OVERVIEW: There are really for all of you. We will:

- Clarify material that may have been difficult in lecture
- Work example problems
- Do some cool optics demonstrations

I encourage all students to email me each week with topics you would like to see covered in more detail during discussion section. There is almost ALWAYS another student that shares your question so discussion is a good time to clarify these issues.

ALWAYS FEEL FREE to stop me & ask questions.

TODAY: Review phase-front time & space evolution

- PIER EXAMPLE
- Review derivation of Snell's Law
- Example HW problem.
- My notes will be available on bspace.
How does light (or any propagating wave) evolve in space \( \frac{1}{2} \) time?

Phase, phase, phase...... OK, one more time phase.

Consider the following propagating wave: (could be light)

\[
\vec{A} = A_0 \sin (kz + wt) \hat{x}.
\]

Characteristics: It is monochromatic. It exists for all time (no envelope), amplitude varies in \( z \) coordinate (const in \( x \) and \( y \)) and in time coordinate.

Imagine that you are standing on a pier looking out at the ocean.

The waves you are looking at are propagating waves. Mathematically we describe this exactly as we would describe the E-field of a light wave, i.e.

\[
\vec{A}_E = A_0 \sin (kz + wt) \hat{x} + B_0 \cos (kz + wt) \hat{y}.
\]

Space evolution \( \hat{x} \)

Time evolution \( \hat{y} \)

What are \( k \) and \( w \)? You should already know \( w \).
Look only at SPATIAL EVOLUTION - need to fix time ⇒ TAKE A PICTURE.

In this PICTURE, time is fixed to one time coordinate. We can describe the wave as:

\[ A(z,t = \text{time of picture}) = A_0 \sin (kz - \text{const.}) \]

The amplitude of the wave changes magnitude \underline{IN SPACE}.

The amount of space (distance) it takes to do one full cycle is the distance \( \lambda \) takes for the argument of \( \sin() \) to change by \( 2\pi \). This distance is \underline{DEFINED as the SPATIAL PERIOD or WAVELENGTH}

\[ k(4\pi = 2) = 2\pi \Rightarrow k = \frac{2\pi}{\lambda} \text{ DEF.} \]

Now, consider another experiment: \underline{FIX SPACE} and let time evolve. Sounds weird, huh.
Imagine holding your hands in front of one of your eye (the other eye is closed) so you see only a tiny sliver of space. Now watch the ocean for a few seconds.

As time evolves, the height of the water bobs up and down in your fixed spatial position.

MATH

\[ A(z=\text{position of hands}, t) = A \sin (\text{const.} \cdot t) \]

The amount of TIME it takes to do one full cycle in the time it takes for the argument of \( \sin(\) to change by \(2\pi\). This time is defined as the TEMPORAL PERIOD, \(T\).

\[ \omega(\Delta t = T) = 2\pi \implies \omega = \frac{2\pi}{T} \quad \text{DEF} \]
Now consider a light source in the point $z = 2 + d_2$, $t = t$, so the phase $\phi = k_2 - \omega t$. Since the wave propagates in the $z$ direction, the phase velocity

$$v = \frac{k_2}{\omega}$$

This is the speed of the light, and the wave crest propagates at this speed.

DEF: The "phase velocity" is the speed (in direction)

$$A(2, t) = A_0 \sin (k_2 z - \omega t)$$

When you combine space and time, you get...
More on the mysterious "k"

"k" is the spatial analog of "w," why?

\[ T = \text{temporal period (seconds)} \]
\[ \lambda = \text{spatial period (meters)} \]

\[ \frac{1}{T} = f = \text{temporal frequency (Hz)} \]
the number
of temporal cycles per second

\[ \frac{2\pi}{T} = 2\pi f = w = \text{(temporal) angular frequency} \]

the radians of temporal phase carved out per second \( \frac{\text{rad}}{\text{sec}} \)
\[
\frac{1}{\lambda} = \nu = \text{spatial frequency} \ (\frac{1}{\text{meters}})
\]

In words: The number of spatial cycles per unit distance.

\[
\frac{2\pi}{\lambda} = 2\pi \nu = k = \text{(spatial angular frequency)}
\]

The radians of spatial phase carried out per unit distance (\(\text{rad/meter}\)).

For some crazy reason, optics people decided to call \(k\) the "wavevector magnitude," which is not a very descriptive name. NOTE: Strictly speaking, we have done all of this in 1-D.

