\[
\left( \frac{1}{2} \left( \frac{z}{r_x} \right) \right)^2 + \left( \frac{z}{r_y} \right)^2 = 1
\]

where \( r_x \) and \( r_y \) are the semi-major and semi-minor axes, respectively.

The equation (2) is analogous to the equation of the orbit of a particle in circular motion around a central force.

The orbital motion is a special case of the general theory of relativity. We wish to derive

\[
\frac{r}{1} = \frac{(2)}{1}
\]
If the origin of the polar curve in the first quadrant is at point $P$, then $r = f(\theta)$ where $f(\theta) = 2\theta + \frac{\cos \theta}{\sin \theta}$.

To find the length of the curve from $0$ to $\pi$, we evaluate:

$$L = \int_0^\pi \sqrt{\left(\frac{dr}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \, d\theta$$

where $r = f(\theta)$. Integrating, we get:

$$L = \frac{\sqrt{3}}{2}$$

**Result:**

The length of the curve is $\frac{\sqrt{3}}{2}$ units.
effective radius of curvature $dz$

$$R(z) = \frac{z^2 + z_0^2}{z}$$

$$w^2(z) = \frac{(z^2 + z_0^2)^2}{Kz_0} = \frac{2z^2z_0^2}{2\pi M z_0}(1 + \left(\frac{z}{z_0}\right)^2)$$

Look at $w^2(z=0) = w$.

$$w^2(z) = w_0^2 \left(1 + \left(\frac{z}{z_0}\right)^2\right) = \frac{1}{e} \text{ width of radio as a function of } z.$$ From math we see that $W^2$ no matter what, $w_0$ smallest at $z=0$

where $W_0 = W^2(z=0) = \frac{2z_0^2}{2\pi \kappa} = \frac{2z_0}{K}$

**Physically, this is the radius of the beam at $z=0$.**

Ok, what the hell does this all mean?

$w_0 = W(z=0)$ is the $\frac{1}{e}$ radius of the beam at $z=0$. Ok, so what? Why is $z=0$ significant?

$$R(z=0) = \frac{z^2 + z_0^2}{z} = \infty !!$$

The radius of curvature is INFINITE at $z=0$
AND the width is THE SMALLEST, IT CAN BE (at least the math says so) at $z=0$.

**LET'S TALK REALITY.** What can we measure? How about $W_0$?
INTERPRETATION:

Under the Mach number

\[ w^2(z) = w_0^2 \left( 1 + \frac{z_0^2}{z^2} \right) \]

\[ w^2(z) = w_0^2 \left[ 1 + \left( \frac{z_0}{w_0^2 \pi} \right)^2 \right] \]

\( z_0 \) is the distance from the min-waist position of the waist where the waist has expanded to \( \frac{1}{2} \) times its original value.

\( z_0 = \frac{w_0^2 \pi}{2} \): the smaller \( w_0 \), the quicker the beam expands to \( \frac{1}{2} \) of its min value.

What about \( R(z) \) the "effective radius of curvature."

\[ R(z) = \frac{z}{1 + \left( \frac{z_0}{z} \right)^2} \]

\[ R(z) = \frac{z}{1 + \left( \frac{\pi w_0^2}{z_0^2} \right)^2} \]

Check limit \( z \rightarrow z_0 \)

\[ R(z) \approx \frac{z}{1 + \left( \frac{z_0}{z} \right)^2} \]

\[ R(z) \approx \frac{z}{1 + \left( \frac{\pi w_0^2}{z_0^2} \right)^2} \]

\( w(z) = \frac{w_0 z}{z_0^2} \) \[ \frac{1}{z_0^2} \] and note \( \frac{d[w(z)]}{dz} = \text{slope @ } z = z_0 \)

\[ \omega = \frac{w_0}{z_0} = \frac{w_0^2 \frac{\pi}{2}}{w_0 \pi} \]

\[ \omega = \frac{w_0}{z_0} = \frac{w_0^2 \frac{\pi}{2}}{w_0 \pi} \]
So, at \( t = 0 \), it's as if you're observing the phase from a point source a dist \( t(\pm \infty) = \infty \) away.

As \( \xi \) increases, \( k(\xi) \) remains very large but quickly approaches the value \( \xi = \) so the form \( \pm \xi \) at always looks like the effective point source rad.in at \( \xi = 0 \).

**EX: At commercial He-Ne laser in air:etched to have a far field divergence half angle of \( 1 \text{ mrad} \). \( \lambda_0 = 632 \text{ nm} \). What is spot size \( W_0 \)?

**Far field \( \frac{\lambda}{2\theta} \) half angle \( \theta = 0.5 \text{ mrad} \).

\[
\frac{1}{\text{mrad}} = \frac{\lambda}{W_0 \theta}
\]

\[
W_0 = \frac{\lambda}{10^{-3} \pi \theta}
\]

\[
= \frac{506 \cdot 10^{-9}}{10^{-3} \pi (0.5)} = \frac{506 \cdot 10^{-6}}{\pi} = 150 \text{ mm}
\]

**Power = \( 5 \text{ mW} \). What is intensity \( \frac{\text{W}}{\text{m}^2} \)?

waist \( (z = 0) \)? Spot size area = \( \pi W_0^2 \)

\[
I = \text{Power} \div \pi W_0^2 \cdot m^2 = \frac{5 \cdot 10^{-3}}{\pi (150^2)(10^{-6})^2}
\]