GSI: Chris Anderson

OH: M & W 11 - 12:30 (After class) in Cory 197 ①

cnanderson@berkeley.edu

<u>OVERVIEW:</u> These are really for all of you. We will:

- Clarify material that may have been difficult in lecture
- Work example problems
- Do some cool optics demonstrations

I encourage all students to email me each week with topics you would like to see covered in more detail during discussion section. There is almost <u>ALWAYS</u> another student that shares your question so discussion is a good time to clarify these issues.

- ALWAYS FEEL FREE to stop me & ask questions.

TODAY:
- Review phasefront time & space evolution
  - PIER EXAMPLE
- Review derivation of Snell's Law
- Example HW problem.
- My notes will be available on bspace :)

HOW DOES LIGHT (OR ANY PROPAGATING WAVE) EVOLVE IN SPACE & TIME?

PHASE, PHASE, PHASE..... OK, ONE MORE TIME <u>PHASE</u>.

Consider the following propagating wave: (could be light)

$\vec{A} = A_0 \sin(kz + \omega t)\,\hat{x}$.

single frequency

Characteristics: It is monochromatic. It exists for all time (no envelope), amplitude varies in z coordinate (const in x and y) and in time coordinate.

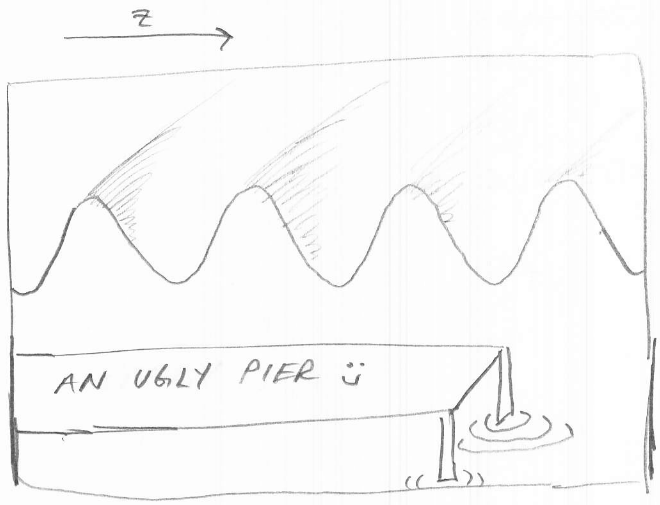
IMAGINE That you are standing on a pier looking out at the ocean.

The waves you are looking ~~at can be described mathematically~~ at are propagating waves. Mathematically, we describe the amp of an ocean wave ~~them~~ exactly as we would describe the E-field of a light wave. I.E.

$$\vec{A}(z,t) = A_0 \sin(\underbrace{kz}_{\text{space evolution}} + \underbrace{\omega t}_{\text{time evolution}})\,\hat{x}\;;$$

What are k and ω? You should already know ω.

Look only at SPATIAL EVOLUTION – need to fix time $\Rightarrow$ TAKE A PICTURE.



In this PICTURE, time is fixed to one time coordinate. We can describe the wave as:

