Lecture 28: Single Stage Frequency response

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Context

In today’s lecture, we will continue to look at the frequency response of single stage amplifiers, starting with a more complete discussion of the CS amplifier, and then looking at the frequency response of CG and the CD connections.

Next: Multi-state amplifiers

Reading

- Reading: We are discussing the frequency response of single stage amplifiers, which isn’t treated in the text until after multi-state amplifiers (beginning of chapter 10). I feel that it is important to get warmed back up on linear circuit analysis for simple circuits before jumping into multi-stage amplifiers.

- We will be starting on chapter 9, multi-state amplifiers, later this week.

Lecture Outline

- Frequency response of the CS as voltage amp
- The Miller approximation
- Frequency Response of a Voltage Buffer
- Frequency Response of Current Buffer
Last Time: CS Amp with Current Input

Calculate the short circuit current gain of device (BJT or MOS)

CS Short-Circuit Current Gain

MOS Case

\[ A(j\omega) = \frac{g_m}{j\omega(C_{gs} + C_{gd})} \]

Note: Zero occurs when all of "gm" current flows into Cgd:

\[ g_m V_{gs} = V_{gs} j\omega C_{gd} \]

Input Impedance

Look at how \( Z_{gd} \) affects the transfer function: find \( Z_{in} \)

\[ Z_{in}(j\omega) = \frac{Z_{gd}}{1 - A_{C_{gd}}} \]

\[ I_t = \frac{V_i - V_{out}}{Z_{gd}} \]

At output node:

\[ V_{out} = (-g_m V_i - I_t) R_{out}^* \approx -g_m V_i R_{out}^* \quad \text{Why?} \]

\[ I_t = \frac{V_i - A_{C_{gd}} V_i}{Z_{gd}} \]
Miller Capacitance $C_M$

Effective input capacitance:

$$Z_{in} = \frac{1}{j\omega C_{M}} = \left(\frac{1}{1 - A_{C_{ds}}}\right) \frac{1}{j\omega C_{gd}} = \frac{1}{j\omega \left(1 - A_{C_{gs}}C_{gd}\right)}$$

**Some Examples**

Common source (emitter) amplifier:

$$A_{C_{ds}} = \text{Negative, large number (-100)}$$

$$C_M = \left(1 - A_{V,C_{gd}}\right)C_{gd} \approx 100C_{gd}$$

→Miller Multiplied Cap has Detrimental Impact on bandwidth

Common drain (collector) amplifier:

$$A_{C_{gs}} = \text{Slightly less than 1}$$

$$C_M = \left(1 - A_{V,C_{gd}}\right)C_{gs} \approx 0C_{gs}$$

"Bootstrapped" cap has negligible impact on bandwidth!

**CE Amplifier using Miller Approx.**

Use Miller to transform $C_{gd}$

$$C_M = C_{gd}(1 + g_mR'_out)$$

Analysis is straightforward now … single pole!

**Comparison**

Miller result (calculate RC time constant of input pole):

$$\omega_{p1}^{-1} = R_s \left\{C_{bs} + (1 + g_mR'_out)C_{gd}\right\}$$

If we hadn’t made the Miller approximation, the result would have been:

$$\omega_{p1}^{-1} = R_s \left\{C_{bs} + (1 + g_mR'_out)C_{gd}\right\} + R'_out C_{gd}$$
Method of Open Circuit Time Constants

- Here is a technique to find the dominant pole of a circuit (only valid if there really is a dominant pole!)
- For each capacitor in the circuit you calculate an equivalent resistor “seen” by capacitor and form a time constant $\tau_i = R_i C_i$
- The dominant pole then is the sum of these time constants in the circuit

\[
\omega_{p,\text{dom}} = \frac{1}{\tau_1 + \tau_2 + \ldots}
\]

Example Calculation: CE input impedance

- Consider the input capacitance $C_i = C_e + C_M$
- Open all other “small” caps (get rid of output cap)
- Turn off all independent sources
- Insert a current source in place of cap and find impedance seen by source

\[
\tau_i = (R_S \parallel R_g) \left\{ C_e + \left( 1 + g_m R_m^* \right) C_e \right\}
\]