The spatial angular frequency (\(k\); also called wave-vector magnitude)

actually has components for each of the 3 dimensions of space.

\[
\mathbf{k} = k_x \mathbf{\hat{x}} + k_y \mathbf{\hat{y}} + k_z \mathbf{\hat{z}} = (k_x, k_y, k_z)
\]

\(k_y\), for example, tells you how many radians of spatial phase is carried out per unit distance propagated in the \(y\) direction.
SUMMARY: Ocean pier example. Fixed space, let time evolve → see height of water bob up & down in space → notion of TEMPORAL PERIOD, T. Time only has ONE dimension, it just goes on indefinitely forward or backwards. TEMPORAL PERIOD has one vector component however SPACEx is 3-D!!! We also fixed time (took a picture) & let space evolve. Observe height of water go up & down in space. NOTION OF SPATIAL PERIOD, x (also called wavelength) BUT we only considered the simple 1D spatial example. In reality, SPACE is 3D so the SPATIAL PERIOD should have 3 VECTOR components.

Ex.: (top view picture of beach) Wave amplitude is out of paper.

\[ A(x, y, t = \text{time of picture}) = A_0 \cos\left(k_x x + k_y y + \omega t + \phi\right) \text{ (wave on coast in space.)} \]

\[ k_x = |k| \cos\phi \]
\[ k_y = |k| \sin\phi \]
EX: Top VIEW of OCEAN waves come in at angle $\theta$

We know a bit what $T_x$ (effective spatial period in $x$) and $T_y$ (effective spatial period in $y$)

\[ \frac{T}{T_x} = \cos \theta \quad \text{and} \quad \frac{T}{T_y} = \sin \theta \]

\[ T_x = \frac{\lambda}{\cos \theta} \]

\[ T_y = \frac{2\pi}{\sin \theta} \]

and \[ k_x = \frac{2\pi}{T_x} = \frac{2\pi}{T} \cos \theta \]

\[ k_y = \frac{2\pi}{T_y} = \frac{2\pi}{T} \sin \theta \]
SNELL'S LAW (MORE DETAIL)

Phase front (surface of constant phase)

At \( t = t' \)
\[ z = z' \]
\[ kz' = \omega t' = \phi \]

Later time, same phase front.
\[ kz'' - \omega t'' = \phi \]

\[ \frac{BC}{V_1} = \Delta t \] when \( V_1 \) = phase velocity in material 1 = \( \frac{\omega}{k_1} \).

In this time \( \Delta t \), RAY 1 travels a longitudinal distance
\[ \Delta t \cdot V_2 \] where \( V_2 = \frac{c}{n_2} \).

So, the trick to find the angle of propagation for the transmitted ray is to always keep the transmitted RAY 1 to the transmitted phase front.
Here we show transmitted RAY 1 and the corresponding
phasefront for three values of $n_2$: 1, 1.2, 1.5
The distance RAY 1 travels in these 3 cases is
\[ \Delta t \cdot \frac{c}{1}, \quad \Delta t \cdot \frac{c}{1.2}, \quad \Delta t \cdot \frac{c}{1.5} \]
respectively.

Always need to force the trans. RAY to be perpendicular to the phasefront. The time \( \Delta t \) multiplied by \( v_2 \), the phase velocity of region 2, gives the distance that RAY 1 travels in region 2.

As \( n_2 \) increases, \( v_2 \) slows down and the "bending" or refraction increases.

CASE 1: \( n_2 = 1 \), NO Bending
CASE 2: \( n_2 = 1.2 \), SOME Bending
CASE 3: \( n_2 = 1.5 \), MORE Bending
\[
\frac{\sin \theta}{\sin \alpha} = \frac{\tan \beta}{\tan \gamma}
\]

\[
\frac{h}{z} = \frac{\tan \theta}{\tan \alpha} = \frac{\tan \theta}{x} = \frac{h}{x}
\]

**Geometry:** \( x = \frac{h}{x} \)

\[\sin \theta \text{ or } \sin \alpha \]

**Use Snell's Law:**

in the water will allow to be \( \frac{2}{3} \) of its true depth.

Looking straight down into a swimming pool, any object

\[ V = 1.35 \]

\[ V = 4 \]

\[ V = 1.33 \]

\[ \frac{3}{4} = 0.75 \text{ or } \frac{3}{4} \]

**Ex 4.2:**