Do "all work" on this exam.

Make your methods clear.

There are 3 problems.
Problem 1) : (30 points)
A plane wave of the form \( \hat{E} = \hat{A} e^{i\omega t - \hat{k} \hat{x}} \) travels through a material with a frequency dependent refractive index given by \( n(\omega) = n_0 + n_1(\omega - \omega_0) \).

a) What is the magnitude of \( \hat{k} \) in terms of the frequency \( \omega \), the speed of light \( c \), \( n_0 \), \( n_1 \), and \( \omega_0 \)?

b) A second plane wave of the form \( \vec{E} = \vec{A} e^{i\omega_1 t - \vec{k}_1 \cdot \vec{r}} \) is now simultaneously present. What is the magnitude of \( \vec{k}_1 \)?

c) What are the phase speeds of the two waves in a) and b)?

d) If \( \omega_1 - \omega \ll \omega \) what is the group speed in terms of the same parameters as in a) and what is it that travels at this speed?
Problem 2) : (30 points)

A simple model of a receiver is a detector (assume p-i-n) in parallel with a resistor of value $R$ and an ideal amplifier having a constant gain $G$ over the bandwidth of interest. It is assumed that the capacitance can be neglected.

a) If it is assumed that $R$ is the input resistance of the amplifier and the noise figure of the amplifier is $F_n$, what is the thermal plus amplifier noise observed at the output of the amplifier?

b) If the optical power is $P_o$, what is the shot noise observed at the output of the amplifier in terms of $P_o$ and the responsivity $R_s$ of the detector?

c) What is the signal to noise ratio at the output of the amplifier?

d) Show that the maximum signal to noise ratio is obtained in the shot noise limit.

d) For a simple amplitude shift keyed digital format for which no power is transmitted for the zero bit and Gaussian noise statistics are assumed, what is the bit error rate in terms of the error function?
Problem 3) (30 points) A diffraction limited laser with a (minimum) beam width equal to \( w \) at \( z = 0 \) has a message imposed upon it (it is modulated). It is transmitted a length \( L \) to a lens of diameter \( D \) which focuses it onto a detector having a diameter \( d \).

a) If the transmitted power from the laser is \( P_{\text{opt}} \), what fraction of \( P_{\text{opt}} \) strikes the detector? Give your answer in terms of the parameters of the problem.

b) What should \( d \) be to collect all the power striking the lens?

c) If an L.E.D. emitting in a cone angle of \( \frac{\lambda}{w} \) with the same power is used instead of the laser how much is the received power reduced? (For the L.E.D. the spectral width is of course much larger which further imposes limits which we neglect here)