EE119 Homework 2

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Due Friday, February 6, 2009

1. For each combination of electric fields, plot the electric field amplitude of x and y components on the same axis as a function of displacement in the direction of propagation at time t=0. Also, plot the electric field amplitude of x and y components on the same axis as a function of time at a fixed point in space (for instance, z=0). Describe the polarization of each type of wave (linear, circular, elliptical). If the polarization is linear, specify the angle of polarization from the x-axis. If the polarization is elliptical, draw the ellipse that's formed by the resultant E-field vector in the x-y plane. Specify the direction in which the electric field is rotating and the angle α that the major axis of the ellipse forms with the x-axis (see Hecht p. 329).

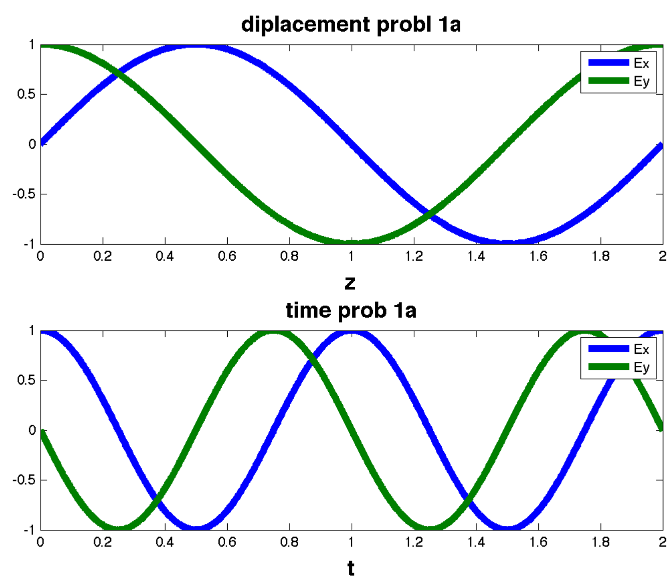

Figure 1: $E_x = \cos(kz - \omega t); E_y = \sin(kz - \omega t)$

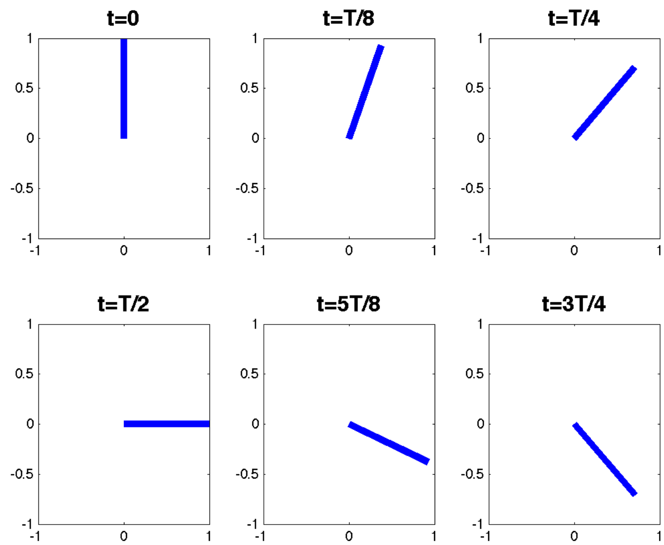

Figure 2: Light in 1a circularly polarized, and is rotating to the right.

