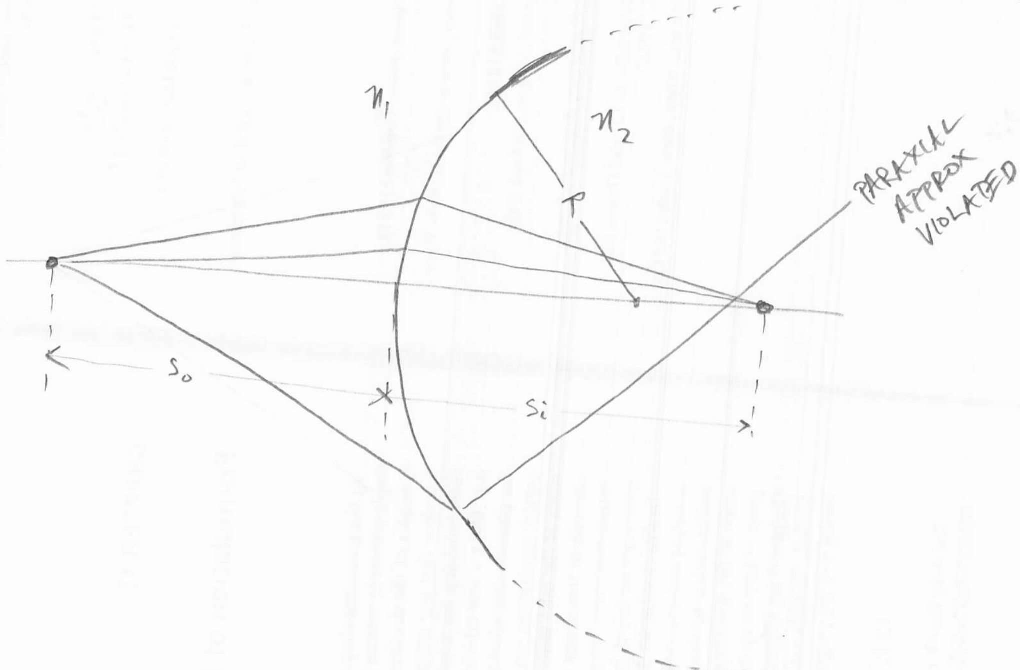
Look at "imaging" a single on-axis point source w/ one refraction at a spherical surface.

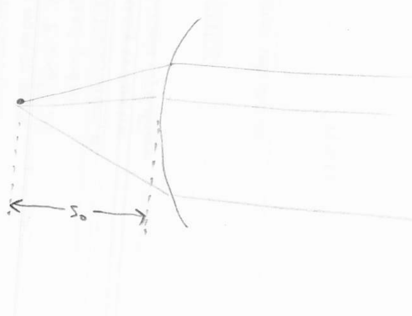
SO FAR (IN CLASS & IN TEXT UP TO 5.2.3) WE HAVE ONLY CONSIDERED ON-AXIS POINT SOURCES. We will get to real "life-sized" (NOT point source) objects soon.



