Carrier Concentration and Potential

- At thermal equilibrium, there are no external bias and we expect both the electron and the hole current densities to be zero:

\[ J_n = 0 = qn_0 \mu_n E_o + qD_n \frac{dn_n}{dx} \]

\[ \frac{dn_n}{dx} = -\left( \frac{\mu_n}{D_n} \right) n_n E_o = -\left( \frac{q}{kT} \right) n_o \left( -\frac{d\phi_0}{dx} \right) \]

\[ d\phi_0 = \left( \frac{kT}{q} \right) \frac{dn_n}{n_0} = V_{th} \frac{dn_n}{n_0} \]

Carrier Concentration and Potential (2)

- If we integrate the above equation we have

\[ \phi_n(x) - \phi_n(x_0) = V_{th} \ln \frac{n_n(x)}{n_n(x_0)} \]

- We define the potential reference at intrinsic Si:

\[ \phi_n(x_0) = 0 \quad n_n(x_0) = n_i \]

\[ \phi_n(x) = V_{th} \ln \frac{n_n(x)}{n_i} \]

Carrier Concentration Versus Potential

- The carrier concentration is thus a function of potential

\[ n_n(x) = n_i e^{\phi_n(x)/V_{th}} \]

- If we do a similar calculation for holes, we arrive at a similar equation

\[ p_0(x) = n_i e^{-\phi_0(x)/V_{th}} \]

- Note that the law of mass action is upheld

\[ n_n(x) p_0(x) = n_i^2 e^{-\phi_n(x)/V_{th}} e^{\phi_0(x)/V_{th}} = n_i^2 \]
Doping Changes Potential

\[ \phi_d(x) = V_{th} \ln \frac{n_d(x)}{n_i(x_0)} = 26 \text{mV} \ln \frac{n_d(x)}{n_i(x_0)} \approx 26 \text{mV} \cdot (\ln 10) \cdot \log \frac{n_d(x)}{10^{10}} \]

\[ \phi_i(x) \approx 60 \text{mV} \log \frac{n_i(x)}{10^{10}} \]

\[ \phi_n(x) \approx -60 \text{mV} \log \frac{p_n(x)}{10^{10}} \]

- Due to the log nature of the potential, the potential changes linearly for exponential increase in doping:
- Quick calculation aid:
  - For a p-type concentration of $10^{16}$ cm$^{-3}$, the potential is -360 mV
  - N-type materials have a positive potential with respect to intrinsic Si

PN Junction (Diode)

- When N-type and P-type dopants are introduced side-by-side in a semiconductor, a PN junction or a diode is formed.

PN Junction: Overview

- Present in most IC structures

Diode’s Three Operation Regions

- In order to understand the operation of a diode, it is necessary to study its three operation regions: equilibrium, reverse bias, and forward bias.
Current Flow Across Junction: Diffusion

- Because each side of the junction contains an excess of holes or electrons compared to the other side, there exists a large concentration gradient. Therefore, a diffusion current flows across the junction from each side.

Depletion Region

- As free electrons and holes diffuse across the junction, a region of fixed ions is left behind. This region is known as the “depletion region.”

Depletion Approximation

- Let’s assume that the depletion region is completely free of electrons or holes (only immobile ions exist)
- Then the charge density is given by
  \[ \rho_0(x) \equiv \begin{cases} +qN_d, & -x_{m0} < x < 0 \\ -qN_a, & 0 < x < x_{p0} \end{cases} \]
- Electric field can be calculated by Gauss Law
  \[ E_0(x) = \int_{-x_{m0}}^{x} \frac{\rho_0(x')}{\varepsilon_s} \, dx' + E_0(-x_{m0}) \]

Depletion Approximation (2)