$$A(z, t = \text{time of picture}) = A_0 \sin(kz - \text{const.})$$

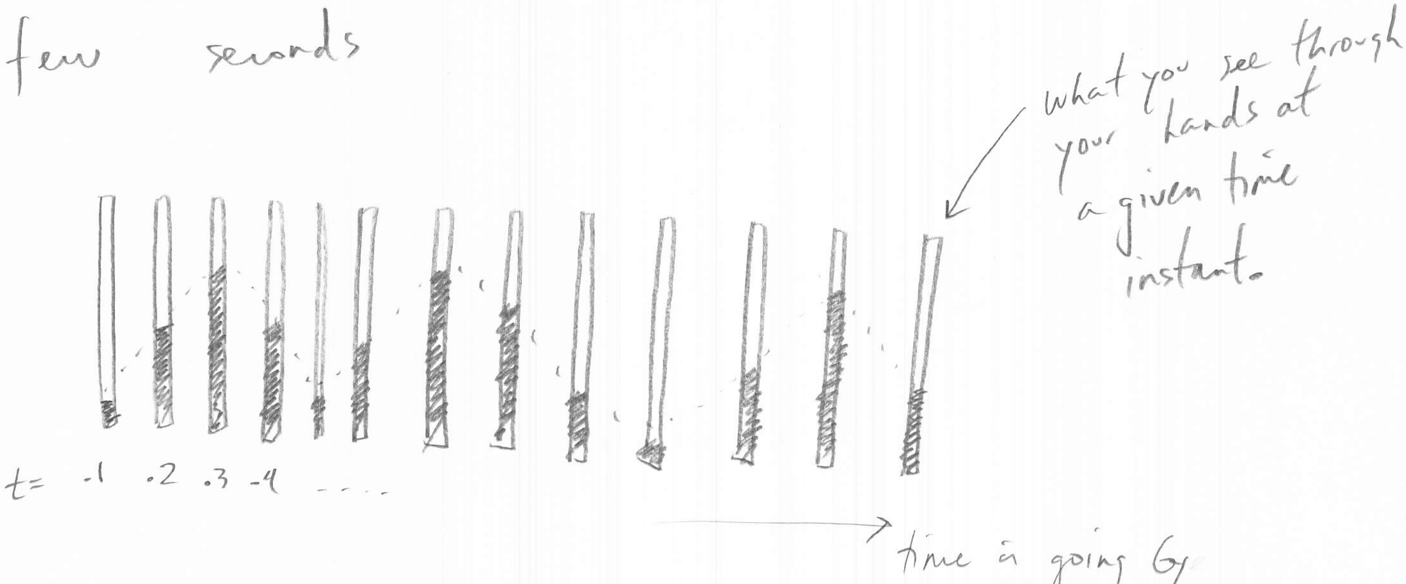
The amplitude of the wave changes magnitude IN SPACE.

The amount of space (distance) it takes to do one full cycle is the distance it takes for the argument of sin( ) to change by 2π. This distance is DEFINED as the SPATIAL PERIOD or WAVELENGTH

$$k(\Delta z = \lambda) = 2\pi \rightarrow \boxed{k = \frac{2\pi}{\lambda} \quad \text{DEF.}}$$

Now consider another experiment: FIX SPACE and let time evolve. Sounds weird, huh.

Imagine holding your hands in front of one of your eye (the other eye is closed) so you see only a tiny sliver of space. Now watch the ocean for a few seconds



As time evolves, the height of the water bobs up & down in your fixed spatial position.

MATH $$A(z = \text{position of hands}, t) = A_0 \sin(\text{const.} - \omega t)$$

The amount of TIME it takes to do one full cycle is the time it takes for the argument of sin( ) to change by 2π. This time is defined as the TEMPORAL PERIOD, T.

$$\omega(\Delta t = T) = 2\pi \quad \therefore \quad \boxed{\omega = \frac{2\pi}{T} \quad \text{DEF.}}$$

When you combine space & time evolution you get a PROPAGATING WAVE.

$$A(z,t) = A_0 \sin(kz - \omega t)$$

DEF: The "phase velocity" is the speed (& direction) that phasefronts (surfaces of constant phase in spacetime) move as the wave propagates. For the OCEAN EXAMPLE, this is the speed a surfer would move toward the beach as he/she rides in on a wave.

HOW TO DETERMINE: Consider an initial spacetime point $z = z'$, $t = t'$ so PHASE $= kz' - \omega t' = \phi$

Now consider a later spacetime point $z = z' + dz$, $t = t' + dt$ and find the phase equal to ϕ.

NOTE:
$$\frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T} = \frac{\text{SPATIAL PERIOD}}{\text{TEMPORAL PERIOD}}$$

$$k(z' + dz) - \omega(t' + dt) = \phi = kz' - \omega t'$$

$$\therefore \quad k\,dz = \omega\,dt \quad \& \quad \boxed{\frac{dz}{dt} = \frac{\omega}{k} = V_{PHASE} \quad \text{DEF.}}$$

so $\underbrace{kz + k\Delta z - \omega t - \omega\Delta t}_{\text{phase later in space \& time}} = \underbrace{kz - \omega t}_{\text{initial phase}}$

$$\Rightarrow \quad k\Delta z = \omega\Delta t$$

$$\boxed{\frac{\Delta z}{\Delta t} = V_{phase} = \frac{\omega}{k} = \frac{2\pi/T}{2\pi/\lambda} = \frac{\lambda}{T} = \frac{\text{SPATIAL PERIOD (meter)}}{\text{TEMPORAL PERIOD (sec)}}}$$

MORE ON THE MYSTERIOUS "K"

"K" is the spatial analog of "ω." why?

