Today, we will continue the discussion of single transistor amplifiers by looking at common source amplifiers with source degeneration (also common Emitter amplifiers with emitter degeneration.

We will then start discussing the frequency response of single stage amplifier, the frequency response of CE amps, and the Miller approximation.

- Emitter Degeneration
- Frequency response of the CE and CS current amplifiers
- Unity-gain frequency \( \omega_r \)
- Frequency response of the CE as voltage amp
- The Miller approximation

- When a transistor biasing circuit is designed, it is important to realize that the characteristics of the transistor can vary widely, and that passive components vary significantly also.
- Biasing circuits, must therefore be designed to produce a usable bias without counting on specific values for these components.
- One example is a BJT base bias in a CE amp. A slight change in the base-emitter voltage makes a very large difference in the quiescent point. The insertion of a resistor will improve sensitivity.
Typical “Discrete” Biasing

- In this scheme, the base bias voltage is given by a voltage divider: 
  \[ V_B = V_{CC} \frac{R_1}{R_1 + R_2} \]
- The emitter will approximately follow the base voltage, so the emitter current is: 
  \[ I_E \approx \left( V_{CC} - V_{BE,dc} \right) / R_E \]

Gain for “Discrete” Design

- \( R_{in} \approx r_b + (\beta + 1)R_E \)
- \( R_{in} \approx (\beta + 1)R_E \)
- \( R_{in} \approx \frac{V_i}{R_E} \)
- \( R_{in2} \approx (\beta + 1)R_E / R_1 \parallel R_2 \)
- Can be made large to couple All of source to input (even with \( R_2 \))

Insensitivity to transistor parameters

- Most of the circuit parameters are independent of variation of the transistor parameters, and depend only on resistance ratios. That is often a design goal, but in integrated circuits we will not want to use so many resistors.

Emitter Degeneration

- The addition of the resistor at the emitter will have several additional effects:
  - The transconductance will be reduced
  - The output resistance will be increased
  - The input resistance will be increased
- Each of these are a result of the negative feedback from the presence of the emitter resistor.
Source Degeneration

- Source degeneration for a FET is not as common as emitter degeneration is for a BJT, because the gain of a FET is already lower than for a BJT, and its input impedance is already $\infty$.
- It is widely used, however, to raise the output impedance of CS amplifiers.

Small signal model

- CS amp with source degeneration:

Source degeneration in a CS amp

- The input impedance of the amplifier is still $\infty$.
- Assume that the body is still connected to ground.

Circuit gain calculation

- From KCL at the source,
  \[ \frac{v_s}{R_S} + \frac{v_i}{r_b} = g_m (v_i - v_s) + g_m (0 - v_s) \]
- At the Drain:
  \[ i_0 + \frac{v_s}{r_b} = g_m (v_i - v_s) + g_m (0 - v_s) \]
- The circuit’s transconductance:
  \[ g_m = \frac{i_0}{v_i} = \frac{g_m}{1 + (g_m + g_m)R_S + \frac{R_S}{r_b}} \]
- Because of the body effect, the output is still dependant on the transistor characteristics.
Output impedance

- To calculate the output impedance, we put in a test current and calculate the voltage:

\[ V_i = V_s + i_r R_s = V_s + r_0 [1 + g_m + g_m] R_S \]

- From which we get the output resistance:

\[ R_0 = \frac{V_s}{i_r} = R_s + r_0 [1 + (g_m + g_m)] R_S \]

Frequency response

- So far, we have modeled the small signal response of the stages for low frequencies (but not so low that we couldn’t neglect the DC blocking coupling capacitors!)

- Now we will put in the parasitic capacitances, and analyze the changes in the transfer functions of the circuits at higher frequencies.

Parasitic Capacitances

- If we look at the small signal model of the FET that we developed a few weeks ago, we have capacitances between the gate and the source, and the gate and the drain.
When we take into account a finite source impedance in a common source amplifier, the capacitances will reduce the voltage swing at the gate at high frequencies.

Parasitic Capacitances

The transfer function will be a low pass filter, with a pole at the frequency determined by the source resistance and the capacitance.

Miller approximation

However, the capacitance from the gate to the drain has a large effect, because the voltage on the drain is amplified, with a negative gain coefficient. This means that the contribution of the capacitance from the gate to the drain is comparable to the same capacitance to ground, multiplied by the gain of the transistor. This is called the Miller effect. We can approximate the effect, by simply putting in a capacitance to ground, multiplied by the low frequency gain. This is called the Miller approximation.

High frequency zero

At very high frequencies, the gain flattens out again, because the capacitor couples from the gate to the drain directly, as a passive circuit.
CE Amplifier with Current Input

Find intrinsic current gain by driving with infinite source impedance and zero load impedance...

