Small Signal Resistance

\[ i_D = I_s (e^{\frac{v_D}{V_T}} - 1) \approx I_s e^{\frac{v_D}{V_T}} \]

Small signal (ac signal)

\[ V_{DD} \]

DC circuit

Bias point \( Q \)

Tangent at \( Q \)

\[ \text{Current} \]

\[ \text{Voltage} \]

For small signal

Replace transfer curve with tangent

\[ \text{For small signal} \]

\[ \text{Replace transfer curve with tangent} \]
\[ I_D \]

\[ \text{Slope} = \frac{\Delta I_D}{\Delta U_D} = \frac{1}{R_d} \]

\[ R_d = \frac{\Delta U_D}{\Delta I_D} = \frac{1}{(\frac{dI_D}{dU_D})} = \frac{1}{I_D} = \frac{V_T}{I_D}. \]

\[ V_T = 26 \text{mV} \]

\[ I_D = I_s e^{\frac{V_T}{V_T}} \]

\[ \frac{dI_D}{dU_D} = I_s \cdot \frac{V_T}{V_T} \cdot e^{\frac{V_T}{V_T}} = \frac{1}{V_T} \cdot I_s \cdot e^{\frac{V_T}{V_T}} = \frac{I_D}{V_T} \]

Symbol convention

- \( I_D \): DC current
- \( i_D \): AC current

If diode is biased at \( I_D = 1 \text{mA} \)

\[ R_d = \frac{V_T}{I_D} = 26 \Omega \]

\[ RC = R_d \cdot C \]
Previously solved by iteration

\[
\begin{align*}
\text{DC solution:} & \quad V_b/V_t \\
\{ I_D &= I_s e^{V_b/V_t} \\
V_s &= I_D R + V_D \\
V_D &= V_b + V_d \\
\text{Small-signal, } & \quad |V_d| \ll |V_b| \\
\text{Total signal:} & \\
I_D &= I_s e^{V_b/V_t} \\
I_D &= I_s + I_d \\
V_D &= V_b + V_d \\
I_D + I_d &= I_s e^{V_b+V_d/V_t} = f(V_b+V_d) = f(V_b) + \left. \frac{df}{dV} \right|_{V=V_b} \cdot V_d \\
\frac{df}{dV} &\approx \frac{I_D}{V_T} \cdot V_d \\
\frac{V_d}{I_d} &= \frac{V_T}{I_D} = \text{small-signal resistance}
\end{align*}
\]
2-step Process

1. Solve DC.
   - e.g. $V_D = 0.72 \text{ V}$
   - $I_D = 1 \text{ mA}$

   ![DC Circuit Diagram]

2. Replace circuit with small-signal equivalent circuit
   - All DC voltage source $\rightarrow$ short circuit
   - All DC current source $\rightarrow$ open circuit
   - Replace diode with $r_d = \frac{V_t}{I_D}$

   ![Small-Signal Equivalent Circuit Diagram]

$$V_0 = \frac{r_d}{R + r_d} V_S$$

HW Problem

- Coupling capacitor, $C$ large
  - $Z_C = \frac{1}{j\omega C} \ll \text{ all resistance}$
  - $\Rightarrow$ Short circuit

$$V_{da} = \frac{r_d}{V_S \cdot R_s + r_d}$$

![HW Problem Diagram]
Photodiodes

"3rd Quadrant"

Normal forward current

Reverse bias

$i_{ph}$

$V$ light

$S_c$ $S_c = S_c$

Light create electron-hole pair

$\bar{i}_{ph} \propto$ light intensity

$\bar{i}_D = I_s (e^{\frac{V_D}{kT}} - 1) + \bar{i}_{ph}$

$i_D$

$V_D$

$V$ reverse-biased mode

$R = 1 \text{ kOhm}$

2 mW

4 mW

6 mW

8 mW

10 mW

current (mA)

voltage (V)
Solar (Photovoltaic, or PV) Cells

• Operating in the 4th quadrant of the I-V curve → It generates power!

• Key parameters:
  – Open circuit voltage, $V_{oc}$
  – Short-circuit current, $I_{sh}$
  – Fill factor

PV ~ 20%
\[ \leq 31\% \text{ for Sc PV} \]

⇒ EE134