BJT Biasing

In the following discussion, we will go over how to design a BJT biasing circuit. The analysis is similar to MOSFETS however there are important differences. The topology shown in figure 1 is designed with the following features:

1) $I_1 \gg I_B$ to lower sensitivity to $\beta$.
2) $V_{RE}$ must be large enough to suppress uncertainties in $V_X$ and $V_{BE}$.

![Figure 1: Robust BJT biasing circuit](image)

Problem
a) Design the circuit in figure 1 so as to provide $V_{RE} = 1V$ and $I_1 \geq 100I_B$. Assume $V_{CC}=5V$, $\beta=100$, and $I_S = 5e-17 A$, and $I_C = 1mA$.
b) Find the input impedance
c) Find the output impedance
d) Find the small signal gain,
a) Design the circuit in figure 1 so as to provide \( V_{RE} = 1V \) and \( I_1 \geq 100I_B \). Assume \( V_{CC} = 5V \), \( \beta = 100 \), and \( I_S = 5e-17 \, A \), and \( I_C = 1mA \).

Find the small signal parameters \( g_m \) and \( r_\pi \).

\[
g_m = \frac{I_C}{V_T} = \frac{1mA}{26mV} = 0.0385S
\]

\[
r_\pi = \frac{\beta}{g_m} = 2600\Omega
\]

Find the values of resistors \( R_1, R_2, R_C, \) and \( R_E \).

\( R_E \): Since we want an output of \( V_{RE} = 1V \), and the current \( I_C = 1mA \), we can calculate the value of \( R_E \).

\[
\frac{V_{RE}}{R_E} = I_E \approx 1mA
\]

\[
R_E = 1000\Omega
\]

\( R_C \): The value of \( R_C \) will be needed to put the transistor in the active region. Recall that active region is defined when \( V_C > V_B \). In order find the minimum \( V_C \), let’s find the value of \( V_X \).

\[
I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)
\]

Using equation 5, we can calculate \( V_{BE} = 0.797V \).

\[
V_X = V_{BE} + V_{RE} = 0.797V + 1V = 1.797V
\]

To be in active region, \( V_C > V_B = V_X \). Now let’s find the maximum \( R_C \) to set the device in active region.

\[
V_C = V_{CC} - I_C R_C
\]

\[
V_{CC} - I_C R_C > V_X = 1.797V
\]

\[
R_C < 3203\Omega
\]

Using equation 7, and plugging it into the boundary condition for active region in equation 8, we can find that the maximum \( R_C \) to keep the device in active region is 3203\( \Omega \). As long as we pick an \( R_C \) above the maximum value, the circuit will work properly. Therefore, we will pick
\[ R_C = 3000\Omega \]  \hspace{1cm} (10)

**R1 & R2:** The value of \( R_1 \) and \( R_2 \) will be determined by the requirement of \( I_I = 100I_B \) and \( V_X \).

\[
\left( \frac{R_2}{R_1 + R_2} \right)V_{CC} = V_X
\]

\hspace{1cm} (11)

\[
I_I = \left( \frac{V_{CC}}{R_1 + R_2} \right) = 100I_B
\]

\hspace{1cm} (12)

\[
I_B = \frac{I_C}{\beta} = 10\mu A
\]

\hspace{1cm} (13)

We can use the voltage divider equation to solve for \( V_X \) as in equation 11. Also, we can use the condition \( I_C = 100I_B \) as in equation 12. Now we have 2 equations and 2 unknowns, solving for \( R_1 \) and \( R_2 \), we get

\[
R_1 = 3203\Omega
\]

\hspace{1cm} (14)

\[
R_2 = 1797\Omega
\]

\hspace{1cm} (15)

**b) Find the input impedance**

![Diagram](image)

**Figure 2: Small Signal Model for finding Rin**

Instead of finding the input impedance by the small signal given in figure 2, we can notice that \( R_1 \) and \( R_2 \) are in parallel with a CE stage with emitter degeneration. The input impedance of an emitter degenerated CE stage is:

\[
r_\pi + (1 + \beta)R_E
\]

\hspace{1cm} (16)

Equation 16 comes from noticing that the current through \( R_E \) is amplified by the current gain, \( \beta \). Therefore the input impedance is enhanced. For more information, refer to the bottom of page 198 in Razavi.
Now, we can easily read off the input impedance as,

\[ R_{in} = R_i \| R_z \| (r_\pi + (1 + \beta)R_E) = 1138.5 \Omega \]  

(17)

c) **Find the output impedance.**

To find the output impedance, we must zero out the source. As a result, the current source also zeros out resulting in an output impedance of

\[ R_{out} = R_C = 300 \Omega \]  

(18)

For more information, refer to page 199 in Razavi.

d) **Find the small signal gain**

From the current loop at the output,

\[ g_m v_\pi = -\frac{v_{out}}{R_C} \]  

(19)

Obtaining,

\[ v_\pi = -\frac{v_{out}}{g_m R_C} \]  

(20)

Applying KVL,

\[ V_t = v_\pi + v_{re} \]  

(21)

\[ V_t = v_\pi + \left(\frac{v_\pi}{r_\pi} + g_m v_\pi\right)R_E \]  

(22)

Substituting equation 20 into equation 22, we can get,

\[ \frac{V_{out}}{V_t} = \frac{g_m R_C}{1 + \left(\frac{1}{r_\pi} + g_m\right)R_E} \approx -\frac{R_C}{1 + \frac{1}{g_m} + R_E} \]  

(23)

\[ \frac{V_{out}}{V_t} = 4.87 \]  

(24)