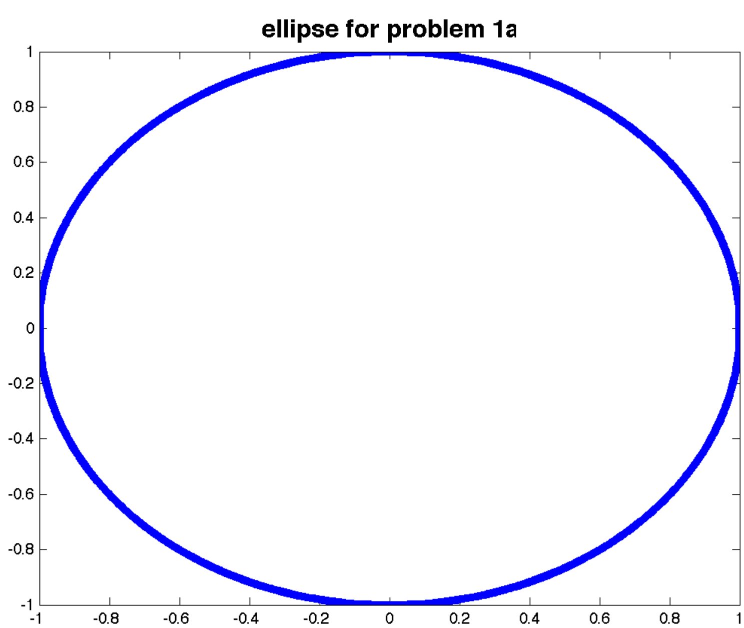

Figure 3: Light in 1a is circularly polarized

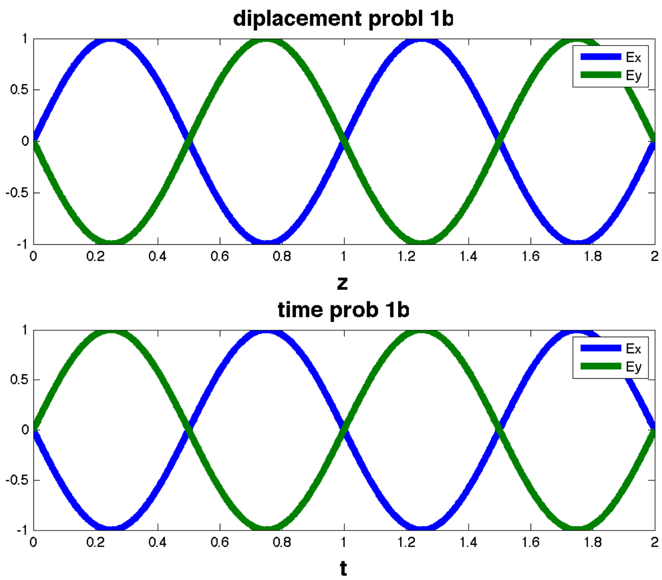

Figure 4: $E_x = \sin(kz - \omega t); E_y = \cos(kz - \omega t + \frac{\pi}{2})$. Light is linearly polarized at an angle of -45°from the x-axis.

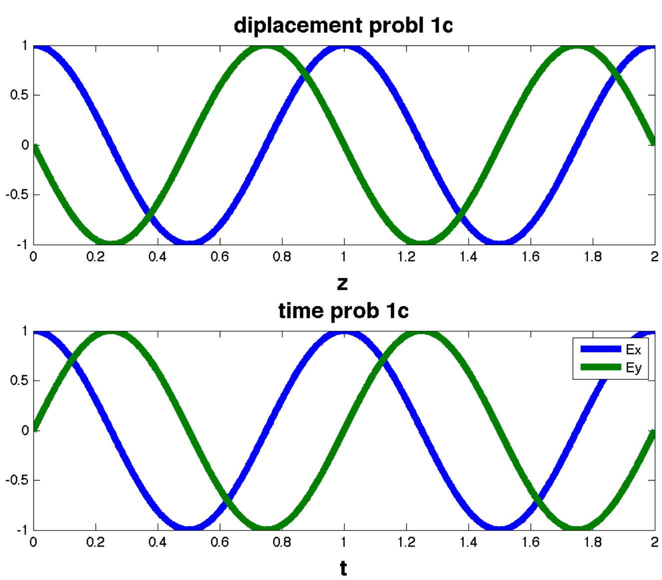

Figure 5: $E_x = \cos(kz - \omega t); E_y = \cos(kz - \omega t + \frac{\pi}{2})$. Light is right-circularly polarized with

