Fiber Optical Amplifiers

In semiconductors, the atoms were excited into a higher state by absorbing energy from an applied current. A different excitation procedure is used in EDFAs. Erbium ions in glass can be excited to move from state 1 to state 3 by a light source (called the pump). In EDFAs designed to amplify 1,550 nm wavelength signals, the pump light usually has a wavelength of either 980 nm or 1,480 nm, and this light is coupled into the Erbium-doped fiber using a simple wave division multiplexer.

Amplification is similar to the optical amplification we saw in a semiconductor laser. The signal light shining on ions in unstable excited states stimulates them to fall back to their lower states and to emit additional light in the process. For very small signal levels, the amount of stimulated emission, and therefore the optical gain of the EDFA, is proportional to the difference in the number of excited state ions in state 2 compared to the number of ions in state 1 and is proportional to the intensity of the 1,550 nm signal light.

However, it is usual to operate EDFAs in a saturated gain mode where the gain is relatively independent of the input signal level.

\[
\text{cross-section} = \frac{\text{W} \cdot \text{B}(\omega)}{c}
\]

\[
gives \quad \beta = 1.25 \times 10^9 \text{ sec}^{-1} \text{ cm}^{-3}
\]

\[
N_3
\]

0.65 μm

0.80 μm

0.98 μm

1.53 μm

1.48 μm pump

Figure 8.17 (a) Energy-level diagram of erbium ions in silica fibers; (b) absorption and gain spectra of an EDFA whose core was codoped with germania to increase the refractive index. (After Ref. [71]. ©1991 IEEE. Reprinted with permission.)
Problem No. 3 will assume that signal - spontaneous beat noise dominates. The (noise current)^2 is then
\[ \text{Optical Power} = \frac{N_a}{N_a - N_1} \]

\[ (L^2) \text{ASE-sig} = \frac{g e^{\frac{1}{2}N_a}}{h \nu} P_{14}(G-1)n_{sp} \Delta f \]

\[ \text{electrical band-width} \]

\[ Q, B \text{ of } \triangle \text{optical frequency} \]

\[ P_{in} \rightarrow P_o = G \cdot P_{in} \text{ Power gain} \]

Assume RIN and Sp-Sp and Spont. Shot Not important.

\[ \text{Lossy Fiber} \rightarrow G \rightarrow \text{Lossy Fiber} \]

\[ \text{Noise Free Signal} \rightarrow \text{Detector} \]

Let \( P_{in} \) be the input power at the amplifier \( = P_{in} e^{-2N_1} \)

All except the thermal noise in involve an exponential decay between the amplifier and detector (the shot current noise) \( \equiv e^{-2N_2} \) and the (signal current noise) \( \equiv e^{-2N_3} \).

Thus,

\[ e^{\frac{2}{2}} \frac{P_{in} e^{\frac{2}{2}} e^{-2N_2}}{(h \nu)^2} \]

\[ \text{SNR} = \frac{(4kT e^{-2N_1}) + (2 \eta e^{-2N_2} P_{in} G e^{-2N_3} \Delta f) + (\frac{4 e^{2N_2} P_{in} G_1 n_{sp} \Delta f \cdot e^{-2N_3}}{h \nu} \text{ thermal} + \text{ shot} + \text{ Spontaneous})}{e^{-2N_2} e^{-2N_3} + 2N_2 n_{sp} G_1 e^{-2N_3} G^2 e^{-2N_3}} \]

\[ K^2 = \text{SNR} = \frac{P_{in} e^{-2N_2} e^{-2N_3}}{(\frac{4kTe^{-2N_1}}{R P_{in} G e^{\eta^2}} + 2 h \nu \Delta f) \text{ thermal} + \text{ shot} + \text{ Spontaneous}} \]
Derivation of Power Gain

(1) \[
\frac{dP}{dz} = g P + g N_{sp} \frac{h \nu \Delta f}{L} \text{ per mode power gain factor}
\]

\[= (h \nu C_r) \frac{N_{sp} \Delta f}{L} \text{ Number of spontaneous emission noise photons}\]

(Note: \( g \propto (N_2 - N_1) \) \( N_2 \) must be \( > N_1 \) for gain)

(Note: Spontaneous Emission only depends upon \( N_2 \), not \( N_1 \); that is why \( N_{sp} = \frac{N_2}{N_2 - N_1} \)

(2) First term in (1) gives signal gain \( (gP) \) and second term Amplified Spontaneous Emission (ASE)

To solve for \( P_0 + \text{Noise} \) move \( gP \) term to left side and write the equation as (after multiplying by \( e^{-gz} \))

\[
\frac{d(P e^{-gz})}{dz} = g N_{sp} h \nu \Delta f e^{-gz}
\]

Integrate from \( z=0 \) to \( z=L \)

\[
P_{out} e^{-gL} - P_{in} = \frac{g N_{sp} h \nu \Delta f (e^{-g} - 1)}{\text{Constant}}
\]

or

\[
P_{out} = P_{in} e^{+gL} + N_{sp} h \nu \Delta f (G-1)
\]

\[
P_{out} = \frac{P_0 + S_{sp} \Delta f}{G} \text{ where } S_{sp} = (G-1)N_{sp} h \nu
\]

Amplified Signal \quad Amplified Spontaneous Emission
Saturation of the Amplifier

In Eq. (1) \[ q \text{P} = \frac{q_0 \text{P}}{1 + \frac{\text{P}}{P_s}} \] where \( P_s = \text{saturation Power} \)

Thus (1) becomes

\[ \frac{d\text{P}}{dz} = \frac{q_0}{1 + \frac{\text{P}}{P_s}} \left( \text{P} + \frac{\text{P}}{P_s} n_{sp} h_{1af} \right) = \frac{q_0}{1 + \frac{\text{P}}{P_s}} \left( \text{P} + n_{sp} h_{1af} \right) \]