$T =$ temporal period (seconds)
$\lambda =$ spatial period (meters)

$\frac{1}{T} = f =$ temporal frequency $\left(\frac{1}{sec}\right)$ IN WORDS: the number of temporal cycles per second

$\frac{2\pi}{T} = 2\pi f = \omega =$ (temporal) angular frequency
the radians of temporal phase carved out per second $\left(\frac{rad}{sec}\right)$

Not "v"

$\frac{1}{\lambda} = \nu = $ spatial frequency $\left(\frac{1}{\text{meters}}\right)$

IN WORDS: The number of spatial cycles per unit distance

$\frac{2\pi}{\lambda} = 2\pi\nu = k = $ (spatial) angular frequency:

the radians of spatial phase carved out per unit distance $\left(\frac{\text{rad}}{\text{meter}}\right)$

For some crazy reason, optics people decided to call k the "wave vector magnitude" which is not a very descriptive name. NOTE: Strictly speaking we have done all of this in 1-D.

The spatial angular frequency (k; also called wavevector magnitude) actually has components for each of the 3 dimensions of space.

NOTE: the temporal angular frequency only has one component b/c time is only one dimension

IE $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z} = (k_x, k_y, k_z)$

k_y, for example tells you how many radians of spatial phase is carved out per unit distance propagated in the y direction.

SUMMARY: Ocean pier example. Fixed space, let

time evolve → see height of water bob up & down in time

→ notion of TEMPORAL PERIOD, T. ~~Space~~ Time only has ONE Dimension, it just goes on indefinately forward or backwards. TEMPORAL PERIOD has one vector component

SPACE however is 3-D !!! We also fixed time (took a picture) & let space evolve. ⇒ observe height of water go up and down in space. NOTION OF SPATIAL PERIOD, λ (also called wavelength) BUT we only considered the simple 1D spatial example.

In reality, SPACE is 3D so the SPATIAL PERIOD should have 3 VECTOR components.

EX: (Top-view picture of beach) wave amplitude is OUT OF PAPER.

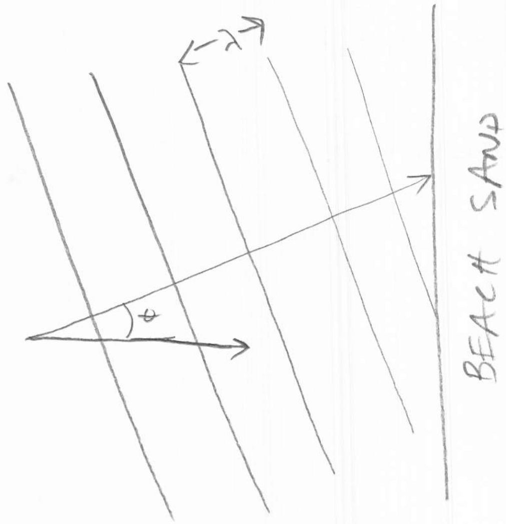
$$A(x, y, t = \text{time of picture}) =$$

$$A_0 \cos\Big(k_x x + k_y y + \underbrace{\omega \cdot t_{picture}}_{\text{const in space.}}\Big)$$