- Since charge density is a constant
  \[ E_0(x) = \int_{-x_{m0}}^{x} \frac{\rho_0(x')}{\varepsilon_s} \, dx' = \frac{qN_d}{\varepsilon_s} (x + x_{m0}) \]
- If we start from the P-side we get the following result
  \[ E_0(x_{p0}) = \int_{x_{m0}}^{x_{p0}} \frac{\rho_0(x')}{\varepsilon_s} \, dx' + E_0(x) = \frac{qN_d}{\varepsilon_s} (x_{p0} - x) + E_0(x) \]
Electric Field Distribution

- E-Field is zero outside depletion region
- The depletion widths on N- and P-side could be asymmetric
- Higher doping → Narrower depletion width
- Peak E-Field occurs at junction

\[ E_0(x) = \frac{qN_d}{\varepsilon_s} (x + x_{no}) \]

Potential Across Junction

- The potential in the N-region is higher than P-region
- The potential has to smoothly transition form high to low in crossing the junction
- Physically, the potential difference is due to the charge transfer that occurs due to the concentration gradient
- We can integrate the field to get the potential:

\[ \phi(x) = \phi(-x_{no}) - \int_{-x_{no}}^{x} \frac{qN_d}{\varepsilon_s} (x' + x_{no}) dx' \]

\[ \phi(x) = \phi_n - \frac{qN_d}{2\varepsilon_s} (x + x_{no})^2 \]

Direction of Current Components Across Junction

- Electron Drift Current
- Electron Diffusion Current
- Hole Drift Current
- Hole Diffusion Current

\[ J_n = qn \mu_n E_0 + qD_n \frac{dn}{dx} \]

\[ J_p = qp \mu_p E_0 - qD_p \frac{dp}{dx} \]

Potential Across Junction

- Potential on N-side (parabolic)

\[ \phi_n(x) = \phi_n - \frac{qN_d}{2\varepsilon_s} (x + x_{no})^2 \]

- Do integral on P-side

\[ \phi_p(x) = \phi_p + \frac{qN_a}{2\varepsilon_s} (x - x_{po})^2 \]

- Potential must be continuous at interface

\[ \phi_n(0) = \phi_n - \frac{qN_d}{2\varepsilon_s} x_{no}^2 \]

\[ = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{po}^2 = \phi_p(0) \]
Solve for Depletion Widths

- We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

\[ \phi_n - \frac{qN_d}{2\varepsilon_s}x_n^2 = \phi_p + \frac{qN_a}{2\varepsilon_s}x_p^2 \]  
\[ (1) \]

\[ qN_a x_n = qN_d x_p \]  
\[ (2) \]

\[ x_{no} = \sqrt{\frac{2\varepsilon_s\phi_b}{qN_d} \left( \frac{N_a}{N_a + N_d} \right)} \]

\[ x_{po} = \sqrt{\frac{2\varepsilon_s\phi_b}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)} \]

\[ \phi_b \equiv \phi_n - \phi_p > 0 \]

Total Depletion Width

- Total depletion width

\[ X_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon_s\phi_b}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} \]

- Due to high electric field, carriers move across region at saturation velocity

- Typical numbers:

\[ X_{d0} \approx \sqrt{\frac{2\varepsilon_s\phi_b}{q} \left( \frac{1}{10^{15}} \right)} \approx 1 \mu \]

\[ E_{pm} \approx \frac{1V}{1 \mu} = 10^4 \frac{V}{cm} \]

Current Flow Across Junction: Equilibrium

- At equilibrium, the drift current flowing in one direction cancels out the diffusion current flowing in the opposite direction, creating a net current of zero.

- The figure shows the charge profile of the PN junction.

Built-in Potential

- The potential difference between the N-side and the P-side:

\[ \phi_0(x) = V_{th} \ln \frac{n_i(x)}{n_i} \]

\[ \phi_n = V_{th} \ln \frac{N_d}{n_i} \quad \phi_p = V_{th} \ln \frac{N_a}{n_i} = V_{th} \ln \frac{n_i}{N_a} \]

\[ V_0 = \phi_b = \phi_n - \phi_p = V_{th} \ln \frac{N_d N_a}{n_i^2} \]

\[ \phi_b = 60mV \cdot \log_{10} \frac{N_d N_a}{n_i^2} \]
Have We Invented a Battery?

