Context

Next week is the last week of lecture, and we will spend those three lectures reviewing the material of the course, and looking at applications of the material.

Today we are going to look at the basics of differential amplifiers.
Reading

- All of the reading assignments are done
- Time to start reviewing for the final!

Lecture Outline

- Why differential signaling important, and becoming more important
- Introduction to differential amplifiers
Device Matching

- One of the things that we depend on in the design of analog integrated circuits is device matching. For example, if we make a current mirror, we are depending on the reference and the mirror device behaving in a very similar fashion. When a gate-source voltage is developed on the reference device, passing a given current, the same voltage appearing across the gate to source on the mirror device will allow the same drain current.

Variations

- The transistors will generally vary due to several causes:
  - Temperature → very similar for devices on the same substrate
  - Implant variations → important for small devices
  - Variations in width → important for narrow devices
  - Variations in length → important for short channel devices
  - Layout variations

Often, analog devices will not be minimum sized devices, so that output resistances will be lower, short channel effects will be smaller, and the effect of variations is reduced.
Differential vs single ended signals

- A voltage is only defined between two points.
- Charge times voltage difference gives the energy needed to move a charge between two points.
- A single ended signal is actually a misnomer, because all voltages are measured between two points, it’s just that single ended signals use “ground” as a reference.

What is “ground”

- In a low frequency circuit, with a big ground plane available, it is possible to approximate that ground is a stable reference which all voltages can be measured with respect to.
- At high frequencies, and at low voltage swings which are needed for reasonable power consumption, the variation of the “ground” can vary by more than the signal level.
- This is even a problem at low frequencies, if devices are separated by any distance. “ground” at one device can easily be several volts different than “ground” at another device. (DC and AC)
Differential signaling

- One solution to this problem is to use differential signaling, that is two wires, close to each other, and in a symmetric configuration, carry the signal as a voltage difference.
- They needed to be routed together and twisted to avoid coupling magnetic fields from other wires or sources.
- They should also be driven with the same impedance, so they pick up the same noise voltages.
- If all of these things are done, then noise picked up by the wires is picked up by both in the same amount and with the same sign—"Common mode".

Differential input

- In order to use a differential signaling scheme, we need to produce an amplified version of the difference between the input voltages.
- This is called a differential amplifier.
- The output of a differential amplifier can either itself be differential, or it could convert the signal into a single ended version with respect to ground.
- If the inputs change in voltage together, swinging in the same direction, this is called common mode.
- A good differential amplifier is designed to cancel the common mode: called common mode rejection.
Differential Amplifiers

Differential amplifiers also solve some of the other problems that we have brushed aside so far—like how to bias the input to the right quiescent voltage.

The Differential Amplifier Concept

The basic idea: amplify the *difference* between two inputs and reject the common component

\[ v_{out, \text{diff}} = A_{v, \text{diff}} (v_{in, \text{diff}}) = A_{v, \text{diff}} (v_{in1} - v_{in2}) \ldots \text{large} \]

\[ v_{out, \text{comm}} = A_{v, \text{comm}} (v_{in, \text{com}}) = A_{v, \text{comm}} [(v_{in1} + v_{in2})/2] \]

\ldots \text{small}
Two single ended amps

- One way you could think of making a differential amplifier is to use two separate single ended amplifiers, as we have been studying for the last several weeks. If the devices happened to be identical voltage amplifiers, we would have:

\[
\begin{align*}
    v_{out1} &= A_1 v_{in1} \\
    v_{out2} &= A_2 v_{in2} \\
    v_{out1} - v_{out2} &= A_1 v_{in1} - A_2 v_{in2} \\
    \Delta v_{out} &= A \Delta v_{in}
\end{align*}
\]

(If the gain is the same \(A = A_1 = A_2\))

A First try at a Differential Amplifier

Notice that this is just two common single transistor amplifiers with resistor pull ups.
Common Mode

- The common mode of a differential signal is the average of the two voltages.
- If we have a voltage $v_+$ and a voltage $v_-$, the differential voltage is:
  \[ \Delta v = v_+ - v_- \]
- And the common mode voltage is:
  \[ v_{CM} = \frac{1}{2}(v_+ + v_-) \]

Amplify the difference, not the common mode

- Since the information is carried only by the difference between the two voltages, and the common mode voltage represents noise, we would prefer not to amplify it.
- Amplifying the common mode voltage wastes the room we have between the rails,
- Minimizing the common mode will also improve our ability to correctly bias the following stages
Two single ended amps

- If we look at our simple model using two separate single ended amplifiers:

\[ v_{out1} = A_1 v_{in1} \]
\[ v_{out2} = A_2 v_{in2} \]
\[ v_{out1} + v_{out2} = A_1 v_{in1} + A_2 v_{in2} \]
\[ v_{outCM} = A v_{inCM} \]

So unfortunately, using two single ended amplifiers amplifies the common mode noise as much as it does the differential signal.