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$ PARAXIAL APPROX

DEF: first focal length – object dist $(s_o = f_o)$ s.t. $s_i = \infty$

$$\frac{n_1}{s_o} + \frac{n_2}{\infty} = \frac{n_2 - n_1}{R}$$



$$f_o = \frac{n_1}{n_2 - n_1} R \qquad (5.9)$$

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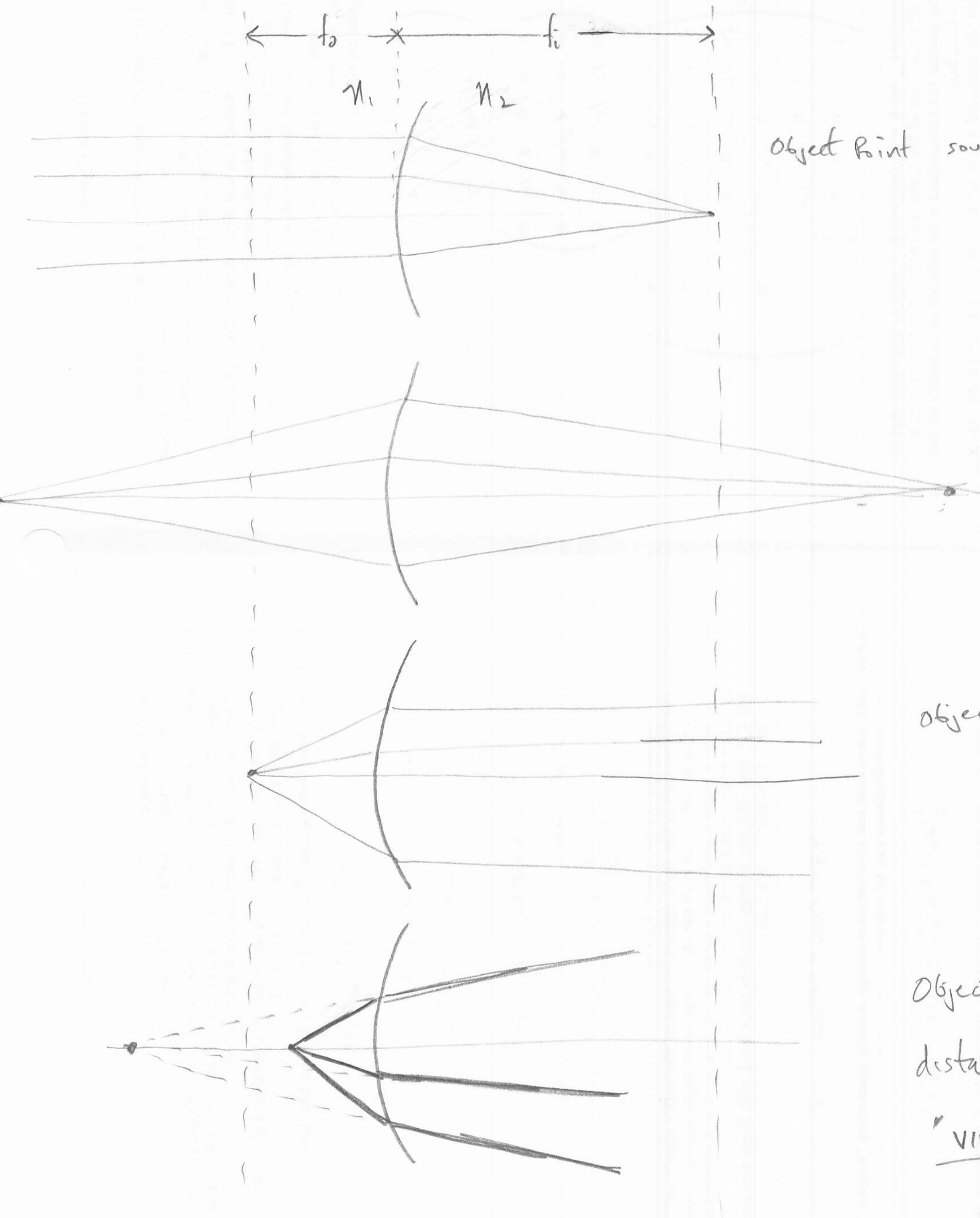
second focal length – axial point where image is formed when $s_o = \infty$.

$$\frac{n_1}{\infty} + \frac{n_2}{s_i = f_i} = \frac{n_2 - n_1}{R}$$

$$\boxed{f_i = \frac{n_2}{n_2 - n_1} R \qquad (5.10)}$$

PICTURES:



