ds} + g_w} \\
\frac{v_{out}}{v_{in}} = \frac{g_m}{g_{ds} + g_w} \approx 1
\]
CD Output Resistance

Sum currents at output (source) node:

$$R_{\text{out}} = r_e \parallel r_o \parallel \frac{v_i}{i_i} = g_m v_i + g_n v_i$$

$$R_{\text{out}} = \frac{1}{g_m + g_n}$$

CD Output Resistance (Cont.)

$r_o \parallel r_e$ is much larger than the inverses of the transconductances → ignore

$$R_{\text{out}} = \frac{1}{g_m + g_n}$$

Function: a voltage buffer
- High Input Impedance
- Low Output Impedance

Bipolar single stage amps.

- BJT Amps
- BJT Biasing
- Common Emitter Amp
- Common Base Amp
- Common Collector Amp
  - AKA Emitter Follower
- $\beta$ Multiplier Concept
- Emitter Degeneration
**Bipolar Amplifiers**

**Common-emitter amplifier:**

Biasing: adjust $V_{\text{BIAS}} = V_{\text{BE}}$ so that $I_C = I_{\text{SUP}}$.

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**Small-Signal Two-Port Model**

Parameters: ($I_C = 1$ mA, $\beta = 100$, $V_A = 3$ V)

- $R_{\text{in}} = r_e = \frac{\beta}{\beta_f} = 100 \times \frac{25 \text{mV}}{1 \text{mA}} = 2.5 \text{kΩ}$
- $R_{\text{out}} = r_e || r_w = \frac{3 \text{V}}{1 \text{mA}} || r_w = 3 \text{kΩ} || r_w = 3 \text{kΩ}$
- $G_m = g_m = \frac{1 \text{mA}}{25 \text{mV}} = \frac{1}{25} \text{S} = 40 \text{mS}$

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**Common-Base Amplifier**

To find $I_{\text{BIAS}}$, note that $I_{\text{BIAS}} = I_E = - (1/\alpha_f) I_C$.

Common-base current gain $A_i = - \alpha_f$.

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**CB Input Resistance**

Summing currents at the input node:

$i_i + \frac{v_x}{r_e} + g_m v_x + \left(\frac{v_x}{r_w}\right) g_m = 0 \quad \Rightarrow \quad i_i = \left(\frac{1}{r_e} + g_m\right) v_i$

$R_{\text{in}} = \frac{v_i}{i_i} = \left(\frac{1}{r_e} + g_m\right)^{-1} = \left(\frac{g_m + r_w}{\beta + r_w}\right) \approx \frac{1}{g_m} \approx \frac{25 \text{mV}}{1 \text{mA}} \approx 25 \Omega$
CB Output Resistance

Same topology as CG amplifier, but with \( r_n \parallel R_S \) rather than \( R_S \)

\[
R_{\text{out}} = r_n \left[ r_n \left( 1 + g_n (r_n \parallel R_S) \right) \right]
\]

\( R_S \gg r_n \)

\[
R_{\text{out}} = r_n \left[ r_n \left( 1 + \beta \right) \right]
\]

Note polarity

Output Impedance Calculation

\[
i = g_m v_x + \frac{v_x - v_i}{r_i} + \frac{v_x - v_i}{r_i} \]

\[
v_x = -i (R_s \parallel r_i)
\]

\[
i = g_m R_s \parallel r_i + \frac{v_x}{R_s \parallel r_i} + \frac{v_x}{R_s \parallel r_i} \]

Output Impedance Details

- First draw small signal equivalent circuit with transistor and simplify as much as possible
- Then (if needed) add the small signal equivalent circuit
- If frequency is low, get rid of caps!

Common-Base Two-Port Model

Why did we consider it a current amp?

Current Amp :

- Unity Current Gain (-1)
- Small Input Impedance
- Large (huge!) Output Impedance
**Common-Collector Amplifier**

DC Bias:
output is one “$V_{BE}$ drop” down from input

“Emitter Follower”


**Common-Collector Output Resistance**

Divider between $v$ and $v_{π}$:

\[ v_z = \frac{r_x}{r_x + r_s} v_i \]

\[ i_z + g_m v_z + v_{π} r_x^{-1} - v_i (r_x \parallel r_s)^{-1} = 0 \]

\[ i_z = v_i \left( g_m + r_x^{-1} \right) \left( \frac{r_x}{r_x + r_s} \right) + v_i (r_x \parallel r_s)^{-1} \]


**Common-Collector Input Resistance**

\[ v_i = i_r + (β+1)i_r \left( R_e \parallel r_x \parallel r_i \right) \]

\[ R_{\text{in}} = r_x + (β+1)(R_e \parallel r_x \parallel r_i) \]

\[ R_{\text{in}} \approx r_x + (β+1)R_e \]


**Common-Collector Output Res. (cont)**

\[ i_z = v_i \left( g_m + r_x^{-1} \right) \left( \frac{r_x}{r_x + r_s} \right) + v_i \left( r_x \parallel r_s \right)^{-1} \]

\[ R_{\text{out}} = \frac{v_i}{i_i} = \frac{r_x + r_s}{β + 1} \]

- Looking into base of emitter follower: load impedance larger by factor $β+1$
- Looking into emitter of follower: “source” impedance smaller by factor $β+1$
Common-Collector Voltage Gain

KCL at the output node: note \( v_{\pi} = v_i - v_{\text{out}} \)

\[
\begin{align*}
    v_{\text{out}} & = \left( g_m + r_e^{-1} \right) v_i \\
    \frac{v_{\text{out}}}{v_i} & = g_m + r_e^{-1} + g_m r_e^{-1} + r_e^{-1}
\end{align*}
\]

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Common-Collector Two-Port Model

Voltage Amp:
- Unity Voltage Gain (+1)
- Large Input Impedance
- Small Output Impedance

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Summary of Two-Port Parameters

<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Controlled Source</th>
<th>Input Resistance</th>
<th>Output Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Emitter</td>
<td>( g_m = g_b )</td>
<td>( r_e )</td>
<td>( g_m r_b )</td>
</tr>
<tr>
<td>Common Source</td>
<td>( g_m = g_b )</td>
<td>infinity</td>
<td>( g_m r_b )</td>
</tr>
<tr>
<td>Common Base</td>
<td>( A_V = 1 )</td>
<td>( 1/g_m )</td>
<td>( r_m )</td>
</tr>
<tr>
<td>Common Gain</td>
<td>( A_V = 1 )</td>
<td>( 1/g_m )</td>
<td>( r_m )</td>
</tr>
<tr>
<td>Common Drain</td>
<td>( A_V = 1 )</td>
<td>( r_m )</td>
<td>( r_m + R )</td>
</tr>
</tbody>
</table>

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