UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
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Problem Set No. 4

Problem Number one) Diffraction
a) Problem 10.22
b) Show that the Fresnel diffraction of a Gaussian distributed transverse electric field with a plane phase front yields the appropriate dispersing Gaussian (Gaussian with the correct curvature and width)
c) Problem 10.25 of text
d) A $10\mu m$ laser beam is propagated between 2 planes separated by 5 cm. Treat the beam as a spherical wave (point source) detected at a point on the receiving plane. A razor blade is inserted into the beam a distance of 5mm perpendicular to the line of sight. By how much is the intensity detected along the line of sight diminished?

Problem Number two) Fourier Transform Property of a Lens
a) Show using the Fresnel integral that the field in the focal plane to the right of a thin lens will be the Fourier transform of the field located at a distance f to the left of the lens. (Ref Ramo, Whinnery, and Van Duzer - Fields and Waves in Communication Electronics)

Problem Number three) The Oscillator model for dielectrics (solid state)
a) Verify that the dipole matrix element squared of the periodic wave functions for a cubic crystal

$$\psi_1 = \frac{u_1(\bar{r})}{\sqrt{V}} e^{i\bar{k}_1 \cdot \bar{r}}$$

and

$$\psi_2 = \frac{u_2(\bar{r})}{\sqrt{V}} e^{i\bar{k}_2 \cdot \bar{r}}$$

where V is the volume of the crystal, is of the form;

$$|M|^2 \frac{(2\pi)^3}{V} \delta^3(\bar{k}_1 - \bar{k}_2)$$

where M is the momentum matrix element over a unit cell.
b) Thus using the oscillator model verify the absorption expression for parabolic bands

$$\alpha = |x|^2 \frac{\omega m q^2}{c\epsilon\hbar^2 2\pi} \sqrt{\frac{m}{\hbar^2}} (\hbar\omega - E_G)^{1/2}$$

where $qx = \frac{qM}{m\omega}$ is the dipole matrix element
c) Explain the difference between this expression and Eq. (15.2-13) of Yariv:

$$\alpha = \frac{\lambda^2}{8\pi^2\tau}(\frac{m}{\hbar})^{3/2}(\omega - E_g/\hbar)^{1/2}[f_c - f_v]$$

Problem Number four) Basic Optical Processing Grating Pair
Consider a parallel grating pair with separation b. Let the periodicity of the grooves be d. Assume both gratings are used in the -1 order.
a) show that the ray path length through the grating pair relative to that without the grating is given by:

$$p = b(1 - cos\theta)/(cos(\gamma - \theta)$$

where θ is the reflection angle of the ray with respect to the incident input ray.
b) Show that the dispersion of the pair is given by:

$$\frac{d^2k}{d\omega^2} = \frac{1}{c}dp/d\omega$$

where p is the ray path length through the grating with respect to the path length in the absence of a grating pair. (Hint : do not forget the phase shift from groove to grove when not working in the zeroth order)
c) Show that this is equal to:

$$dp/d\omega = \frac{4\pi^2c^2b}{\omega^3d^2}[1 - (\frac{2\pi c}{\omega d} - sin\gamma)^2]^{-3/2}$$

where γ is the incident angle on the input grating.
d) Show that such a pair can be used to compress a positively chirped pulse (one that has a frequency sweep that goes from low frequency to high frequency across the pulse) (ref Yariv). Use a grating spacing of .22 cm, a groove spacing $d^{-1} = 1300$ lines per millimeter and an incident angle $\gamma = 60\,\text{deg}$ Assume a Gaussian optical pulse:

$$E(z,t) = E_0e^{i(\omega t + (B/2)t^2) - t^2/\tau^2}$$

at a wavelength of $1\mu n$ at the entrance of the grating pair. Let $B = 8 \cdot 10^{13}Hz/psec$ and let $\tau = 5.0psec$