Object Point source @ ∞; $s_i = f_i$

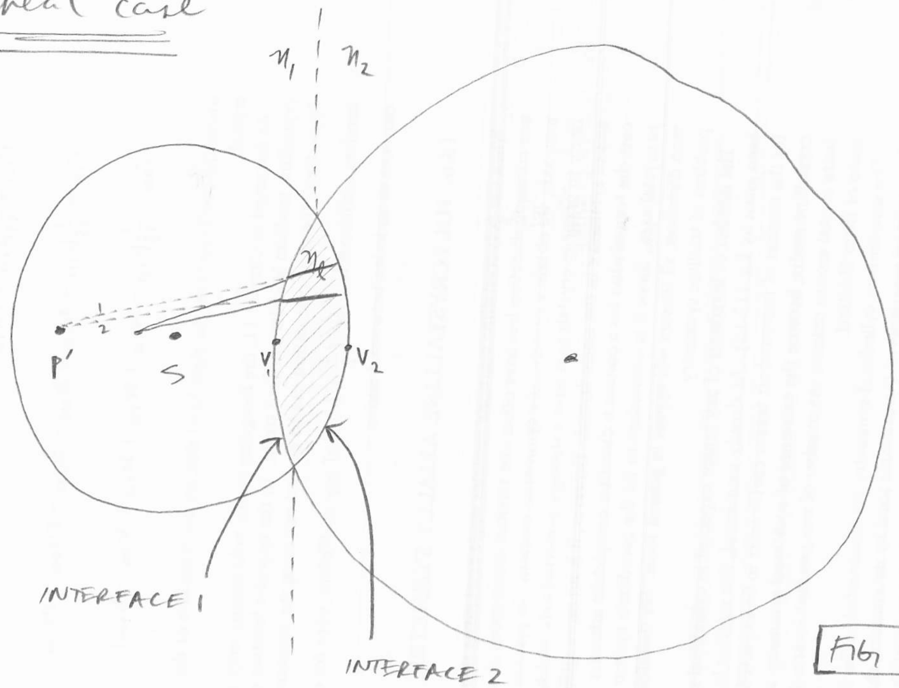
object point source outside of f_o
$f_i < s_i < \infty$

object point source @ f_o,
$s_i = \infty$

Object (point source) inside f_o, image distance is NEGATIVE
"VIRTUAL IMAGE"

FIGURE 5-13
IN TEXT.

Thick lens equation (Paraxial approximation) for the general case (3)



Talk about problem w/ sign conventions in Hecht.

Fig 5.14 TEXT

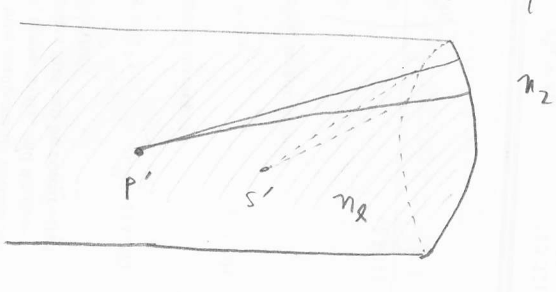
STEP 1: Conside object on left of lens inside of the first focal length. will form virtual image (image dist will be negative).

$$\frac{n_1}{s_{o_1}} + \frac{n_\ell}{s_{i_1}} = \frac{n_\ell - n_1}{R_1} \qquad (5.11)$$

In region 1 all distances are w.r.t. V_1

The refracted rays 1 & 2, "appear" to have → originated at P′ and as far as the second surface of the lens is concerned it "sees" a point P′ inside of index n_ℓ.

rays emerging from



~~[illegible]~~

What interface 2 "observes" or "perceives"

~~We have an object~~

STEP 2: Look at interface 2 → Starts w/ object a distance $-s_1 + d$ from interface point V_2

