EE119 Homework 10 Solutions: More on Lasers: Broadening, Gaussian Beams

Professor: Jeff Bokor GSI: Julia Zaks

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1. Identify each of the following broadening mechanisms as homogeneous or inhomogeneous. Explain your answer.

   (a) Collisions between atoms in a gas
   Solution:
   This is homogeneous broadening because all the atoms experience the same effect

   (b) Randomly spaced impurities in a semiconductor crystal
   Solution:
   This is inhomogeneous broadening because the amount of broadening you get from this effect varies depending on the distance from an impurity, which varies from atom to atom.

   (c) Temperature differences between different regions of the gain medium.
   Solution:
   This is also inhomogenous because different areas of the gain medium have different amounts of this broadening.

   (d) Vibrational relaxation within an energy band of an atom or semiconductor (this is the same thing as dissipation of electronic energy into phonons within an energy band).
   Solution:
   This is homogeneous, because each atom undergoes this process in the same way.

2. Show that the magnitude of the radius of curvature of a Gaussian beam is changed upon reflection from a spherical mirror, unless (a) the mirror has infinite radius of curvature, or (b) the radius of curvature of the mirror equals that of the Gaussian beam.

   Solution:

   A mirror with radius of curvature r has a focal length of -r/2, so the phase shift for a mirror of radius of curvature R is

   $$\phi(x,y) = -k\frac{x^2+y^2}{2}\frac{1}{f} = +k\frac{x^2+y^2}{r}$$

   At point z, the wavefronts of a gaussian beam with a radius of curvature R(z) all appear to be spherically diverging fronts coming from a point a distance R(z) away from the lens. Consequently, the mirror "sees" spherical wavefronts, and reflects them as thought they were spherical wavefronts. Using our results from homework 3, we can write the phase of the incident wave fronts as

$$\psi_i(x,y) = -k\frac{x^2+y^2}{2R}$$

Adding to this the phase shift from the mirror, we get a resultant reflected phase of

$$\psi_r(x,y) = -k\frac{x^2+y^2}{2R} + k\frac{x^2+y^2}{r} = -k\frac{(x^2+y^2)(2R-r)}{2Rr}$$

If R=r then ψ_i will equal ψ_r and the wavefront will be unchanged, leading to resonance. Otherwise, the beam will change upon reflection and that particular Gaussian mode will not be reinforced by the cavity.

3. A TEM00 He-Ne laser (λ = 632.8 nm) has a cavity that is 0.34 m long, a fully reflecting mirror of Radius R = 10m (concave inward), and an output mirror of radius R = 10 m (also concave inward).

   (a) From the symmetry of mirror geometries and the boundary condition that wavefront and mirror cavities match at the mirrors, determine the location of the beam waist in the cavity. Set z = 0 at this location to be the reference plane.

   Solution:

   The two mirrors have the same radius of curvature, so the waist will be in the center of the cavity, 0.17 m from each mirror.

   (b) Determine the size of the beam waist (w_0). Because the radius of curvature of the beam must be unchanged at the end mirrors, we know that R(0.17)=10m. The Rayleigh Range, and from it the beam waist, is fully determined from this information. An expression for the radius of curvature as a function of displacement from the waist is

   $$R(z) = z + \frac{z_R^2}{z}$$

   Plugging in numbers and solving for the Rayleigh range z_r:

   $$10 = 0.17 + \frac{z_R^2}{0.17}$$

   $$z_R = \sqrt{1.7 - 0.0289} = 1.3\text{meters}$$

   and since the waist size is related to the Rayeleigh range through

   $$z_R = \frac{\pi w_0^2}{\lambda}$$

   we find that

   $$w_0 = \sqrt{\frac{1.3\times 632.8\times 10^{-9}}{\pi}} = 5.1\times 10^{-4} = 0.5\text{mm}$$

   (c) Determine the beam spot size $w(z)$ at the left and right cavity mirrors.