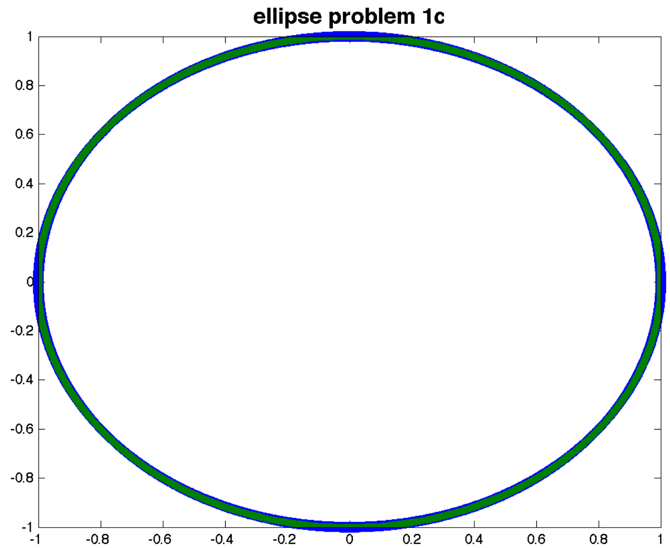

Figure 6: Light in part c is circularly polarized, rotating to the left

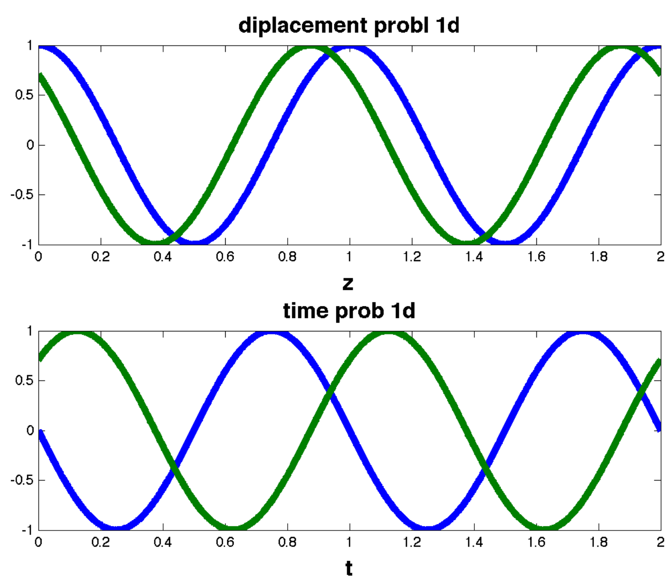

Figure 7: $E_x = \cos(kz - \omega t); E_y = \cos(kz - \omega t + \frac{\pi}{4})$

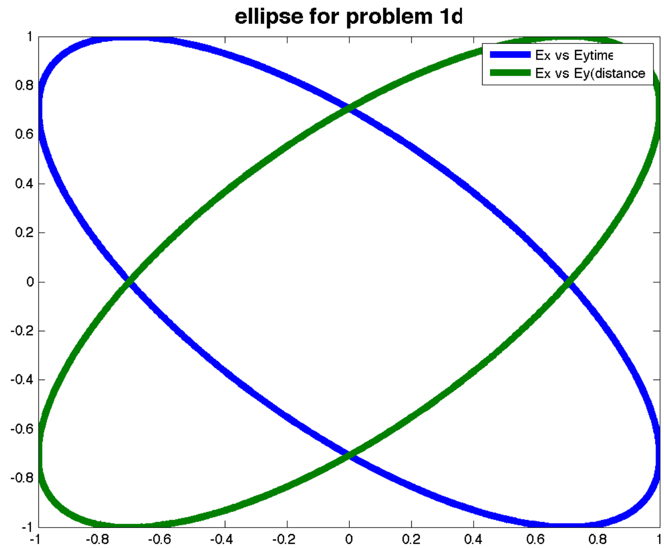

Figure 8: Light is elliptically polarized, rotating to the right, with α =+45°.