RECALL: s_1 is negative so the additional negative sign makes a positive number. I KNOW THIS IS ANNOYING! :)

$$\frac{n_l}{(-s_{i_1}+d)} + \frac{n_2}{s_{i2}} = \frac{n_2 - n_l}{R_2} \qquad (5.13)$$

$$(5.13) + (5.11) \Rightarrow \frac{n_1}{s_{o1}} + \frac{n_2}{s_{i2}} + \underbrace{\frac{n_l}{(-s_{i_1}+d)} + \frac{n_l}{s_{i_1}}}_{\text{GET COMMON DENOMINATOR.}} = \underbrace{\frac{n_2 - n_l}{R_2} + \frac{n_l - n_1}{R_1}}$$

$$\frac{n_l s_{i_1}}{(-s_{i_1}+d)(s_{i_1})} + \frac{n_l(-s_{i_1}+d)}{(-s_{i_1}+d)\,s_{i_1}} = \frac{n_l d}{s_{i_1}(-s_{i_1}+d)}$$

$$n_l\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \left(\frac{n_1}{R_1} - \frac{n_2}{R_2}\right)$$

GENERALIZED $\boxed{\text{PARAXIAL}}$ THICK LENS IMAGING EQUATION:

$$\frac{n_1}{s_{o1}} + \frac{n_2}{s_{i2}} = n_l\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \left(\frac{n_1}{R_1} - \frac{n_2}{R_2}\right) - \underbrace{\frac{n_l d}{s_{i_1}(-s_{i_1}+d)}}_{\text{Drop this when dealing with "thin enough" lens.}}$$

WE WILL deal w/ this later.

⟹ DO "SIMPLE" CASE: $n_1 = n_2 = n$; "THIN LENS"
MOST COMMON USE.

$$\frac{1}{s_{o1}} + \frac{1}{s_{i2}} = \frac{(n_l - n)}{n}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{f_n} \leftarrow$$

Focal length "in" the material with index n.

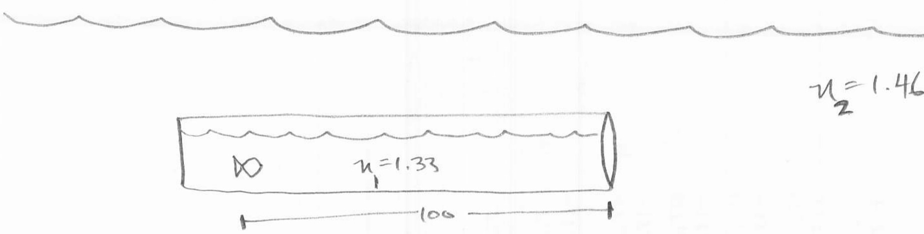
SIMPLIFIED PARAXIAL THIN LENS IMAGING EQUATION. INSIDE OF INDEX n

~~which is the (extremely useful) simplified but often THIN LENS IMAGING EQUATION~~

EX: A thin, bi-convex lens (), $R_1 = -R_2$, $|R_1| = R$, has a focal length of 10-cm in air.

The lens is placed on the end of a long tube to seal it containing fresh water ($n = 1.33$) and in the tube is a fish 100 cm from the lens. You (& the tube) jump in the Mediterranean sea ($n = 1.46$). ~~Where~~ Where is the image of the 1cm Guppy?

This may seem overwhelming, but let's start w/ what we know. A PICTURE:



SINCE SIZE GUPPY $\ll S_o$, can consider GUPPY a ON-AXIS POINT SOURCE!!

GIVEN: $$\frac{1}{10} = \frac{1}{f_{AIR}} = (n_\ell - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (n_\ell - 1)\frac{2}{R}$$

Cannot do this in general.

Can we use SIMPLIFIED THIN LENS PARAXIAL IMAGING EQUATION? NO must have same index on both sides of the lens to use "SIMPLIFIED EQN"

→ USE GENERALIZED PARAXIAL THIN LENS IMAGING EQUATION:

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = n_\ell\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \left(\frac{n_1}{R_1} - \frac{n_2}{R_2}\right)$$

For $|R_1| = |R_2| = R$, $R_1 = -R_2$ this simplifies to:

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = n_e\left(\frac{2}{R}\right) - \frac{(n_1+n_2)}{R}$$

$$= \left[n_e - \frac{(n_1+n_2)}{2}\right]\left(\frac{2}{R}\right)$$

use $\frac{1}{10} = (n_e - 1)\left[\frac{2}{R}\right] \therefore \left(\frac{2}{R}\right) = \frac{1}{10(n_e-1)}$

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = \left[n_e - \frac{(n_1+n_2)}{2}\right]\left(\frac{1}{10(n_e-1)}\right)$$

You are basically done from here.

