# Basic Pulse Compression



Assuming an input Gaussian pulse, the phase profile in the vicinity of the pulse peak is quadratic and thus the spectrum has a linear frquency sweep from a down-shift at the leading edge to an up-shift on the trailing edge. If we use a quadratic approximation for the phase, the Fourier transform of the electric field coming out of the fiber, $F(\omega)$ is given by:

$$F(\omega) = \int_{-\infty}^{\infty} exp(-j\omega t + \left(\frac{t}{t_p}\right)^2 + jBt^2)dt \tag{10.3.1}$$

by completing the square and letting $u = t + \frac{j\omega}{2(\frac{1}{t_p}^2 - jB)}$ we get,

$$F(\omega) = e^{\left(\frac{-\omega^2}{4(\frac{1}{t_p}^2 - jB)}\right)} \int_{-\infty}^{\infty} e^{(-\frac{1}{t_p}^2 + jB)u^2} du = \sqrt{\frac{\pi}{\frac{1}{t_p}^2 - jB}} e^{\left(\frac{-\omega^2}{4(\frac{1}{t_p}^2 - jB)}\right)} \tag{10.3.2}$$

$$F(\omega) = \sqrt{\frac{\pi}{\frac{1}{t_p}^2 - jB}} exp\left(\frac{-\omega^2}{4t_p^2(\frac{1}{t_p^4} + B^2)}\right) exp\left(\frac{-jB\omega^2}{4(\frac{1}{t_p^4} + B^2)}\right) \tag{10.3.3}$$

To compress the pulse a linear dispersive element with the opposite quadratic frequency sweep is used to cancel the chirped phase component due to SPM, and therefore its form should be:

$$G(\omega) = de^{-j\frac{a}{2}\omega^2} \tag{10.3.4}$$

where d is a constant and a is the quadratic dispersion parameter exactly equal in size but opposite in sign to the chirp factor in $F(\omega)$, that is

$$a = \frac{1}{2B}$$

The bandwidth of the spectrally broadened pulse emerging from the fiber can be estimated by:

$$(\Delta\omega)^2 = 4t_p^2(\frac{1}{t_p^4} + B^2) \qquad or \qquad \Delta\omega = 2Bt_p \tag{10.3.6}$$

if it is assumed that $B >> \frac{1}{t_p^2}$, that is that the chirp is large compared to the initial bandwidth of the pulse divided by initial pulse duration.

There are several ways to implement the dispersive delay line; Gratings , bromobenzene, interferometers, Bragg scattering, resonant molecular vapors, prisms and even the fiber itself (operating in the negative group dispersion regime and yielding soliton solutions). Gratings act as a frequency selective mirror by reflecting different frequencies at different angles. A parallel pair of gratings can be arranged in such a way as to make lower frequencies travel further than shorter frequencies. while spatially separating them. Alternatively a grating or prism pair, when arranged properly, can impose a chirp equal in magnitude but opposite in sign to the chirp caused by SPM and thus the chirp acquired by SPM can be canceled by the chirp imposed by the grating pair. The difference between prism pairs and grating pairs arises from a different inherent angular dispersions. In general, grating pairs are more dispersive than prism pairs, however their angular dispersion is a function of the groove spacing. The advantage of prisms over gratings lies in their greater efficiency especially when employing Brewster prisms.

The two-stage pulse compression technique has been used extensively for radar applications, eventually being adapted for optical pulse compression. The principle underlying chirped radar, optical pulse compression and some novel electromagnetic short pulse generation techniques, are the same.

## 10.4 Theory of Compression by Dispersive Elements- Gratings

The compression of pulses with spectra broadened by SPM was first accomplished by Treacy using the grating pair, which has since seen broad application in the optical signal processing area. The anomalous dispersion characteristics arises from a combination of a spectrally dependent path delay and phase shift due to the grating momentum component. It is anomalous because the spectral rate of change of the phase is negative as we will show. The general grating arrangement is illustrated in fig.(9), taken from the initial development given by Tracey . Phase delay arising from different