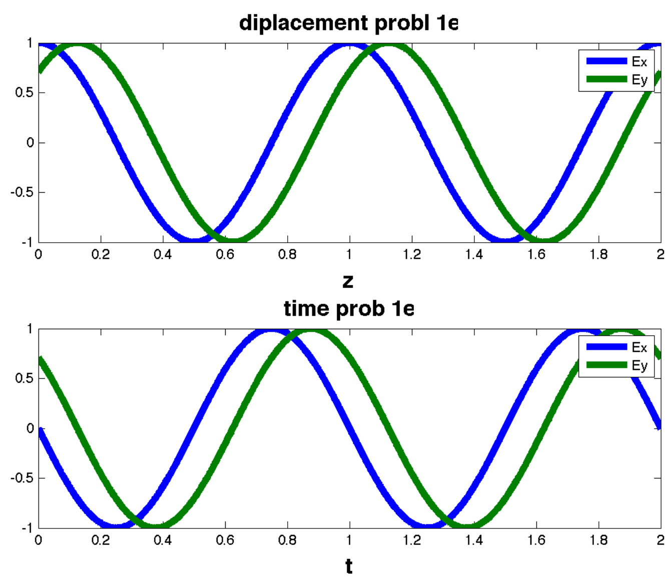

Figure 9: $E_x = \cos(kz - \omega t); E_y = \cos(kz - \omega t - \frac{\pi}{4})$

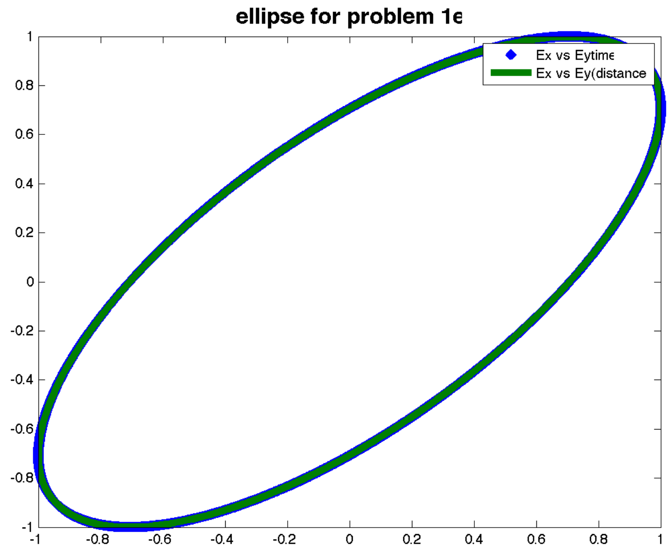

Figure 10: Light is elliptically polarized, rotating to the left, with $\alpha = +45°$.

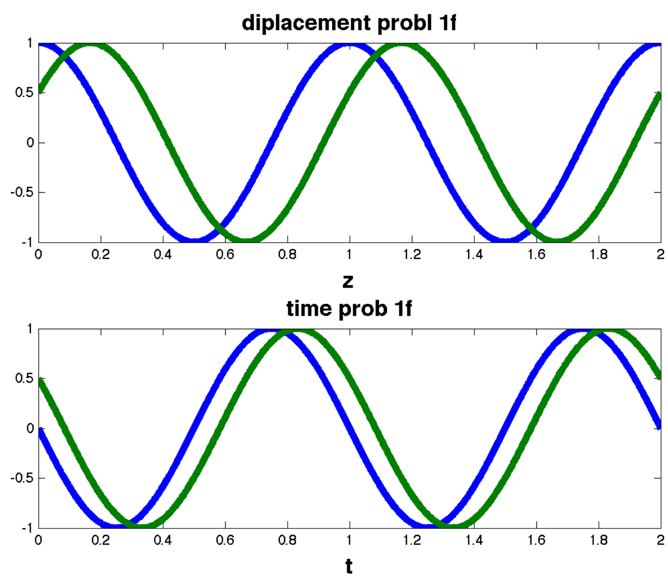

Figure 11: $E_x = \cos(kz - \omega t); E_y = \cos(kz - \omega t - \frac{\pi}{3})$

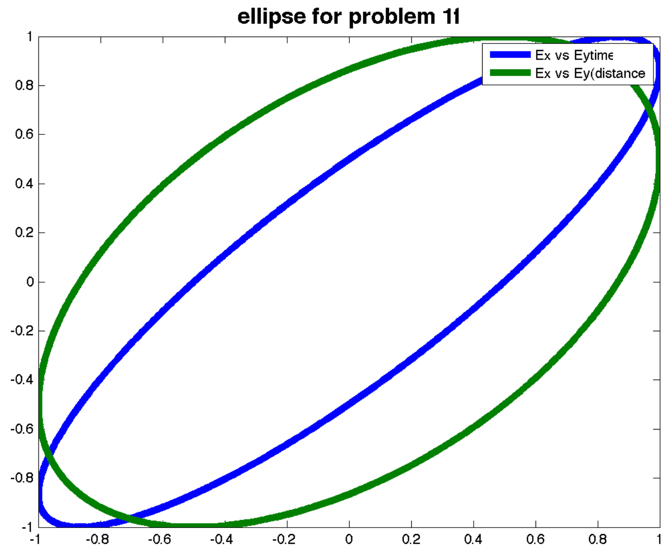

Figure 12: Light is elliptically polarized, rotating to the left, with α =+45°.

