EE119 Homework 1

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1. The Electromagnetic Spectrum and Units

   Fill in the table below:

   Solution:

   For conversion of wavelengths to Hz: $\lambda = \frac{c}{\nu} = 3 \times 10^8 m/s \times \nu \frac{1}{s}$

   There are 1.6×10^{-19} Joules in an electron Volt (eV). so

$$E = \frac{h\nu \text{Joules}}{1.6 \times 10^{-19} \text{Joules/eV}} = \frac{6.62 \times 10^{-34} \times \nu}{1.6 \times 10^{-19}} \text{eV} = (4.14 \times 10^{-15} \times \nu) eV$$

	wavelength(nm)	energy (eV)	frequency (Hz)
Silicon bandgap	1128	1.1	2.659×10^{14}
infrared light	1200	1.034	2.5×10^{14}
red ligth	630	1.97	4.762×10^{14}
yellow light	575	2.159	5.217×10^{14}
green light	530	2.342	5.66×10^{14}
blue light	450	2.758	6.667×10^{14}
ultraviolet light	100	12.41	3×10^{15}
x-rays light	5	248.3	6×10^{16}
Power Transmission	5×10^{15}	2.483×10^{-13}	60
KDFC Radio Station	2.938×10^9	4.224×10^{-7}	102.1 MHz

2. Unpolarized sunlight is reflected off the surface of a lake (n_{water}=1.33) at an angle of 70° to normal. What fraction of the total incident sunlight is reflected? What fraction of the reflected light is s-polarized? What fraction is p-polarized?

   Solution:

   Since the sunlight is unpolarized, we can assume that half of it is s-polarized and half of it is p-polarized. The fraction of incident light that is reflected will be the average of the reflection coefficients for the two polarizations. We can use the fresnel coefficients of p. 7 of notes 1. At an incident angle of 70°, θ_2 for n=1.33 will be 0.746 radians ≈ 45 °. So we get that $R_s = 0.2180$ and $R_p = 0.0473$, so the fraction of total incident light that will be reflected is 0.0775=7.75%. Of this light, 0.2180/(0.2180+0.0473)=82.2% will be s-polarized and 0.0473/(0.2180+0.0473)=17.8% will be p-polarized.

3. (a) Unpolarized light passes through two linear polarizers. The second polarizer is aligned so that its transmission axis is 80° from that of the first polarizers. What fraction of the incident is transmitted through the two polarizers? How is this light polarized?

Solution:

The intensity of light passing through a polarized is given by $I = I_0\cos^2(\theta)$ where θ is the angle between the polarization of the light and the transmission axis of the polarizer. Since unpolarized light has light polarized equally in all directions, we can guess that half of the light will be transmitted in a specific polarization direction. We can also do this more rigorously by assuming a uniform angular distribution:

$$I = I_0 \int_0^\pi \cos^2((\theta_{pol} - \theta_{inc}))\frac{1}{\pi}d\theta_{inc} = -I_0(\frac{1}{2\pi}(\theta_{pol} - \theta_{inc}) + \frac{1}{4\pi}\sin(2(\theta_{pol} - \theta_{inc}))|_0^\pi\frac{1}{\pi}$$

$$= -I_0(\frac{\theta_{pol} - \pi - (\theta_{pol} - 0)}{2\pi} + \frac{\sin(2(\theta_{pol} - \pi)) - \sin(2(\theta_{pol} - 0))}{4\pi}) = I_0\frac{\pi}{2\pi} = \frac{I_0}{2}$$

and get, after all that work, that half of the intensity of unpolarized light is transmitted through a polarizer. This light is polarized along the axis of the first polarizer. When the light is transmitted through the second polarized, it emerges polarized at 80°from the direction of the first polarizer. The intensity of the transmistted light will be $\frac{I_0}{2}\cos^2(80) = 0.015I_0$, so 1.5% of the initial light intensity will be transmitted through the two polarizers.

(b) You position a third linear polarizer between the first two. At what angle should the transmission axis of this polarizer be aligned to maximize the total intensity of transmitted light?