   Solution:

   The spot size at distance z from the waist is given by

   $$w(z) = w_0\sqrt{1+(z/z_R)^2}$$

   so at z=0.17,

   $$w(z) = 0.5\sqrt{1+(\frac{0.17}{1.3})^2} = 0.5\text{mm}$$

(d) Determine the half-angle beam divergence (θ) for this laser.

Solution:

In the paraxial regime,

$$\theta_d = \frac{\lambda}{\pi w_0} = 4 \times 10^{-6}$$

(e) Where is the far field for this laser if you use the criterion $z_{FF} \geq 50(\pi w_0^2/\lambda)$?

Solution:

$$\frac{\lambda}{\pi w_0^2} = 1.29$$

So the far field region occurs when $z \geq$64.6 meters.

(f) If the laser emits a constant beam of power 5mW, what is the average irradiance at the position where $z_{FF} \geq 50(\pi w_0^2/\lambda)$?

Solution:

At $z = 64.6$ meters, the waist will be $w(z) = 0.5\sqrt{1 + (64.6/1.3)^2} = 24.6\text{mm}{=}0.0246$ m. The area of the beam is then $\pi \times 0.0246^2{=}0.0019{\approx}\ 2 \times 10^{-2}\text{m}^3$. The average intensity is then $5 \times 10^{-3}/2 \times 10^{-3}{=}2.5$ Watts per square meter.

4. Compare the irradiances at the retina that result when looking:

(a) Directly into the sun. The sun subtends an angle of 0.5 degrees. At the earths surface, the suns irradiance is 1kW/m^2. Assume that the pupil of the bright-adapted eye is 2mm in diameter with focal length of 22.5mm.

Solution:

The radius of the pupil is 1 mm, so the area is $\pi \times 10^{-6}$ square meters. The intensity entering the eye is

$$P = 1 \times 10^3 \times \pi \times 10^-6 = \pi \times 10^{-3} = 3.14\text{mW}$$

The sun subtends an angle of 0.5 degrees at the pupil, and the light travels 22.5 mm to the retina, so the radius of the image of the sun ends up being 22.5 mm$\times \tan(0.5) =$ 0.1964mm. The area of this image is 0.12 mm^2. The intensity at the eye is then

$$I_{\text{sun}} = \frac{3.14 \times 10^{-3}}{1.2 \times 10^{-7}} = 2.4 \times 10^4 \text{Watts/m}^2 = 2.4\text{Watts/cm}^2$$

(b) Into a 1mW He-Ne laser. Assume the beam waist of the laser is 1mm, and the laser is located 1m from the eye.

Solution:

The rayleigh range of the laser is

$$z_r = \frac{\pi w_0^2}{\lambda} = \frac{3.14 \times 1 \times 10^{-6}}{632.8 \times 10^{-9}} = 4.96\text{meters}$$

The beam radius of curvature at 1m from the waist (at the eye) is

$$w(1) = 1 \times 10^{-3}\sqrt{1 + (1/4.96)^2} = 1.02\text{mm}$$

The area of the beam is 3.77×10^{-6} m^2 . The radius of curvature of the beam is

$$R(z = 1\text{m}) = z + z_R^2/z = 1 + 4.96^2 = 25.6\text{m}$$

After passing through the eye with focal length 22.5 mm, the radius of curvature of the beam will be

$$\frac{1}{R_eye} = \frac{1}{f} + \frac{1}{R(z)} = \frac{1}{0.0225\text{m}}$$

(Notice that the curvature of the beam is so small (radius so big) that you can ignore it). Now we have a Gaussian beam traveling into the eye, with radius of curvature 0.0225 meters at z=0.0225. We can solve for the beam waist:

$$w_0 \approx \frac{R_{\text{eye}}\lambda}{\pi w_{\text{lens}}} = \frac{0.0225 \times 632 \times 10^{-9}}{\pi 0.00102} = 4.44 \times 10^{-6}$$

So the area of the beam at the retina is 6.2×10^{-11}m^2. The intensity, then, is So the intensity is

$$I_{\text{laser}} = \frac{1 \times 10^{-3}}{6.2 \times 10^{-11}} = 1.61 \times 10^7 \text{Watts/m}^2 = 1.61 \times 10^3 \text{ Watts/cm}^2$$

(c) Which one will damage your eye? Eye-damaging intensities are in the range of 10 μW/cm^2. Both will damage your eye. But the LASER will cause MORE DAMAGE!!!