Equivalent Resistance “Seen” by Capacitor

- For each “small” capacitor in the circuit:
  - Open-circuit all other “small” capacitors
  - Short circuit all “big” capacitors
  - Turn off all independent sources
  - Replace cap under question with current or voltage source
  - Find equivalent input impedance seen by cap
  - Form RC time constant
- This procedure is best illustrated with an example…

Common-Drain Amplifier

\[
I_{DS} = \mu C_m \frac{W}{L} \left( V_{GS} - V_T \right)^2
\]

\[
V_{GS} = V_T + \sqrt{\frac{2I_{DS} W}{\mu C_m \frac{W}{L}}}
\]

Weak $I_{DS}$ dependence
**CD Voltage Gain**

\[ v_{\text{out}} \approx g_m \approx 1 \]

\[ \frac{v_{\text{out}}}{v_{\text{in}}} \approx \frac{g_m}{g_{\text{mb}} + g_{\text{ss}}} \approx 1 \]

**CD Output Resistance (Cont.)**

\[ r_o \parallel r_{\text{oc}} \text{ is much larger than the inverses of the transconductances } \rightarrow \text{ ignore} \]

\[ R_{\text{out}} = \frac{1}{g_m + g_{\text{mb}}} \]

Function: a voltage buffer
- High Input Impedance
- Low Output Impedance

**Add capacitors**

Procedure:
- Start with small-signal two-port model
- Add device (and other) capacitors

\[ C_{\text{in}} \]

\[ C_{\text{out}} \]

Sum currents at output (source) node:

\[ R_{\text{out}} = r_o \parallel r_{\text{oc}} \approx \frac{V_s}{I} \]

\[ i = g_m v_r + g_{\text{oc}} v_r \]

\[ R_{\text{out}} = \frac{1}{g_m + g_{\text{oc}}} \]
Common-Collector Amplifier

Voltage Gain $A_{VC,\pi}$ Across $C_\pi$

\[ A_{VC,\pi} \approx \frac{R_{\text{out}}}{(R_{\text{out}} + R_L)} \approx 1 \quad R_{\text{out}} = \frac{1}{g_m} \]

Note: this voltage gain is neither the two-port gain nor the "loaded" voltage gain.

\[ C_{\text{in}} = C_\mu + C_M = C_\mu + (1 - A_{VC,\pi})C_\pi \]

\[ C_{\text{in}} = C_\mu + \frac{1}{1 + g_m R_L} C_\pi \]

\[ C_{\text{in}} \approx C_\mu \]

Find Miller capacitor for $C_\pi$ -- note that the base-emitter capacitor is between the input and output.

Two-Port CC Model with Capacitors

Bandwidth of CC Amplifier

Input low-pass filter’s –3 dB frequency:

\[ \omega_p^{-1} = \left( R_s \parallel R_L \right) \left( C_\mu + \frac{C_\pi}{1 + g_m R_L} \right) \]

Substitute favorable values of $R_S$, $R_L$:

\[ R_s \approx 1/g_m \quad R_L >> 1/g_m \]

\[ \omega_p^{-1} \approx \left( 1/g_m \right) \left( C_\mu + \frac{C_\pi}{1 + B/L} \right) \approx C_\mu / g_m \]

\[ \omega_p \approx g_m / C_\mu > \omega_t \]

Model not valid at these high frequencies.
Common Gate Amplifier

DC bias:

\[ I_{SUP} = I_{BIAS} = I_{DS} \]

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CG Input Resistance

We found the approximation:

\[ R_{in} \approx \frac{1}{g_{m} + g_{mbl}} \]

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CG Output Resistance

\[ R_{out} \approx r_{oc} \left[ r_{o} + g_{m} r_{o} R_{S} \right] = r_{oc} \left[ r_{o} (1 + g_{m} R_{S}) \right] \]
The function of the CG amp was a current buffer:
• Low input impedance
• High output impedance

No Miller-transformed capacitor!
The only parasitic capacitances are directly across the input and output; frequency response can be directly determined.

Unity-gain frequency is on the order of $\omega_T$ for small $R_L$.

Same procedure: start with two-port model and capacitors.

CS, CE: suffer from Miller-magnified capacitor for high-gain case
CD, CC: Miller transformation $\Rightarrow$ nulled capacitor $\Rightarrow$ “wideband stage”
CG, CB: no Millerized capacitor $\Rightarrow$ wideband stage (for low load resistance)