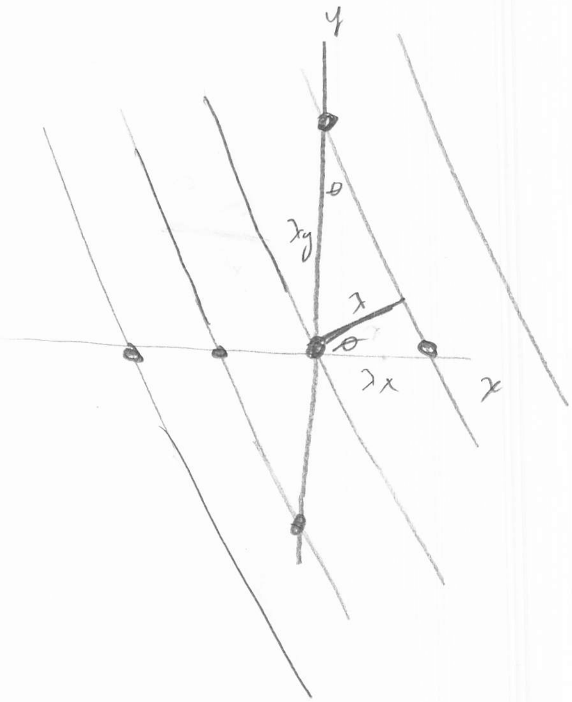
$$k_x = |\vec{k}| \cos\theta$$

$$k_y = |\vec{k}| \sin\theta$$

EX: TOP VIEW OF OCEAN waves come in @ angle θ



we know λ but what λ_x (effective SPATIAL PERIOD IN x) and λ_y (effective spatial period in y)



$$\frac{\lambda}{\lambda_x} = \cos\theta \quad ; \quad \frac{\lambda}{\lambda_y} = \sin\theta$$