Solution:

The intensity of the transmitted light will be $I_t = I_0\cos^2((\theta 1 - \theta_2))\cos^2((\theta 2 - \theta 3))$. The relative angle between θ_1 and θ_3 is fixed, so we want to find θ_3 to maximize this I_t. We can define the direction of θ_1 to be $\theta = 0$. We find the maximum of the transmitted intensity by taking derivatives:

$$\frac{\partial I_t}{\partial \theta_2} = -2\sin\theta_2\cos\theta_2\cos^2(\theta_2 - \theta_3) - 2\cos^2(\theta_2)\sin(\theta_2 - \theta_3)\cos(\theta_2 - \theta_3) = 0$$

$$-\sin\theta_2\cos(\theta_2 - \theta_3) - \cos\theta_2\sin(\theta_2 - \theta_3) = 0$$

$$-\sin\theta_2\cos(\theta_2)\cos(\theta_3) - \sin^2(\theta_2)\sin\theta_3 - \cos\theta_2\sin(\theta_2)\cos(\theta_3) + \cos^2(\theta_2)\sin(\theta_3) = 0$$

$$-2\sin\theta_2\cos(\theta_2)\cos(\theta_3) + (\cos^2(\theta_2) - \sin^2(\theta_2))\sin\theta_3 = 0$$

$$\sin(2\theta_2)\cos(\theta_3) - \cos(2\theta)\cos\theta_3 = 0$$

so

$$\tan(2\theta_2) = \tan(\theta_3)$$

so the most light is translated when $\theta_2 = \theta_3/2$, when the axis of the second polarizer is positioned halfway between the axis of the first and third polarizer. See figure 1.

4. (a) Superman has been told that he is faster than a speeding bullet, but he has never checked whether this is actually true. He does an experiment and finds he is indeed faster than a speeding bullet! In fact, his friend Spiderman found that it only took him 2 seconds to fly around the earth (40,000km). Superman decides to try and impress Lois Lane by showing her that he is faster than light! He is looking for a material to build a 100m tube in which he can pipe light into, and race it for 100m.

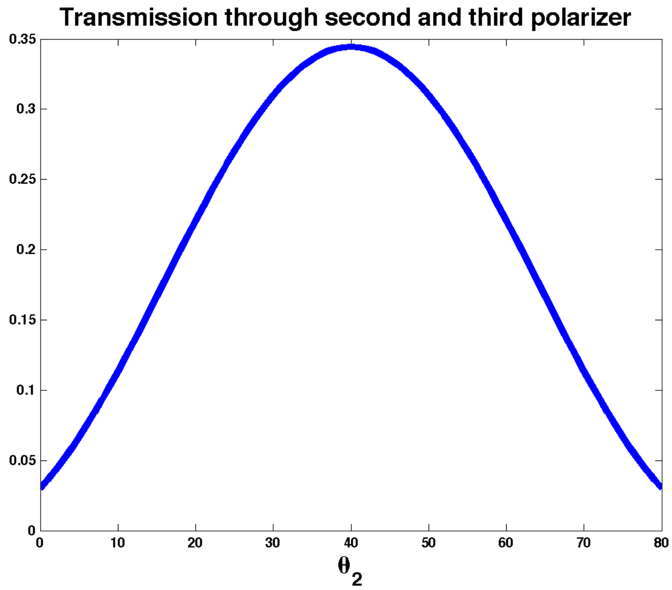

Figure 1: The fraction of light transmitted through polarizer 3 as a function of the angle between polarizer 2 and polarizer 1 (problem 3b).

What is the index of refraction he needs the tube to be made out of if he is to tie the 100m race?

Solution:

Superman's speed was $\frac{40{,}000\text{km}}{2\text{seconds}} = 2 \times 10^7 \frac{\text{m}}{\text{s}}$. Light travels at a speed of $c = 3 \times 10^8 \frac{\text{m}}{\text{s}}$ in a vacuum. In a material with index of refraction n, light travels a a speed of $v = c/n$, so Superman needs to find a material that will slow light down to his speed. So we solve for n:

$$n = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^7} = 1.5 \times 10^1 = 15$$

So he needs to make his tube out of a material that has a refractive index of at least 15. Notice that the length of the tube was irrelevant.

(b) Superman is happy with his win at the race and decides to make a cup of tea. He uses his heat vision to send infrared radiation (λ= 1000nm) at an angle of 75° from the normal towards a cup of water to heat it up. The index of refraction of water is 1.33. Assuming his eyes are unpolarized (equal s-polarization and p- polarization), what is the ratio of the energies of p-polarized to s-polarized light transmitted into the water? If his eyes were only p-polarized, what is the optimal angle to heat the water at, such that the infrared radiation is fully transmitted? What about if his eyes were only s-polarized?

Solution:

We can solve for the transmission using the Fresnel equations given on p. 7 of Notes 1. For p-polarized light, the fraction of incident light intesity that is transmitted is

$$T_p = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1} \frac{4\sin^2(\theta)\cos^2(\theta_1)}{\sin^2(\theta_1 - \theta_2)\cos^2(\theta_1 - \theta_2)}$$

We can calculate θ_2 using Snell's law. Snell's law tell us that at an interface, $n_1 \sin\theta_1 = n_2 \sin theta_2$. so if the light enters the water at $\theta_1 = 75$, we get that

$\theta_2 = \sin^{-1}(\frac{1}{n_2}\sin 75 \times \pi/180) = 46.6°$. At 75 degrees from normal,

$$T_p = \frac{1.33 \times 0.6874}{0.2588}\left[\frac{2 \times 0.9659\cos^2(\theta_1)}{\sin^2(\theta_1-\theta_2)\cos^2(\theta_1-\theta_2)}\right]^2$$

The ratio of p to s polarized light is 1.293.

