Cascode Stages and Current Mirrors

- 9.1 Cascode Stage
- 9.2 Current Mirrors

PMOS Cascode Stage

\[ R_{out} = \left(1 + g_{m1}r_{O2}\right)r_{O1} + r_{O2} \]

\[ R_{out} \approx g_{m1}r_{O1}r_{O2} \]

Example: Parasitic Resistance

\[ R_{out} = (1 + g_{m1}r_{O2})(r_{O1} || R_p) + r_{O2} \]

- \( R_p \) will lower the output impedance, since its parallel combination with \( r_{O1} \) will always be lower than \( r_{O1} \).
Short-Circuit Transconductance

- The short-circuit transconductance of a circuit measures its strength in converting input voltage to output current.

\[ G_m = \left. \frac{i_{out}}{v_{in}} \right|_{v_{out}=0} \]

Transconductance Example

\[ G_m = g_{m1} \]

Derivation of Voltage Gain

- By representing a linear circuit with its Norton equivalent, the relationship between \( V_{out} \) and \( V_{in} \) can be expressed by the product of \( G_m \) and \( R_{out} \).

\[ v_{out} = -i_{out} R_{out} = -G_m v_{in} R_{out} \]

\[ \frac{v_{out}}{v_{in}} = -G_m R_{out} \]

MOS Cascode Amplifier

\[ A_v = -G_m R_{out} \]

\[ A_v \approx -g_{m1} \left[ (1 + g_{m2} r_{O2}) r_{O1} + r_{O2} \right] \]

\[ A_v \approx -g_{m1} r_{O1} g_{m2} r_{O2} \]
**Improved MOS Cascode Amplifier**

Similar to its bipolar counterpart, the output impedance of a MOS cascode amplifier can be improved by using a PMOS cascode current source.

\[
R_{on} \approx g_{m2}r_{O2}r_{O1} \\
R_{op} \approx g_{m3}r_{O3}r_{O4} \\
R_{out} = R_{on} || R_{op}
\]

**Temperature and Supply Dependence of Bias Current**

Since \(V_T, I_S, \mu_n, \) and \(V_{TH}\) all depend on temperature, \(I_1\) for both bipolar and MOS depends on temperature and supply.

\[
I_1 = \frac{1}{2} \mu_n C_w \left( \frac{R_2}{R_1 + R_2} \right) \left( \frac{V_{DD} - V_{TH}}{-V_{BE}} \right)^2
\]

**Concept of Current Mirror**

The motivation behind a current mirror is to sense the current from a “golden current source” and duplicate this “golden current” to other locations.

**MOS Current Mirror**

The same concept of current mirror can be applied to MOS transistors as well.
Bad MOS Current Mirror Example

- This is not a current mirror since the relationship between \( V_x \) and \( I_{REF} \) is not clearly defined.
- The only way to clearly define \( V_x \) with \( I_{REF} \) is to use a diode-connected MOS since it provides square-law I-V relationship.

Example: Current Scaling

- Similar to their bipolar counterpart, MOS current mirrors can also scale \( I_{REF} \) up or down (\( I_1 = 0.2 \text{mA}, I_2 = 0.5 \text{mA} \)).

CMOS Current Mirror

- The idea of combining NMOS and PMOS to produce CMOS current mirror is shown above.

Skip this part until we cover bipolar transistors
Boosted Output Impedances

\[ R_{out1} = \left[ 1 + g_m (R_E \parallel r_{\pi}) \right] r_O + R_E \parallel r_{\pi} \]
\[ R_{out2} = (1 + g_m R_S) r_O + R_S \]

Bipolar Cascode Stage

\[ R_{out} = [1 + g_m (r_{O2} \parallel r_{\pi1})] r_{O1} + r_{O2} \parallel r_{\pi1} \]
\[ R_{out} \approx g_m r_{O1} (r_{O2} \parallel r_{\pi1}) \]

Maximum Bipolar Cascode Output Impedance

\[ R_{out,max} \approx g_m r_{O1} r_{\pi1} \]
\[ R_{out,max} \approx \beta_1 r_{O1} \]

- The maximum output impedance of a bipolar cascode is bounded by the ever-present \( r_{\pi} \) between emitter and ground of Q1.