$$\therefore \quad \lambda_x = \frac{\lambda}{\cos\theta}$$

$$\lambda_y = \frac{\lambda}{\sin\theta}$$

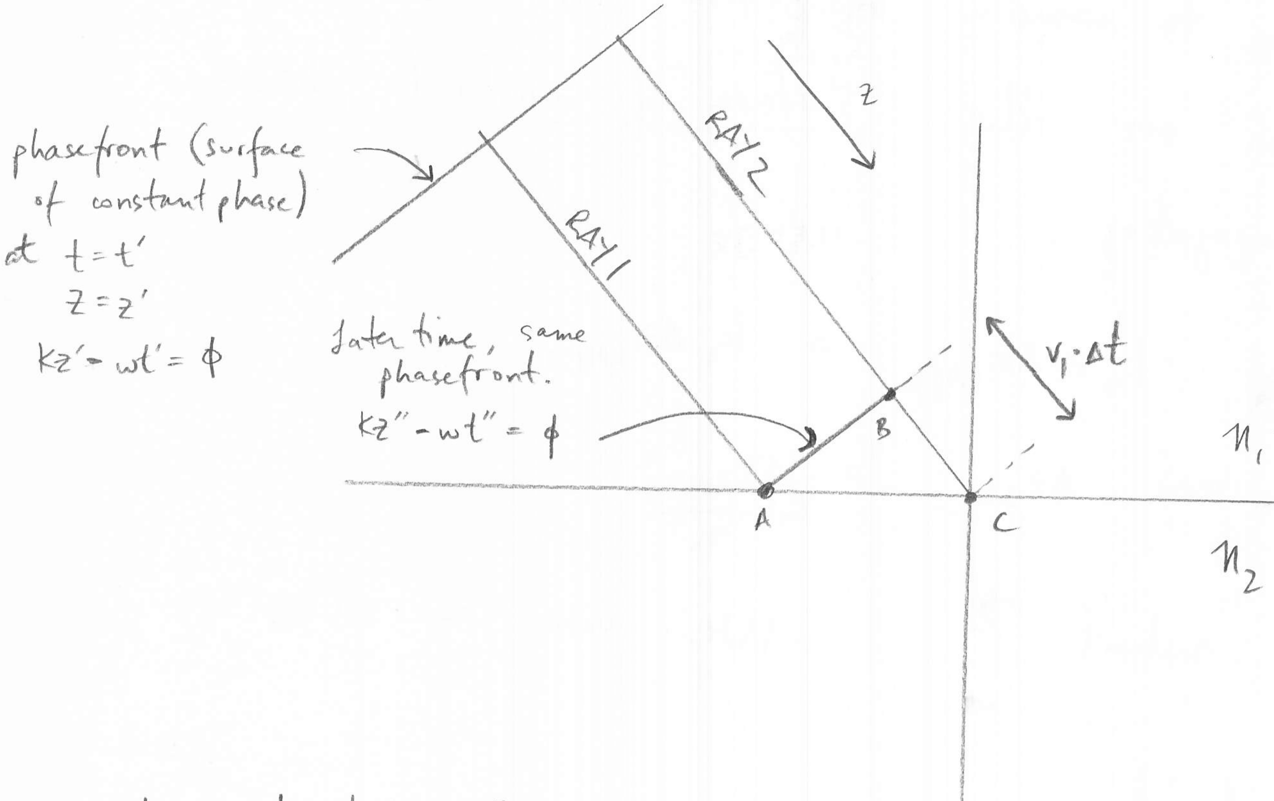
and

$$k_x = \frac{2\pi}{\lambda_x} = \frac{2\pi}{\lambda}\cos\theta$$

$$k_y = \frac{2\pi}{\lambda_y} = \frac{2\pi}{\lambda}\sin\theta$$

$\cancel{v_x = \frac{\omega}{k_x} = \frac{v}{\cos\theta}}$

SNELL'S LAW (MORE DETAIL)



Ok, at time t'', RAY 1 enters material 2 (n_2).

the time Δt before RAY 2 enters material 2 is

$$\frac{BC}{v_1} = \Delta t \quad \text{where } v_1 = \text{phase velocity in material 1} = \frac{\omega}{k_1} = c$$

In this time Δt, RAY 1 travels a longitudinal distance

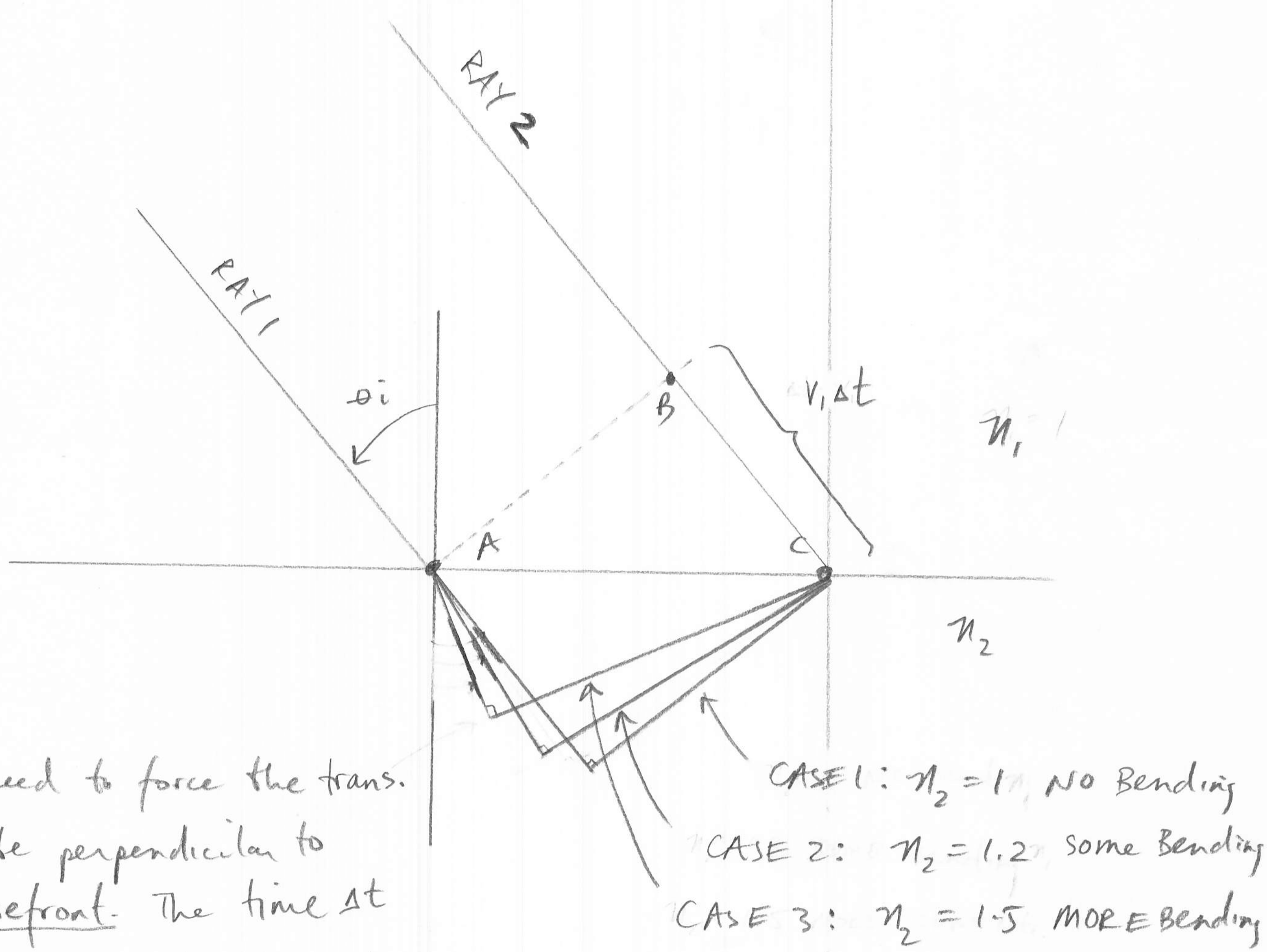
$\Delta t \cdot v_2$ where v_2 (phase velocity in region 2) $= \frac{c}{n_2}$. So, the trick

