Your Name: ______________________________
Student ID Number: ________________________

Guidelines

Closed book and notes; there are some useful formulas in the end of the exam.
You may use a calculator.
You can unstaple the pages with formulas, but do not unstaple the exam.
Show all your work and reasoning on the exam in order to receive full or partial credit.
Time: 80 minutes = 1 hour, 20 minutes.

Score

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points Possible</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
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<tr>
<td>2</td>
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<td>3</td>
<td>18</td>
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<td>Total</td>
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1. MOS current source [20 points]
For the current source shown in Figure 1, dimensions of MOS transistors M1 and M2 are \((W/L)_1 = 2\), \((W/L)_2 = 10\). \(V_{DD} = 2.5\) V. \(\mu_n C_{ox} = 100\) \(\mu\)A/V^2, \(V_{Tn} = 0.5\) V, \(\lambda_n = 0.05\) V^{-1}, \(\gamma = 0\).

a) [4 points] Find the value of \(R_{REF}\) such that \(I_{REF} = 100\mu\)A. You can ignore channel length modulation in this part.

\[ R_{REF} = \]
(b) [4 points] Find the value of $R_1$ that gives $I_{OUT} = 40\mu A$? Assume that $M_2$ is in saturation. You can ignore channel length modulation in this calculation.

$$R_1 =$$

(c) [4 points] Find the lowest output voltage $v_{OUT}$ for which the circuit in Figure 1 still acts as a current source. You can ignore channel length modulation.

$$v_{OUT, \text{min}} =$$
(d) [6 points] Find the small-signal output resistance of the current source in Figure 1.
(e) [2 points] If body effect parameter, \( \gamma > 0 \), would it increase or decrease the value of output resistance from part (d)? Explain your answer.

\[ R_{out} \text{ increases / decreases (circle one)} \]
2. MOS amplifiers [14 pts]
For the MOS amplifier in Figure 2, \((W/L)_1 = 10\), \((W/L)_2 = 20\), \((W/L)_3 = 10\), \(I_{\text{BIAS}} = 50\mu\text{A}\).
\(V_{DD} = 2.5\text{V}\). \(\mu_nC_{ox} = 100\ \mu\text{A/V}^2\), \(\mu_pC_{ox} = 30\ \mu\text{A/V}^2\), \(V_{Tn} = V_{Tp} = 0.5\ \text{V}\), \(\lambda_n = \lambda_{np} = 0.05\text{V}^{-1}\), \(\gamma = 0\). \(C_{GS2} = 2C_{GS1} = 2C_{GS3} = 100\text{fF}\). \(C_{GD2} = 2C_{GD1} = 2C_{GD3} = 10\text{fF}\).
Input voltage \(v_S\) has negligible input resistance and contains a DC and an AC component.

Figure 2.

(a) [2 points] Find the bias current of transistor \(M_1\).

\[I_{M1} = \]
(b) [4 points] Find the small-signal voltage gain $A_v = \frac{v_{out}}{v_s}$.
(c) [4 points] Find the maximum and the minimum voltage at the output of this amplifier.

\[
V_{\text{out, max}} = \quad \text{V}; \quad V_{\text{out, min}} = \quad \text{V}
\]

(d) [4 points] Find the frequency of the dominant pole of this amplifier.

\[
\omega = \quad \text{rad/s}
\]
3. Amplifier frequency response [18 points]
An amplifier has all of its poles and zeros in the left-hand frequency plane (it is a stable, minimum-phase system) and an amplitude frequency response, as shown in Figure 3.

<table>
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<th>Figure 3</th>
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a) [6 points] Write the transfer function that produces this response.

b) [6 points] Draw the phase response that corresponds to this amplitude response.
(c) [6 points] Write the transfer function that produces the response from Figure 4.

Figure 4.
Some equations

Mass-action law \( n \times p = n_i^2(T) \)

Resistivity:
\[
\rho_n = \frac{1}{\sigma_n} = \frac{1}{q \mu_n N_d \text{eff}}
\]

Resistance:
\[
R = \frac{\rho L}{W t} = \left( \frac{\rho}{t} \right) \left( \frac{L}{W} \right) = R_{sq} \left( \frac{L}{W} \right)
\]

Total current (\( e^- \)):
\[
J = J_{drift} + J_{diff} = q \mu_n n E + q D_n \frac{dn}{dx}
\]

Gauss’s law:
\[
\oint E \cdot dS = \frac{Q}{\epsilon} \quad Q = CV \quad E = -\frac{d\phi}{dx}
\]

