Example Receiver S/N Ratio Calculation

Basic Equivalent Circuit

Reference on Noise:
*Noise in Receiving System*
Example Small Signal Analysis

\[ \frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} \]

\[ C = C_D + C_S + C_A \]

\[ V_i = \frac{R \cdot M \cdot I(t)}{1 + j \cdot 2 \pi f \cdot C \cdot R} \]

\[ V_o = G(f) \cdot V_i = \frac{G(f) \cdot R \cdot M \cdot I(t)}{1 + j \cdot 2 \pi f \cdot C \cdot R} \]

For "Equalized" Amplifier \( G(f) = G_0 (1 + j \cdot 2 \pi f \cdot R \cdot C) \) and

a) Output is independent of frequency

\[ V_o = G_0 \cdot M \cdot R \cdot I \]

b) \( V_o \) has the same functional relationship as a transimpedance amplifier.
Various Noise Sources

\[(I_{sh}^2)^2 = \Delta f \left( 2 e I_n M^2 F \right) \text{ --- Photo diode Shot Noise} \]

Excess Noise Due to \( M \)

\[(I_{th}^2) = \Delta f \left( \frac{4 k T}{R} \right) = 4 k T \left( \frac{1}{R_A} + \frac{1}{R_B} + \frac{G_D}{R_D} \right) \Delta f \]

Equivalent Noise Circuit:

\[(I_T^2) = (I_{sh}^2) + (I_{th}^2) + (I_A^2) \]

\[C = C_S + C_A + C_D \]

\[\frac{1}{R} = \frac{1}{R_D} + \frac{1}{R_A} + \frac{1}{R_B} \]

\[V_N^2 = G_0^2 \int |G(f)|^2 (V_A^*)^2 \, df + \int \frac{G^2 R^2 (I_T^*)^2}{\left| 1 + j 2 \pi f CR \right|^2} \, df \]

\[0 \to \Delta f \]

(over the band width)
\[ V_N^2 = G_0^2 \int_0^\infty \left[ 1 + 4 \pi^2 \delta_c^2 R_N^2 \right] (V_A^*)^2 + R^2 (I_T^*)^2 \, d\omega \]

Thus

\[ \frac{V_0}{V_N} = \frac{G_0 \, M \, I \, R}{\left[ (\Delta f + \frac{4 \pi^2}{3} (\Delta f)^2 c^2 R_N^2)(V_A^*)^2 + \Delta f R^2 (I_T^*)^2 \right]^{\frac{1}{2}}} \]

\[ K = \frac{I}{\sqrt{\Delta f} \left[ \left( \frac{V_A^*)^2}{M^2} \left( \frac{1}{R^2} + \frac{4 \pi^2}{3} (\Delta f)^2 c^2 \right) + 2eI F + \frac{4 kT (I_T^*)^2}{R^2} \right]^{\frac{1}{2}}} \]

Notes:

1) \( S/N \) increases with \( M \) until shot noise dominates.

2) Increasing \( R \) excellent as long as (a) \( \& \) (d) significant. If \( R \) too large equalization is needed to assure band-width.

3) Shot noise causes \( K \) to be signal dependent.

4) At high frequencies \( b \) dominates (increases as \( c^2 \) - thus important to minimize \( C \)).

5) Assumes noise sources uncorrelated.
Given \( K \) (12 for BER \( \approx 10^{-9} \)) can
solve for \( I \)
\[
I^2 - 2p I - q = 0
\]
\[
eF K^2 \Delta f
\]
\[
k^2 \Delta f (I_n)^2 + \frac{k^4 \Delta T \Delta f}{M^2 R} + \frac{k^2 \Delta f}{M^2} \left[ \frac{V_n^2}{R^2} \left(1 + \frac{q^2}{3} \right) \right]
\]
\[(a) \quad (b)
\]
\[(c) \quad (d)
\]
Solution:

\[
I = p \pm \sqrt{p^2 + q}
\]

Relevant sign is + (Otherwise noise can \( \to 0 \))

\[
= p \left\{ 1 + \left( 1 + \frac{q}{p^2} \right)^{1/2} \right\}
\]

Note:

Minimum

\( I = 2p \to \text{shot noise limit} \)

\[
= k^2 2e \Delta f F
\]

Example Si-APD

\( \eta = 0.75 \quad \beta = 0.5 \text{A/W} \quad F = 6 \)

\( B = 2 \Delta f = 1 \text{MHz} \quad K = 12 \)

\[
I = K^2 e \left( 2 \Delta f \right) F
\]

\[
= 144 \times 1.6 \times 10^{-19} \times 10^6 \times 6
\]

\[
= 1.4 \times 10^{-10} \text{amps}
\]

Show that when thermal noise dominates and

\[
\Delta f \ll \frac{1}{2\pi RC}
\]

\( M^2 \gg \frac{2\pi kT C}{e^2 F^2 K^2} \)

assures the shot noise limit