to find the the angle of ~~the~~ propagation for the transmitted ray is to always keep the transmitted RAY $\perp$ to the transmitted phasefront

Here we show transmitted RAY 1 and the corresponding phasefront for three values of n_2 : 1, 1.2, & 1.5

The distance RAY 1 travels in these 3 cases is

$$\Delta t \cdot \frac{c}{1}\ , \quad \Delta t \cdot \frac{c}{1.2}\ , \quad \Delta t \cdot \frac{c}{1.5} \quad \text{respectively}$$

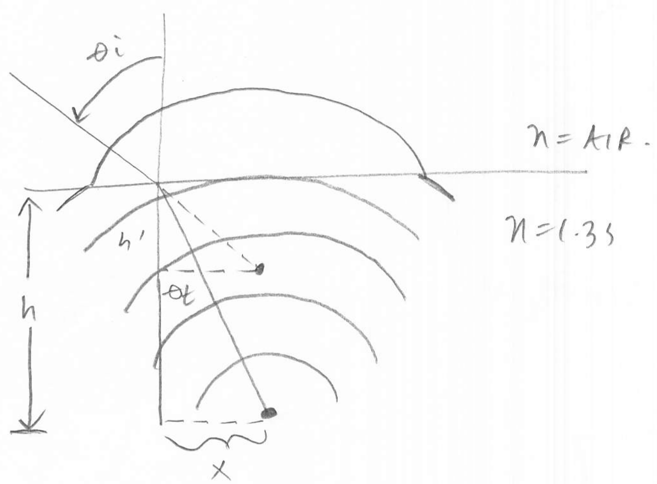


always need to force the trans. RAY to be perpendicular to the phasefront. The time Δt multiplied by v_2, the phase velocity of region 2, gives the distance that RAY 1 travels in region 2.

As n_2 increases, v_2 slows down and the "bending" on refraction increases.

EX #2: $n_{H_2O} = 1.33 = \frac{4}{3}$. Prove that to someone looking straight down into a swimming pool any object in the water will appear to be $\frac{3}{4}$ of it's true depth.

USE SNELL's LAW:



Snell @ interface: $\underbrace{(1)\sin(\theta_i)}_{AIR} = \underbrace{\frac{4}{3}\sin(\theta_t)}_{H_2O} \Longrightarrow \boxed{\frac{\sin\theta_t}{\sin\theta_i} \approx \frac{3}{4}}$

GEOMETRY: $\frac{x}{h} = \tan\theta_t \quad \therefore \quad x = h\tan\theta_t$

and $\frac{x}{h'} = \tan\theta_i \quad \therefore$

$$h' = \frac{x}{\tan\theta_i} = \frac{h\tan\theta_t}{\tan\theta_i} \approx \frac{3}{4}h$$

we have used

$$\frac{\tan a}{\tan b} \approx \frac{\sin a}{\sin b}$$