5. Gaussian beam/lens The laser resonator shown in the figure below with z = 0 located at the flat mirror and its output impinges on a lens of focal length 10cm. Assume the beam waist size, w_0=0.5mm; laser wavelength, λ = 632.8nm; and distance of the lens to laser output mirror, d=50 cm.

   (a) What is the far-field beam divergence in mrad?
   Solution:
   The far field divergence is $\lambda/(\pi w_0) = 4 \times 10^{-4}$ radians.

   (b) What are the spot size and radius of the curvature of the output laser beam on the lens?
   Solution:
   The Rayleigh range of the laser is

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{\pi 25 \times 10^{-8}}{632.8 \times 10^{-9}} = 1.24\text{meters}$$

   the radius of curvature is

$$R(z) = z + \frac{z_R^2}{z} = 0.5 + 3.08 = 3.58\text{meters}$$

$$w(z = 0.5\text{m}) = w_0\sqrt{1 + [z/z_R]^2} = 5 \times 10^{-4} \times (1 + (0.5/1.24)^2) = 5.4 \times 10^{-4}\text{meters}$$

   (c) What is the radius of the curvature after passing through the lens?
   Solution:
   The "source" of the beam is to the left, so its radius of curvature is negative.

$$\frac{1}{R_o} = \frac{1}{f} + \frac{1}{R(z)} = \frac{1}{0.1} - \frac{1}{3.58} = \frac{1}{.103}$$

   The radius of curvature of the beam after passing through the lens is 10.03 cm.

(d) What is the spot size at the focal point after the lens if the clear aperture of the lens is 1.5 cm in radius? And what is the far-field beam divergence with the lens?

Solution:

The clear aperture of the lens is larger than the beam (1.5¿0.54) so the radius of the beam at the lens is 0.58 mm. The spot size at the focal length is

$$w_0 = \frac{R_0 \lambda}{\pi w_{\text{lens}}} = \frac{0.103 \times 632.8 \times 10^{-9}}{\pi \times 0.5 \times 10^{-4}} = 3.63 \times 10^{-5} \text{meters}$$

The spot size at the focal point is 0.0363 mm in radius.

(e) What is the beam radius if the laser beam is propagated 1m further after the focal point? The rayleigh range is

$$z_R = \frac{\pi w_0^2}{\lambda} = 0.065 \text{ meters}$$

So the beam spot size at 1 meter from the focal point is

$$w_z = w_0 \sqrt{1 + (z/z_R)^2} = 15.4 \times 3.64 \times 10^{-5} = 5.6 \times 10^{-4}$$

The beam radius is 1.7 mm at 1 meter past the focal length. And (you didn't have to solve for this) the radius of curvature is

$$R = 1 + 0.065^2/1 = 1.00 \text{ meters}$$

The far-field beam divergence with the lens is

$$\theta_d = \frac{\lambda}{\pi \times w_0} = 0.0055 \text{ radians}$$

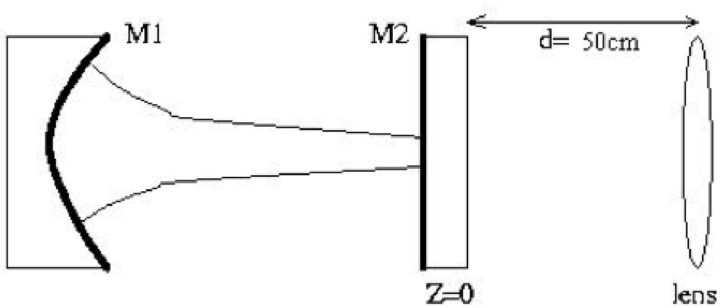

Figure 1: Laser Producing Gaussian Beam and 10cm focal length lens