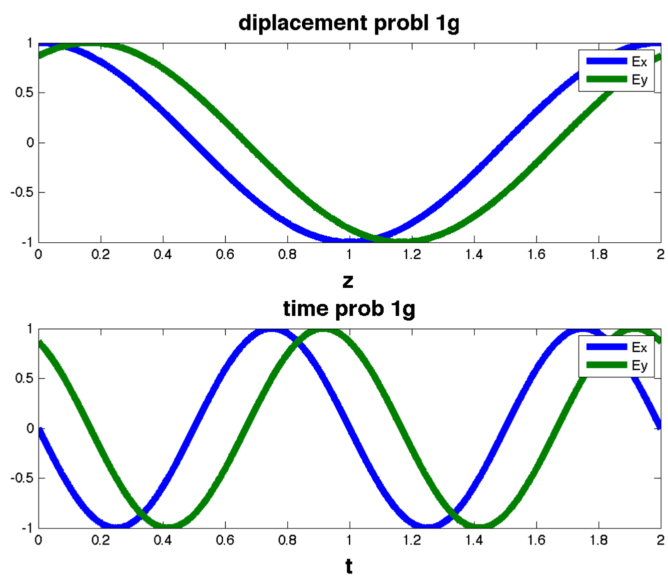

Figure 13: $E_x = \cos\left(kz - \omega t\right); E_y = \cos\left(kz - \omega t - \frac{\pi}{6}\right)$

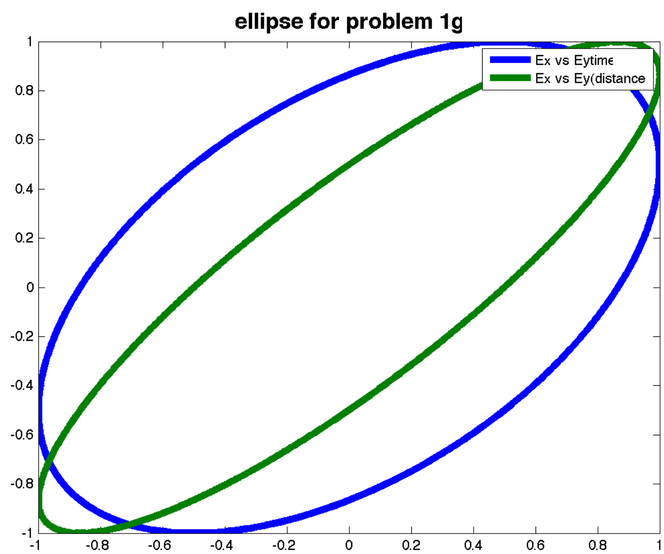

Figure 14: Light is elliptically polarized, rotating to the left, with α =+45°.

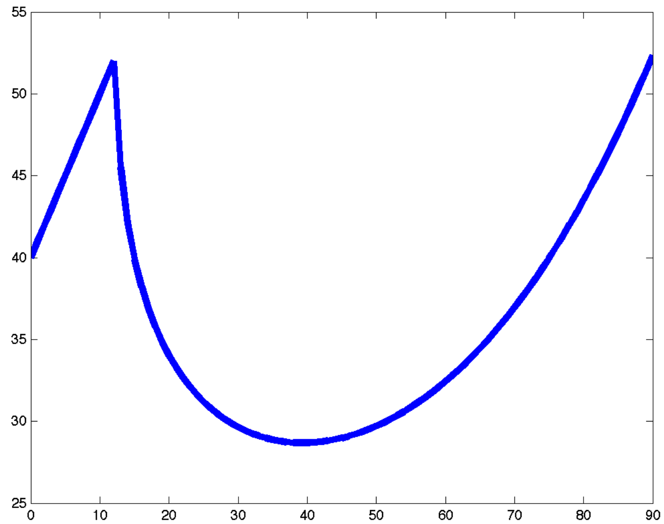

Figure 15: Angle of deviation of a prism for α =60°.