* NOTE: As determined in section, you need ~~[illegible]~~ n_e to complete this problem :)

BUT WHAT ABOUT REAL (NOT ~~[illegible]~~ POINT SOURCE) OBJECTS?

This is actually an EASY EXTENSION

you can rotate ~~coordinate~~ system around center of curvature & NOTHING CHANGES!

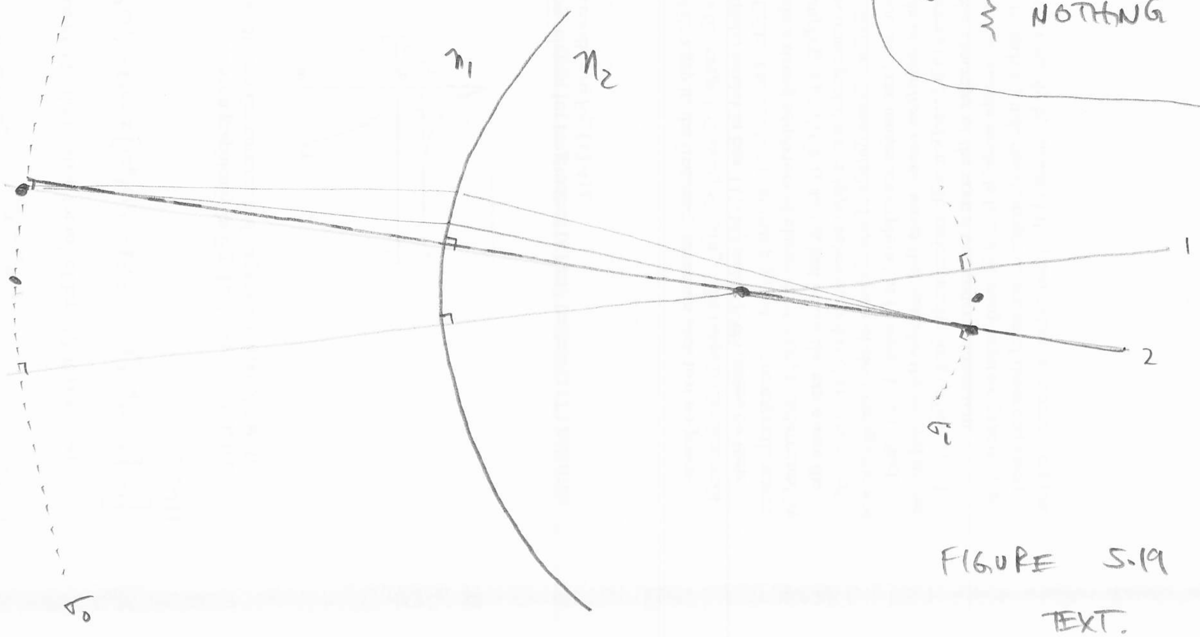


FIGURE 5.19 TEXT.

Lines 1 & 2 start from center of curvature and are $\perp$ to surface of spherical interface (no refraction) and extend forever in both directions.

Each object point on spherical object surface σ_o will be imaged to an image point on spherical image surface σ_i

WHY? B/c ~~you can create a new effective optical axis for~~ EACH conjugate point pair on the spherical surfaces σ_o & σ_i ~~[illegible]~~ satisfies the paraxial refraction equation by symmetry.

$$\frac{n_1}{s_i} + \frac{n_2}{s_o} = \frac{n_2 - n_1}{R}$$