Consider the signal term on the right side of this equation:

\[ \frac{d\text{P}}{dz} = \frac{q_0}{P} \frac{\text{P}}{1 + \frac{\text{P}}{P_s}} \quad \text{or} \quad \frac{d\text{P}}{dz} \left( 1 + \frac{\text{P}}{P_s} \right) = q_0 dz \]

Integrating gives:

\[ \ln \left( \frac{\text{P}}{\text{Pin}} \right) = \ln \frac{\text{P}}{\text{P}_s} + \frac{\text{P}}{\text{Pin}} \]

or \[ \frac{\text{P}}{\text{Pin}} = e^{-\frac{(\text{P}_0 - \text{Pin})}{\text{P}_s} \cdot G_0} \]

where \( G_0 = e^{q_0 L} \)

Letting \( \text{P}_0 = G \cdot \text{Pin} \) we have \[ \frac{\text{P}_0 - \text{Pin}}{\text{P}_s} = \frac{\text{P}_0 - \text{P}_0 / G}{\text{P}_s} = \frac{\text{P}_0 (1 - \frac{1}{G})}{\text{P}_s} \]

Thus \[ \frac{\text{P}_0}{\text{Pin}} = G = e^{-\left( 1 - \frac{1}{G} \right) \frac{\text{P}_0}{\text{P}_s} \cdot G_0} \]

Implicit Relation for the Saturated Gain

when the noise term is included this becomes

\[ \text{P}_0 = \text{Pin} \cdot G + (G - 1) n_{sp} h_{1af} \]

where \( G = e^{-\left( 1 - \frac{1}{G} \right) \frac{\text{P}_0}{\text{P}_0 - h_{1af} n_{sp}}} \cdot G_0 \) with \( G_0 = e^{(q_0 L) h_{1af} n_{sp}} \)
Spontaneous - Signal Beat Noise - Simplified

Detector current $I(t) = \frac{n_0 P_0 e}{\hbar} = \Delta R \frac{P_0}{\hbar}$ (1)

For two fields Electric Field $\rightarrow \vec{E}_S + \vec{E}_N$

$\vec{E}_S = E_1 \cos(\omega_1 t + \phi_1) = \frac{E_1 e^{i(\omega_1 t + \phi_1)}}{2} + \text{c.c.}$

$\vec{E}_N = E_2 \cos(\omega_2 t + \phi_2) = \frac{E_2 e^{i(\omega_2 t + \phi_2)}}{2} + \text{c.c.}$

$P_0 = \left( \frac{E_S^2 + E_N^2}{\sqrt{2}/\epsilon} \right)^{\text{Low Frequency Terms}}$

$= \left( \frac{E_S^2 + E_N^2}{\sqrt{2}/\epsilon} \right)^{\text{Low Frequency Terms}}$

$= \left( \frac{1}{\sqrt{2}/\epsilon} \right) \left( \frac{1}{2} E_1^2 + \frac{1}{2} E_2^2 + \frac{1}{2} \vec{E}_1 \vec{E}_2 \cos((\omega_1 - \omega_2)t + (\phi_1 - \phi_2)) \right)$

But $P_1^{\text{Low Frequency}} = \frac{1}{2} E_1^2 / \sqrt{2}/\epsilon$; $P_2^{\text{Low Frequency}} = \frac{1}{2} E_2^2 / \sqrt{2}/\epsilon$

Thus $P_0 = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos((\omega_1 - \omega_2)t + (\phi_1 - \phi_2)) - (2)$

Now $P_1 = G P_{\text{in}}$ and $P_2 = n_{\text{sp}} (G - 1) \hbar f \Delta f$

The $\cos((\omega_1 - \omega_2)t + \phi_1 - \phi_2)$ term when squared and averaged over a random $\phi_2$ gives $\frac{1}{2}$. Thus

$\frac{I^2}{N_{\text{sp}}^2} = \left( \frac{n_0^2}{2} + G P_{\text{in}} (G - 1) \hbar f n_{\text{sp}} \Delta f \right)^2$ electrical

Band width used
Dominant Noise Terms When An Optical Amplifier is Used with a Detector

Shot Noise (Signal) \[ \frac{\overline{\Delta^2}}{8} = q \frac{2 q \eta G P_{in} \Delta f}{h_f} \]

Spontaneous - Spontaneous

Beat Noise \[ \frac{\overline{\Delta^2}_{sp-sp}}{h_f} = h_f (q \eta G P_{in})^2 \Delta f_{opt} (\Delta f h_f) \]

Signal - Spontaneous Beat Noise

\[ \frac{\overline{\Delta^2}_{sig-sp}}{h_f} = 4 \frac{(q \eta G P_{in})(q \eta (G-1) N_{sp}) h_f^2 \Delta f}{h_f} \]

Usually Dominant  (Approximately 2G x shot noise)

Noise Figure of an Optical Amplifier

Let \( R = \frac{q \eta}{h_f} \) = Responsivity

when \( G = 1 \) the dominant noise (minimum)

is the shot noise

\[ (SNR)_{in} = \frac{(R P_{in})^2}{2 q R P_{in} \Delta f} = \frac{P_{in} \eta}{2 h_f \Delta f} \]

with the amplifier Sp-Sig Noise dominant

\[ (SNR)_{out} = \frac{(R G P_{in})^2}{4 R^2 G P_{in} (G-1) N_{sp} h_f \Delta f} \]

\[ = \frac{G P_{in}}{4 (G-1) N_{sp} h_f \Delta f} \]

Thus \( F_n = \frac{(SNR)_{in}}{(SNR)_{out}} = 2 N_{sp} (G-1) \to 2 \) at best