- Can we harness the PN junction and turn it into a battery?
  \[ \phi_{bi} = \phi_n - \phi_p = V_{th} \left( \ln \frac{N_D}{n_i} + \ln \frac{N_A}{n_i} \right) = V_{th} \ln \frac{N_D N_A}{n_i^2} \]

- Numerical example:
  \[ \phi_{be} = 26 \text{mV} \ln \frac{N_D N_A}{n_i^2} = 60 \text{mV} \times \log \frac{10^{15} \times 10^{15}}{10^{20}} = 600 \text{mV} \]

Contact Potential

- The contact between a PN junction creates a potential difference
- When a metal-semiconductor junction is formed, a contact potential forms as well
- If we short a PN junction, the sum of the voltages around the loop must be zero:

Diode in Reverse Bias

- When the N-type region of a diode is connected to a higher potential than the P-type region, the diode is under reverse bias, which results in wider depletion region and larger built-in electric field across the junction.

Reverse-Biased Diode’s Application: Voltage-Dependent Capacitor

- The PN junction can be viewed as a capacitor. By varying \( V_{R} \), the depletion width changes, changing its capacitance value; therefore, the PN junction is actually a voltage-dependent capacitor.
Voltage Dependence of Depletion Width

- The expression of depletion width under reverse width is similar to that at thermal equilibrium except we replace the built-in potential with the effective reverse bias:

\[
x_a(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_a}} \left(\frac{N_a}{N_a + N_d}\right) = x_d\sqrt{1 - \frac{V_D}{\phi_{bi}}}
\]

\[
x_p(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_a}} \left(\frac{N_d}{N_a + N_d}\right) = x_p\sqrt{1 - \frac{V_D}{\phi_{bi}}}
\]

\[
X_d(V_D) = x_p(V_D) + x_a(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{q}} \left(\frac{1}{N_a + \frac{1}{N_d}}\right)
\]

\[
X_d(V_D) = x_d\sqrt{1 - \frac{V_D}{\phi_{bi}}}
\]

Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

\[
Q_J(V_D) = -qN_a x_p(V_D) = -qN_a x_p 0 \sqrt{1 - \frac{V_D}{\phi_{bi}}}
\]

- Charge is not a linear function of voltage

\[\rightarrow\] This is a nonlinear capacitor

- We can define a small-signal capacitance by breaking up the charge into a DC and an AC terms:

\[
Q_J(V_D + v_D) = Q_J(V_D) + q(v_D)
\]

Derivation of Small Signal Capacitance

\[
Q_J(V_D + v_D) = Q_J(V_D) + \frac{dQ_J}{dV} \bigg|_{v_D} v_D + \cdots
\]

\[
C_J = C_J(V_D) = \frac{dQ_J}{dV} \bigg|_{v_D} = \frac{d}{dV} \left(-qN_a x_p 0 \sqrt{1 - \frac{V_D}{\phi_{bi}}}\right) \bigg|_{v_D = 0}
\]

\[
C_J = \frac{qN_a x_p 0}{2\phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}}} = \frac{C_J 0}{C_J 0} = \frac{qN_a x_p 0}{2\phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}}}
\]

Physical Interpretation of Depletion Cap

\[
C_J 0 = \frac{q\varepsilon_s N_a N_d}{2\phi_{bi} N_a + N_d}
\]

- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium

\[
C_J 0 = \varepsilon_s \sqrt{\frac{q}{2\varepsilon_s \phi_{bi}} \left(\frac{1}{N_a} + \frac{1}{N_d}\right)^{-1}} = \frac{\varepsilon_s}{X_d 0}
\]

- This looks like a parallel plate capacitor!

\[
C_J(V_D) = \frac{\varepsilon_s}{X_d(V_D)}
\]
A Variable Capacitor (Varactor)