5. (a) On a sunny day in Berkeley at noon, the sun shines from directly overhead with an intensity of approximately 10^{21} photons per square meter per second. You want to boil a liter (1000 grams) of water in a pot that has a diameter of 10 cm. Since water only absorbs infrared light (at approximately 1000 nm), each photon will deliver the same amount of energy to the water (This is, of course, a simplification, but it will give you an upper limit on how fast direct sunlight can boil water). Remember that it takes 1 calorie of energy to raise 1 gram of water 1 degree C, and that there are 4.18 Joules in a calorie. Water has an index of refraction of 1.33. What fraction of the incident sunlight will enter the water? (Hint: reflected and transmitted light intensity added together equal the incident light intensity). Assume for simplicity that all the light that enters the water is absorbed. How long will it take to bring the water to a boil from room temperature (25° C)?

Solution:

The pot of water has a surface area of $\pi * (0.05)^2 = 0.0079$ square meters. The number of photons striking the surface of this pot per second is 10^{21}photons/($\text{m}^2\times$second) $\times 7.9\times 10^{-3}\ \text{m}^2 = 7.9\times 10^{18}$ photons per second. Of these photons, only the ones that are transmitted into the water will heat it. At normal incidence ("directly overhead") the transmission is equal to

$$T_{\theta=0} = 1 - R_{\theta=0} = 1 - \frac{(n_2-n_1)^2}{(n_2+n_1)^2} = \frac{4n_1n_2}{(n_2+n_1)^2} = \frac{4\times 1.3}{2.3^2} = 0.983$$

so 98% of the incident light enters the water. This means that $0.983\times 7.9\times 10^{18} = 7.76\times 10^{18}$ photons will be absorbed by the water. The energy in a photon of 1000 nm is E=hν=h$\frac{c}{\lambda} = \frac{6.62\times 10^{-34}\times 3\times 10^8}{10^{-6}} = 1.987\times 10^{-19}$ Joules/photon. So each second, the water is absorbing $1.987\times 10^{-19}\times 7.76\times 10^{18} = 1.54$ Joules per second.

We want to raise the temperature of 1000 grams of water from 25 to 100 degrees Celcius, which so we need to put 75 degrees$\times 4.18\frac{\text{Joules}}{degree\times gram}\times$1000 grams $= 3.13\times 10^5$ Joules of energy into the water. This means we need to wait $\frac{3.13\times 10^5 \text{ Joules}}{1.54 \text{ Joules/second}} = 2.033\times 10^5$seconds. This is about 56 hours.

(b) Now assume that instead of "thermalizing" each photon and only getting 1000 nm worth of energy from it, the water is a black body that can absorb all the energy contained in a photon. Assume that the energies of incoming photons are uniformly distributed between infraredred (1000nm) and violet (400 nm) (remember that energy is proportional to frequency, not wavelength!). Now how long will it take to boil a liter of water in the container?

Solution:

Now the energies of the photons are uniformly distributed between 1.99 $\times 10^{-19}$ Joules/photon and $\frac{6.62\times 10^{-34}\times 3\times 10^8}{4\times 10^{-7}} = 4.97\times 10^{-19}$ Joules/photon. Since we're assuming that photon energies are uniformly distributed, the average photon energy absorbed by the water is just the average of these two numbers, 3.47×10^{-19} Joules/photon.

Since the rate of photons entering the water is unchanged, the water is now absorbing $3.47 \times 10^{-19} \times 7.76 \times 10^{18} = 2.67$ Joules per second. we need to wait $\frac{3.13 \times 10^5 \text{Joules}}{2.67 \text{Joules/second}} = 1.1719 \times 10^5$seconds. This is about 32 hours now.

6. A narrow beam of white light passes through a prism at an angle of 70° from normal. The apex angle of the prism is 50°. The index of refraction of the prism is 1.51 for red light and 1.56 for violet light.

   (a) At what angle does the light emerge? What is the angular spread of the emerging light?