Depletion layer:
\[
X_{d0} = X_{p0} + X_{n0} = \sqrt{\frac{2 \epsilon_s \phi_{bi} \left( 1 + \frac{1}{N_d} \right)}{q \left( N_a + \frac{1}{N_d} \right)}} \quad X_d(V_D) = X_{d0} \sqrt{1 - \frac{V_D}{\phi_{bi}}}
\]

pn depletion layer capacitance:
\[
C_j = \frac{C_{j0}}{2 \phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_{bi}}}}
\]

pn diffusion current \( J_{diff} \):
\[
J_{diff} = q n_i^2 \left( \frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p} \right) \left( \frac{q V_D}{e kT} - 1 \right) i_D = I_S \left( \frac{q V_D}{e kT} - 1 \right)
\]

Diffusion capacitance:
\[
C_d = \frac{1}{2} \frac{q l_D}{kT} \tau
\]
Threshold voltage (NMOS)
\[ V_{Tn} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}} \sqrt{2q\varepsilon_s N_A (-2\phi_p)} \]
\[ \phi_p = -\frac{kT}{q} \ln \frac{N_A}{n_i} \]
\[ V_{Tn} = V_{Tn0} + \gamma \left( \sqrt{V_{SB} - 2\phi_p} - \sqrt{-2\phi_p} \right) \]

NMOS equations:
\[ i_D = \frac{W}{L} \mu C_{ox} \left( v_{GS} - V_{Tn} - \frac{V_{DS}}{2} \right) v_{DS} (1 + \lambda V_{DS}), \quad V_{GS} > V_{Tn}, V_{DS} < V_{GS} - V_{Tn} \]
\[ i_D = \frac{W}{L} \mu C_{ox} \left( v_{GS} - V_{Tn} \right)^2 (1 + \lambda V_{DS}), \quad V_{GS} > V_{Tn}, V_{DS} > V_{GS} - V_{Tn} \]

MOS capacitances in saturation \quad \[ C_{gs} = \frac{2}{3} W L C_{ox} + C_{ov} \quad C_{ov} = L D W C_{ox} \]

MOS signal parameters:
\[ g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{V_{DS}} = \mu C_{ox} \frac{W}{L} (v_{GS} - V_{Tn}) (1 + \lambda V_{DS}) \approx \mu C_{ox} \frac{W}{L} (v_{GS} - V_{Tn}) \]
\[ r_0 = \left. \left( \frac{\partial i_D}{\partial v_{DS}} \right) \right|_{V_{GS}}^{-1} \approx \frac{1}{\lambda J_{DS}} \]
\[ g_{mb} = \left. \frac{\partial i_D}{\partial v_{BS}} \right|_Q = \frac{\gamma g_m}{2 \sqrt{-V_{BS} - 2\phi_p}} \]

Power: \[ \langle P \rangle = \frac{|V|^2}{2} \cos(\angle I - \angle V) = \text{Re} \{ V \cdot V^* \} \]

Canonic transfer function:
\[ H(\omega) = G_0(j\omega)^K \frac{(1 + j\omega \tau_1)(1 + j\omega \tau_2) \cdots (1 + j\omega \tau_n)}{(1 + j\omega \tau_p)(1 + j\omega \tau_p) \cdots (1 + j\omega \tau_{pm})} \]

Bode plots
Amplitude
\[ |H(j\omega)|_{dB} = 20 \log|G_0(j\omega)^K \frac{(1 + j\omega \tau_1)(1 + j\omega \tau_2) \cdots (1 + j\omega \tau_n)}{(1 + j\omega \tau_p)(1 + j\omega \tau_p) \cdots (1 + j\omega \tau_{pm})}| \]
Phase
\[ \angle H(j\omega) = \angle G_0(j\omega)^K \frac{(1 + j\omega \tau_1)(1 + j\omega \tau_2) \cdots (1 + j\omega \tau_n)}{(1 + j\omega \tau_p)(1 + j\omega \tau_p) \cdots (1 + j\omega \tau_{pm})} \]

Second order TF:
\[ H(j\omega) = \frac{N(j\omega)}{\omega_0^2 + (j\omega)^2 + j\omega \omega_0} \frac{Q}{Q} \]

LC:
\[ \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{\sqrt{LC}}{C R} = \frac{L}{C R} = \frac{Z_0}{R} \quad Q = \frac{1}{2\zeta} \quad \omega_0^2 = \frac{1}{LC} \quad \tau = \frac{1}{\omega_0} \]