The equations that describe the voltage-dependent capacitance are shown above.

\[ C_f = \frac{C_{j0}}{1 + \frac{V}{V_0}} \]

\[ C_{j0} = \frac{\varepsilon q N_A N_D}{2 (N_A + N_D) V_0} \]

Voltage-Controlled Oscillator (VCO)

A very important application of a reverse-biased PN junction is VCO, in which an LC tank is used in an oscillator. By changing \( V_R \), we can change \( C \), which also changes the oscillation frequency.

\[ f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \]

Forward-Biased Diode

When the N-type region of a diode is at a lower potential than the P-type region, the diode is in forward bias.

The depletion width is reduced and the built-in electric field decreased.

Minority Carrier Profile in Forward Bias

Under forward bias, minority carriers in each region increase due to the lowering of built-in field/potential. Therefore, diffusion currents increase to supply these minority carriers.
Diffusion Current in Forward Bias

\[
\Delta n_p \approx \frac{N_D}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right) \quad \Delta p_n \approx \frac{N_A}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right)
\]

\[
I_{D\text{c}} \propto \frac{N_D}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right) + \frac{N_A}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right)
\]

\[
I_{D\text{c}} = I_s \left( \exp \frac{V_F}{V_T} - 1 \right)
\]

- Diffusion current will increase in order to supply the increase in minority carriers. The mathematics are shown above.

Minority Charge Gradient

- Minority charge profile should not be constant along the x-axis; otherwise, there is no concentration gradient and no diffusion current.
- Recombination of the minority carriers with the majority carriers accounts for the dropping of minority carriers as they go deep into the P or N region.

Forward Bias Condition: Summary

- In forward bias, there are large diffusion currents of minority carriers through the junction. However, as we go deep into the P and N regions, recombination currents from the majority carriers dominate. These two currents add up to a constant value.

IV Characteristic of PN Junction

- The current and voltage relationship of a PN junction is exponential in forward bias region, and relatively constant in reverse bias region.
Diode Small-Signal Model

- The I-V relation of a diode can be linearized

\[ I_D + i_D = I_s \left( e^{\frac{q(V_D + v_d)}{kT}} - 1 \right) \approx I_s e^{\frac{qV_D}{kT}} e^{\frac{qv_d}{kT}} \]

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \]

\[ I_D + i_D \approx I_D \left( 1 + \frac{q(V_D + v_d)}{kT} + \cdots \right) \]

\[ i_D = I_D \frac{qv_d}{kT} \]

Small-Signal Resistance (or Dynamic Resistance)

\[ r_D = \frac{v_d}{i_D} = kT \frac{1}{q I_D} = \frac{1}{r_T} \]

Parallel PN Junctions

- Since junction currents are proportional to the junction’s cross-section area. Two PN junctions put in parallel are effectively one PN junction with twice the cross-section area, and hence twice the current.

Constant-Voltage Diode Model

- Diode operates as an open circuit if VD < VD, on and a constant voltage source of VD, on if VD tends to exceed VD, on.

Example: Diode Calculations

\[ I_X = I_X R_I + V_D = I_X R_I + V_T \ln \left( \frac{I_X}{I_S} \right) \]

\[ I_X = 2.2mA \quad \text{for} \quad V_X = 3V \]

\[ I_X = 0.2mA \quad \text{for} \quad V_X = 1V \]

- This example shows the simplicity provided by a constant-voltage model over an exponential model.
- For an exponential model, iterative method is needed to solve for current, whereas constant-voltage model requires only linear equations.
Reverse Breakdown

- When a large reverse bias voltage is applied, breakdown occurs and an enormous current flows through the diode.

Zener vs. Avalanche Breakdown

- Zener breakdown is a result of the large electric field inside the depletion region that breaks electrons or holes off their covalent bonds.
- Avalanche breakdown is a result of electrons or holes colliding with the fixed ions inside the depletion region.