10.4.1

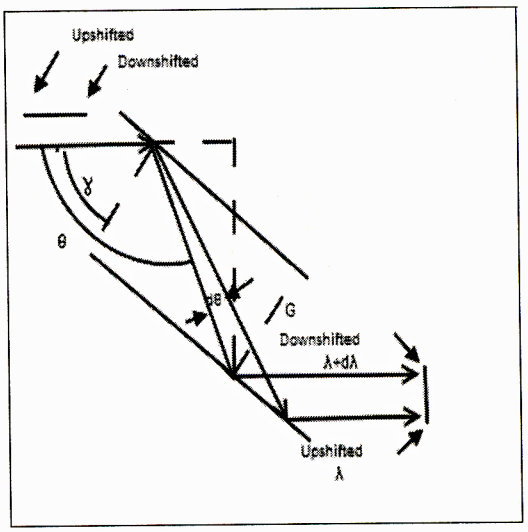


Figure 10.4.1: The Treacy Grating Pair

optical paths traveled by the different frequencies is given by:

$$\Phi(\omega) = \frac{\omega}{c}P - 2\pi\frac{G}{\Lambda}tan\theta_r \tag{10.4.1}$$

The second term accounts for a $2\pi$ phase shift per ruling in the first order diffraction. This term can be deduced by referring to The incident angle on the first grating is $\theta_i = \gamma$ and the reflected angle is $\theta_r = \theta - \gamma$ for wavelength $\lambda$. For a wavelength shift of $d\lambda$ the reflection angle is shifted by $d\theta_r = d\theta$. The path length between the gratings is $G/cos(\theta_r)$. The distance between the reflection points on the second grating at $\lambda$ and $\lambda + d\lambda$ is then $dl = (G/cos(\theta_r)d\theta)/cos(\theta_r)$. The phase shift $d\phi$ is thus $-dl\frac{2\pi}{\Lambda}$. Thus

$$d\phi/d\theta_r = -\frac{2\pi G}{\Lambda}/(cos(\theta_r)^2) \tag{10.4.2}$$

Integrating this gives the required extra phase term above. The optical path P can be expressed as the distance between the path across the grating pair minus this path length projected onto the direction of the incident beam.

$$P = \frac{G}{cos\theta_r}(1 + cos(\theta_r + \theta_i)) \tag{10.4.3}$$

The second term is negative since $\theta_r + \theta_i = \theta$ is greater than $\pi/2$. $\theta_r$ and $\theta_i$ are related through the grating equation:

$$sin\theta_r = sin\theta_i + \frac{n\lambda}{\Lambda} \tag{10.4.4}$$

where n is the order of the grating reflection and $\Lambda$ is the spatial grating period, but here $n = -1$. Since we are interested in dispersion parameter $a = -\frac{\partial^2\Phi}{\partial\omega^2}$, taking second derivative of $\Phi(\omega)$ in eq.(2.2.1) yields:

$$\frac{\partial\Phi}{\partial\omega} = \frac{P}{c} + \frac{\omega}{c}\frac{\partial P}{\partial\omega} - 2\pi\frac{d}{\Lambda}\frac{\partial}{\partial\omega}tan\theta_r = \frac{P}{c} \tag{10.4.5}$$

since one finds that the second and third terms cancel. Thus

$$\frac{\partial^2\Phi}{\partial\omega^2} = \frac{1}{c}\frac{\partial P}{\partial\omega} = \frac{G}{c}\left(\frac{\lambda}{cos^2\theta_r\Lambda}\right)\frac{\partial\theta_r}{\partial\omega} \tag{10.4.6}$$

Taking the derivative of the grating equation ( eq.(2.2.2)),

$$\frac{\partial\theta_r}{\partial\omega} = -\frac{\lambda}{\omega\Lambda cos\theta_r} \tag{10.4.7}$$

plugging the above result into eq.(2.2.4), yields an expression for the parameter a:

$$a = \frac{G\lambda^2}{c\omega\Lambda^2\,(cos^2\theta)^{\frac{3}{2}}} = \frac{4\pi^2 Gc}{\omega^3\Lambda^2}\left(1 - (sin\theta_i + \frac{\lambda}{\Lambda})^2\right)^{-\frac{3}{2}} \tag{10.4.8}$$