   Solution:

   On p. 10 of the notes we are given that $\delta = \theta_1 - \alpha + \sin^{-1}\left[\sin\alpha\sqrt{n^2 - \sin^2(\theta_1)} - \cos\alpha\sin\theta_1\right]$ For both colors, the incident angle $\theta_1 = 70°$. For red light, n=1.51, so we get

$$\delta_{\text{red}} = 70 - 50 + \sin^{-1}\left[\sin 50\sqrt{1.51^2 - \sin^2(70)} - \cos 50 \sin 70\right] = 37.54°$$

$$\delta_{\text{violet}} = 70 - 50 + \sin^{-1}\left[\sin 50\sqrt{1.56^2 - \sin^2(70)} - \cos 50 \sin 70\right] = 40.48°$$

   So the light emerges at an angle of approximately 37 °. The angular spread of the emerging light is 40.48.9-37.54=2.94°.

   (b) If you place a screen 10 cm away from the prism, by what distance will the red light be separated from the violet light?

   Solution:

   The angular spread of the emerging light is 2.9°. If this angle is the tip of an isosceles whose height is 10 cm, the base of the triangle is equal to D=2*10×tan(2.9/2) = 0.51cm.

   (c) You want to disperse the white light without using a prism, so you decide to use a piece of glass. How thick a piece of glass would you need to place in the beam of white light to separate the red and violet light by the same distance that you calculated in part 6b, using the same incident angle?

   Solution:

   The equation for displacement in glass is given on p. 11 of the notes and is equal to

$$D = t\sin\theta_i\left[1 - \sqrt{\frac{1-\sin^2(\theta_i)}{n^2 - \sin^2(\theta_i)}}\right]$$

   So the distance between two colors will be

$$d = D_{\text{violet}} - D_{\text{red}} = t\left|\sin\theta_i\left[1 - \sqrt{\frac{1-\sin^2(\theta_i)}{n^2_{\text{violet}} - \sin^2(\theta_i)}}\right] - \sin\theta_i\left[1 - \sqrt{\frac{1-\sin^2(\theta_i)}{n^2_{\text{red}} - \sin^2(\theta_i)}} - \right]\right|$$

   Plugging in numerical values, we get that the thickness of the glass must be ≈ 37.5 cm.

7. **Hecht 4.58** A glass block having an index of 1.55 is covered with a layer of water of refractive index 1.33. For light traveling in the glass, what is the critical angle at the interface?

Solution:

If the angle of refracted (transmitted) light is greater than 90 degrees from the surface normal, then light cannot pass into the medium. We can use Snell's law to solve for the incident angle at which the refracted angle would be 90 degrees:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

θ_2 is 90 at the critical angle, and $\sin 90 = 1$ so $n_1 \sin\theta_c = n_2$ so we get that $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$. For the glass-water interface in the problem, $n_1 = 1.55$ and $n_2 = 1.33$ so $\frac{n_2}{n_1} = 0.86$. So we get that $\theta_c = \sin^{-1}(0.86) = 59.1°$

8. The critical angle of a prism is the angle of incidence at which no light exits the opposite side of the prism from the apex angle. A right angle prism (see notes) has two critical angles that correspond to the two different apex angles. What are these critical angles?

   Solution:

   There are four angles of interest in analyzing a prism, labeled in the notes (p.13) as $\theta_1, \theta_1', \theta_2,$ andθ_2'. The critical angle is the angle θ_1 that gives a $\theta_2' = 90$, which is equivalent to a $\theta_2 = \sin^{-1}(\frac{n_{air}}{n_{glass}}) \approx 42°$. We use n$_{glass}$ ≈1.5 as given in other problems.

   $$\theta_2 = \alpha - \theta_1' = \alpha - \sin^{-1}\frac{\sin(\theta_1)}{n_{glass}}$$

   So, the critical angle is the angle at which

   $$\sin^{-1}(\frac{n_{air}}{n_{glass}}) = \alpha - \sin^{-1}\frac{\sin(\theta_c)}{n_{glass}}$$

   $$\theta_c = \sin^{-1}\left[n_{glass} \sin\left(\alpha - \sin^{-1}(\frac{n_{air}}{n_{glass}})\right)\right]$$

   Now we can plug in numbers. The right angle prism shown in the notes has apex angles of 90 and 45, and glass has a refractive index of 1.5. For apex angle $\alpha = 45°$, $\theta_c = 4.78°$. Notice that there is no transmission for all angles *below* the critical angle. There is no critical angle for the 90°prism because there is no transmission for any angle; that is why the right angle prism in the notes is drawn with all internal lines reflecting out.

9. Green light has a wavelength of 530 nm in air. What wavelength will it have in glass that has an index of refraction of 1.52?

   Solution:

   The speed of light in a material of refractive index n is $v = \frac{c}{n}$ where c is the speed of light in a vacuum. The frequency of the light does not change, so the wavelength must go down in order to maintain the relationship that wavelength×frequency=velocity. So the wavelength of green light in a material of refractive index 1.53 is 530/1.52=348.69 nm.