EE119 Introduction to Optical Engineering
Spring 2009
Midterm Exam

Name: Solutions

Signature: ____________________________________________

SID: ___________________________

CLOSED BOOK. ONE 8 1/2" X 11" SHEET OF NOTES, AND SCIENTIFIC POCKET CALCULATOR PERMITTED.

TIME ALLOTTED: 50 MINUTES

Fundamental constants you might need:

Planck’s constant, $h = 6.62 \times 10^{-34} \text{ J-s}$
Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
Permeability of free space, $\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$
Speed of light in vacuum, $c = 2.998 \times 10^8 \text{ m/s}$
Electron charge, $e = 1.6 \times 10^{-19} \text{ C}$
Free electron mass, $m_e = 9.1 \times 10^{-31} \text{ kg}$
Electron volt, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
1) [15 points] The distance between an object and a screen is 1 meter. A thin biconvex lens located at 0.75 meters from the object forms an image 10 cm in front of the screen. You can add another lens right next to the original lens to change the effective focal length. What should be the focal length of this second lens in order to cause the image to be formed on the screen?

\[
\frac{1}{f} = \frac{1}{d_i} - \frac{1}{d_o} = \frac{1}{75} + \frac{1}{75} = \frac{2}{75} \Rightarrow f = \frac{75}{2} = 37.5 \text{ cm}
\]

We want \(d_i = 25 \text{ cm}\)

\[
\frac{1}{f'} = \frac{1}{25} + \frac{1}{75} = \frac{4}{75} \Rightarrow f' = 18.75 \text{ cm}
\]

So we need \(-\frac{3}{25} = \frac{1}{f_c}\)

Corrector lens \(f_c = -\frac{75}{2} = -37.5 \text{ cm}\)
2) [20 points] The left end of a long glass rod of index 1.5 is ground and polished to a convex spherical surface of radius 2. cm. A small object is located in the air to the left of the rod, and on the optical axis 10 cm from the vertex. The image is formed inside the rod. Find the
a) [10 points] front and back focal lengths,
b) [5 points] image distance, and
c) [5 points] transverse magnification.

\[
\frac{n' l'}{l'} - \frac{n}{l} = \frac{n' - n}{R}
\]

a) Back focal length: \( l' \to -\infty \)

\[
\frac{n'}{l'} = \frac{n' - n}{R}
\]

\[
l' = \frac{n'}{n' - n} R = 6 \text{ cm}
\]

so bffl is 6 cm.

Front focal length: \( l' \to -\infty \)

\[
-\frac{n}{l'} = \frac{n' - n}{R}
\]

\[
l = \frac{n}{n' - n} R = -4 \text{ cm}
\]

so fffl is 4 cm.

b) \( l = -10 \text{ cm} \)

\[
\frac{l'}{l} = \frac{0.5}{2} - \frac{1}{10} = 0.25 - 0.1 = 0.15
\]

\[
l' = \frac{1.5}{0.15} = 10 \text{ cm}
\]

c) \( M = \frac{n l'}{n' l} = \frac{10}{1.5(-10)} = -\frac{1}{1.5} = -\frac{2}{3} \)
3) [15 points] A beam of natural (unpolarized) light in air ($n_a=1$) is incident on a glass surface ($n_g=1.5$).
   a) [5 points] Is there a critical angle in this case? Explain your answer. If so, calculate it.
   b) [10 points] Is there a Brewster's angle in this case? Explain your answer. If so, calculate the Brewster's angle for this situation, sketch a ray incident on the surface at Brewster's angle, and show the direction of polarization for which the reflected beam disappears.

\[ \Theta_B = \tan^{-1}(n_g) = 56.3^\circ \]

\[ \text{Air beam for p-polarized light} \]

\[ \text{Glass} \]

\[ \text{p-polarization} \]
4) **[10 points]** Light of wavelength 600nm in vacuum enters a block of glass where $n_e=1.5$.
   a) **[5 points]** Compute the wavelength in the glass.
   b) **[5 points]** What is the photon energy of the light in vacuum (in eV)? What is the photon energy of the light in glass (in eV)?

   
   a) \[ \lambda_{\text{glass}} = \frac{\lambda_{\text{vac}}}{n} = \frac{600 \text{ nm}}{1.5} = 400 \text{ nm} \]

   b) \[ h\nu_{\text{vac}} = 2.066 \text{ eV} \]

     In glass, $h\nu$ is the same
5) [15 points] A diver looking straight up toward the smooth surface of a pond sees a circle of light filled with an image of the sky, birds, and whatever else is up there. This bright circular field is surrounded by darkness, even though it is bright daylight out there. Explain what is happening and compute the cone angle for the bright circle from the point of view of the diver. Use \( n=4/3 \) for the water. Sketch a diagram illustrating the situation.

\[
\sin \theta_c = \frac{n_1}{n_2} = \frac{3}{4} \\
\theta_c = 48.6^\circ \\
\text{full cone angle} = 2\theta_c = 97.2^\circ
\]
6) [25 points] The size of the sensor in a digital camera is an important property of the camera. Generally, more expensive cameras have larger sensors. In this question, we explore the effect of sensor size on various imaging properties of the camera. Consider a photographer with two cameras, Camera A with a smaller sensor, and Camera B with a larger sensor. The sensor in the Camera A is 1.6 times smaller than the sensor in Camera B. Each camera has a zoom lens. Suppose the photographer is taking a picture of a group of people at a distance of 20 feet. With each camera, he adjusts the zoom so that the group just fills the frame, and he sets the lens f-stop to f8 for both cameras.

a) [10 points] Will the zoom lens on each camera be set to the same focal length? Explain your answer.

b) [15 points] Will the diffraction limited resolution on the object-side be the same or different for the two photos? If different, which camera will produce the greater object-side diffraction limited resolution? Explain your answer. [Hint: This question is tricky! Think it through very carefully. A diagram will be helpful.]

a) Image size is different for the same object size. Therefore the lens focal length must be different for Camera A and Camera B.

b) On the image side, the f# or NA is the same. So the resolution on the image side is the same in both cameras. But the magnification is different.

\[
M^A = \frac{h_s^A}{h_o^A} \\
M^B = \frac{h_s^B}{h_o^B}
\]

We have \( h_o^A = h_o^B \) since the object size is the same, also \( h_s^A = \frac{h_s^B}{1.6} \) \( \Rightarrow M^A = M^B / 1.6 \).

Camera A has smaller magnification and thus a larger object-side diffraction-limited spot size. Camera B has the smaller object-side spot and we refer to that as a higher resolution.