Example: Output Impedance

- Typically \( r_{\pi} \) is smaller than \( r_O \), so in general it is impossible to double the output impedance by degenerating Q2 with a resistor.
### PNP Cascode Stage

\[ R_{out} = [1 + g_m (r_{o2} \parallel r_{\pi1})] r_{o1} + r_{o2} \parallel r_{\pi1} \]
\[ R_{out} \approx g_m r_{o1} (r_{o2} \parallel r_{\pi1}) \]

### Another Interpretation of Bipolar Cascode

\[ R_{out} = r_{o2} \]

### False Cascodes

- When the emitter of Q₁ is connected to the emitter of Q₂, it’s no longer a cascode since Q₂ becomes a diode-connected device instead of a current source.

\[ R_{out} = \left[ 1 + g_m \left( \frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{\pi2} \right) \right] r_{o1} + \frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{\pi1} \]
\[ R_{out} \approx \left( 1 + \frac{g_m}{g_{m2}} \right) r_{o1} + \frac{1}{g_{m2}} \approx 2r_{o1} \]

### MOS Cascode Stage

\[ R_{out} = \left( 1 + g_m r_{o2} \right) r_{o1} + r_{o2} \]
\[ R_{out} \approx g_m r_{o1} r_{o2} \]
Another Interpretation of MOS Cascode

- Similar to its bipolar counterpart, MOS cascode can be thought of as stacking a transistor on top of a current source.
- Unlike bipolar cascode, the output impedance is not limited by $\beta$.

Example: Voltage Gain

$A_v = -g_{m1}r_{o1}$

Comparison between Bipolar Cascode and CE Stage

- Since the output impedance of bipolar cascode is higher than that of the CE stage, we would expect its voltage gain to be higher as well.

Voltage Gain of Bipolar Cascode Amplifier

$G_m \approx g_{m1}$
$A_v \approx -g_{m1}r_{o1}g_{m1}(r_{o1} || r_{π2})$

- Since $r_o$ is much larger than $1/g_{m1}$, most of $I_{CQ1}$ flows into the diode-connected $Q_2$. Using $R_{out}$ as before, $A_v$ is easily calculated.
**Alternate View of Cascode Amplifier**

A bipolar cascode amplifier is also a CE stage in series with a CB stage.

**Practical Cascode Stage**

Since no current source can be ideal, the output impedance drops.

\[ R_{out} \approx r_{O3} \parallel g_{m2}r_{O2}(r_{O1} \parallel r_{\pi2}) \]

**Improved Cascode Stage**

In order to preserve the high output impedance, a cascode PNP current source is used.

\[ R_{out} \approx g_{m3}r_{O3}(r_{O4} \parallel r_{\pi3}) \parallel g_{m2}r_{O2}(r_{O1} \parallel r_{\pi2}) \]

**Bipolar Current Mirror Circuitry**

The diode-connected Q_{REF} produces an output voltage \( V_1 \) that forces \( I_{copy} = I_{REF} \) if \( Q_1 = Q_{REF} \).

\[ I_{copy} = \frac{I_{S1}}{I_{S,REF}} I_{REF} \]
Bad Current Mirror Example I

Without shorting the collector and base of $Q_{REF}$ together, there will not be a path for the base currents to flow, therefore, $I_{copy}$ is zero.

Bad Current Mirror Example II

Although a path for base currents exists, this technique of biasing is no better than resistive divider.

Multiple Copies of $I_{REF}$

Multiple copies of $I_{REF}$ can be generated at different locations by simply applying the idea of current mirror to more transistors.

Current Scaling

By scaling the emitter area of $Q_j$ $n$ times with respect to $Q_{REF}$, $I_{copy,j}$ is also $n$ times larger than $I_{REF}$. This is equivalent to placing $n$ unit-size transistors in parallel.
Example: Scaled Current

A fraction of $I_{REF}$ can be created on $Q_1$ by scaling up the emitter area of $Q_{REF}$.

Example: Different Mirroring Ratio

Using the idea of current scaling and fractional scaling, $I_{copy2}$ is 0.5mA and $I_{copy1}$ is 0.05mA respectively. All coming from a source of 0.2mA.

Mirroring Error Due to Base Currents

\[ I_{copy} = \frac{nI_{REF}}{1 + \frac{1}{\beta}(n+1)} \]
Improved Mirroring Accuracy

Because of $Q_F$, the base currents of $Q_{REF}$ and $Q_1$ are mostly supplied by $Q_F$ rather than $I_{REF}$. Mirroring error is reduced $\beta$ times.

Example: Different Mirroring Ratio Accuracy

\[ I_{\text{copy}} = \frac{nI_{\text{REF}}}{1 + \frac{1}{\beta^2} (n+1)} \]

\[ I_{\text{copy1}} = \frac{I_{\text{REF}}}{4 + \frac{15}{\beta^2}} \]

\[ I_{\text{copy2}} = \frac{10I_{\text{REF}}}{4 + \frac{15}{\beta^2}} \]

PNP Current Mirror

PNP current mirror is used as a current source load to an NPN amplifier stage.

Generation of $I_{\text{REF}}$ for PNP Current Mirror
Example: Current Mirror with Discrete Devices

- Let $Q_{REF}$ and $Q_1$ be discrete NPN devices. $I_{REF}$ and $I_{copy1}$ can vary in large magnitude due to $I_S$ mismatch.