However, with a simple, very similar circuit, we can solve most of these problems.

A good, simple Differential Amplifier

By having the current from both transistors competing for the same current, we can minimize the amplification of the common mode signal.
Let's say something tries to decrease both sides of the output voltage.

The current coming through the pull up resistors would then both have to increase—but that can't happen because the only place for the current to go is through the same current sink.
If one current goes up, the other current has to go down to keep the same current in the current sink. This is perhaps the most common two transistor configuration in linear integrated circuits.

Biasing

If our resistors or transistors are not completely identical, or if the current source has a finite impedance, then we won’t remove all of the common mode signal, but we will reduce it.

The biasing problem is also reduced, in that a common mode steady state voltage can usually be avoided. Internal stages can be isolated from common mode offsets at the inputs.
In order to bias this circuit, first look at the output voltages. It is desirable to keep the output voltages close to the middle of the rails, so we want:

\[ V_{1out} = V_{2out} \approx 0 \]

\[ I_{d1} = I_{d2} = +V / R_D \]

\[ I_{bias} = I_{d1} + I_{d2} = 2(+V) / R_D \]

Notice that with our idealization of the current source and identical transistors and resistors, with a zero differential input, the output will be zero on both sides over the accommodation range of the differential amplifier.

What is the accommodation range of this differential amplifier? (The accommodation range is the range of common mode input voltages over which the amplifier will continue to work satisfactorily.)
Accommodation limits

- If both the inputs go low enough so that the drop across the current source is not sufficient to keep it in its operating range, the transistors will both go into cut off and both sides of the output will rise to the positive rail, which also reduces the gain for the differential mode signal.

- If the inputs both go high enough, the transistors will go into the triode mode, and the gain for the differential mode will be reduced. This limit can be designed to happen only for inputs higher than $V_{dd}$ by some amount.

Terminology

- This configuration is called a differential source coupled pair.
- The common current source (or resistor) at the transistor’s source terminals is called the tail.
DC characteristics

- Let’s assume that the resistors have been chosen small enough so that neither transistor goes into the triode mode if the inputs are separately less than $V_{DD}$, $r_0 \rightarrow \infty$.
- We can then write: $V_{in1} - V_{gs1} = V_{in2} - V_{gs2}$

$$V_{gs1} = V_{Tn} + \frac{2I_{d1}}{C_{ox}\mu_n(W/L)_1}$$

$$V_{gs2} = V_{Tn} + \frac{2I_{d2}}{C_{ox}\mu_n(W/L)_2}$$

Rearranging, with identical transistors:

$$\Delta V_{in} = V_{in1} - V_{in2} = V_{gs1} - V_{gs2}$$

$$\Delta V_{in} = \frac{\sqrt{I_{d1}} + \sqrt{I_{d2}}}{\sqrt{\frac{C_{ox}\mu_n}{2} \left( \frac{W}{L} \right)}}$$

We also know: $I_{d1} + I_{d2} = I_{tail}$

Plugging in, rearranging, and using the quadratic formula, we get:

$$I_{d1} = \frac{I_{tail}}{2} \pm \frac{C_{ox}\mu_n}{4} \left( \frac{W}{L} \right) (\Delta V_{in}) \sqrt{\frac{4I_{tail}}{C_{ox}\mu_n(W/L)} - (\Delta V_{in})^2}$$
Similarly:

\[ I_{d2} = \frac{I_{\text{tail}}}{2} \left( 3 \frac{C_{ox} \mu_n}{L} \right) \left( \frac{W}{L} \right) (\Delta V_{in}) \sqrt{\frac{4I_{\text{tail}}}{C_{ox} \mu_n \left( \frac{W}{L} \right)}} - (\Delta V_{in})^2 \]

\[ \Delta V_{out} = (V_{DD} - I_{d1} R_D) - (V_{DD} - I_{d2} R_D) \]

\[ \Delta V_{out} = -(I_{d1} - I_{d2}) R_D \]

\[ \Delta V_{out} = -R_D \left( \frac{C_{ox} \mu_n}{2} \left( \frac{W}{L} \right) (\Delta V_{in}) \sqrt{\frac{4I_{\text{tail}}}{C_{ox} \mu_n \left( \frac{W}{L} \right)}} - (\Delta V_{in})^2 \right) \]

Notice that the voltages are not being offset through this differential amplifier, so that these stages can be directly coupled, without problems with the bias voltages, a big advantage.