Short-Circuit Current Gain

Pure input current ($R_s = 0$ $\Omega$)

Small-signal short circuit (could be a DC voltage source)

Substitute equivalent circuit model of transistor and do the "math"

Small-Signal Model: $A_i$

Note that $r_o$ and $C_{cs}$ play no role (shorted out)
Phasor Analysis: Find $A_i$

KCL at the output node:

$$I_{out} = g_m V_s + (0 - V_s)/Z_p$$

KCL at the input node:

$$I_i = V_s / Z_s + (V_s - 0)/Z_p$$

Solve for $V_s$:

$$V_s = (1/Z_s + 1/Z_p)^{-1} I_i$$

Phasor Analysis for $A_i$ (cont.)

$$I_{out} = (g_m - j\omega C_m) V_s$$

Substituting for $V_s$

$$A_i(j\omega) = \frac{g_m - j\omega C_m}{(1/Z_s) + j\omega C_m}$$

Substituting for $Z_s = r_s \parallel (1/j\omega C_m)$

$$Z_s = \frac{r_s}{1 + j\omega r_s C_s}$$

$$A_i(j\omega) = \frac{\beta_s (1 - j\omega C_m / g_m)}{1 + j\omega r_s (C_s + C_m)}$$

Short-Circuit Current Gain Transfer Function

Transfer function has one pole and one zero:

$$A_i(j\omega) = \frac{\beta_s (1 - j\omega / \omega_p)}{(1 + j\omega / \omega_z)}$$

Note: Zero Frequency much larger than pole:

$$\omega_z = \frac{g_m}{C_s} = \frac{1}{r_s C_s} > \beta \frac{1}{r_s (C_s + C_m)} = \beta \omega_p$$

Magnitude Bode Plot

- **Pole**: $\omega_p$
- **Zero**: $\omega_z$
- **Unity current gain**: $1/j\omega / \omega_p$

Note: Zero Frequency much larger than pole.
Transition Frequency $\omega_T$

$\omega_T = \beta_m \omega_p = \frac{g_m}{C_x + C_p}$

Dependence on DC collector current:

$C_x = C_{je} + C_{diff}$

$C_{diff} = g_m \tau_F = \tau_F q T / kT$

Limiting case: $f_T = \frac{\omega_p}{2\pi} \rightarrow \frac{1}{2\pi \tau_F}$

CS Short-Circuit Current Gain

Transfer function:

$A(j\omega) = \frac{g_m (1 - j\omega C_{gs}/g_m)}{j\omega(C_{gs} + C_{gd})}$

MOS Unity Gain Frequency

- Since the zero occurs at a higher frequency than pole, assume it has negligible effect:

$A_s = \frac{g_m}{j\omega(C_{gs} + C_{gd})} = 1 \quad \rightarrow \quad \omega_s = \frac{g_m}{(C_{gs} + C_{gd})}$

$\omega_s = \frac{g_m}{C_{gs}} - \frac{\mu C_{ox} W (V_{gs} - V_T)}{L} = \frac{3\mu (V_{gs} - V_T)}{2}$

Performance improves like $L^2$ for long channel devices!

For short channel devices the dependence is like $\sim L^1$

$\omega_s = \frac{3\mu (V_{gs} - V_T)}{2} \approx \frac{V_{gs} - V_T}{L} = \frac{\mu E_{ox}}{L} = \frac{V}{L} = \tau_L$ Time to cross channel
Miller Impedance

- Consider the current flowing through an impedance $Z$ hooked up to a “black-box” where the voltage gain from terminal to the other is fixed (as you can see, it depends on $Z$)

\[
A_v = \frac{v_2}{v_1}
\]

\[
I = \frac{v_1 - A_v v_1}{Z} = \frac{v_1 - A_v v_1}{Z} = v_1 \left(1 - A_v\right)
\]

- Notice that the current flowing into $Z$ from terminal 1 looks like an equivalent current to ground where $Z$ is transformed down by the Miller factor:

\[
I = v_1 \frac{1 - A_v}{Z} \rightarrow Z_{M,1} = \frac{Z}{1 - A_v}
\]

- From terminal 2, the situation is reciprocal

\[
-I = \frac{v_2 - v_1}{Z} = \frac{v_2 - A_v^{-1} v_1}{Z} = v_1 \frac{1 - A_v^{-1}}{Z}
\]

\[
Z_{M,2} = \frac{Z}{1 - A_v^{-1}}
\]

Miller Equivalent Circuit

- We can “de-couple” these terminals if we can calculate the gain $A_v$ across the impedance $Z$
- Often the gain $A_v$ is weakly dependent on $Z$
- The approximation is to ignore $Z$, calculate $A_v$, and then use the decoupled miller caps

\[
Z_{M,1} + Z_{M,2} = Z
\]

\[
Z_{M,1} = \frac{Z}{1 - A_v}
\]

\[
Z_{M,2} = \frac{Z}{1 - A_v^{-1}}
\]