2. (a) The angle of deviation for a prism is given in the notes. Plot the angle of deviation (δ) versus incident angle for n=1.5 and apex angle of 60°.

   (b) The angle of deviation should have a minimum. Derive a general expression for the incidence angle at which the deviation δ is minimized. Your answer should be given as a function of the apex angle and refractive index of the prism.

   Solution:

   Following Hecht p. 187-188: Start with the formulas

$$\begin{aligned} \alpha &= \theta_1' + \theta_1 & (1) \\ \delta &= \theta_1 + \theta_2' - \alpha & (2) \\ \sin(\theta_1) &= n \times \sin(\theta_1') & (3) \\ \sin(\theta_2') \quad t &= n \times \sin(\theta_2) & (4) \\ & & (5) \end{aligned}$$

   We want to find the value of θ_1 for which δ is minimized. We can take the derivative of δ and set it to zero:

$$\frac{d\delta}{d\theta_1} = 1 + \frac{d\theta_2'}{d\theta_1} = 0$$

   so this means that at the position of minimum deviation,

$$\frac{d\theta_2'}{d\theta_1} = -1$$

   We can re-express the left hand side as

$$\frac{d\theta_2'}{d\theta_1} = \frac{d\theta_2'}{d\theta_2}\frac{d\theta_2}{d\theta_1'}\frac{d\theta_1'}{d\theta_1}$$

We can find these intermediate expressions by differentiating Snell's law at both ends of the prism. At the first interface,

$$\cos(\theta_1)d\theta_1 = n\cos(\theta_1^{'})d\theta_1^{'}$$

giving

$$\frac{d\theta_1^{'}}{d\theta_1} = \frac{\cos(\theta_1)}{n\cos(\theta_1^{'})}$$

and at the second interface,

$$\cos(\theta_2)d\theta_2 = n\cos(\theta_2^{'})d\theta_2^{'}$$

giving

$$\frac{d\theta_2^{'}}{d\theta_2} = \frac{n\cos(\theta_2)}{\cos(\theta_2^{'})}$$

and we also know that

$$\frac{d\theta_1^{'}}{d\theta_2} = -1$$

So

$$\frac{d\theta_2^{'}}{d\theta_1} = \frac{n\cos(\theta_2)}{\cos(\theta_2^{'})}(-1)\frac{\cos(\theta_1)}{n\cos(\theta_1^{'})} = -1$$

$$\frac{n\cos(\theta_2)}{\cos(\theta_2^{'})}\frac{\cos(\theta_1)}{n\cos(\theta_1^{'})} = 1$$

$$\frac{\cos(\theta_2)}{\cos(\theta_2^{'})} = \frac{\cos(\theta_1^{'})}{\cos(\theta_1)}$$

Unless $n = 1$ (in which case $\theta_1 = \theta_1^{'}$), this relationship will hold only when $\theta_1 = \theta_2^{'}$. This implies that $\theta_1^{'} = \theta_2$ by Snell's law. Since $\theta_1^{'} + \theta_2 = \alpha$, we get that at the position of least deviation, $\theta_2 = \alpha/2$. Applying Snell's law gives us that

$$\theta_1 = \sin^{-1}(n\sin\frac{\alpha}{2})$$

(c) Prove that the ray for which the deviation is a minimum traverses the prism symmetrically. This means that the incident angle and the exit angle are the same. This position for the prism is also approximated in most spectral instruments because it gives the highest spectral resolution (part 2e), also the largest diameter beam to pass through a given prism and also produce the smallest amount of loss due to surface reflections.

Solution:

This was shown in the derivation above.

(d) In the case when the angle of deviation is a minimum, find an expression for a refractive index of the prism. Express the refractive index in terms of the minimum angle of deviation and the apex angle. This equation forms the basis of one of the most accurate techniques for determining the refractive index of a transparent substance.

Solution:

In part b, we showed that

$$\theta_1 = \sin^{-1}(n\sin\frac{\alpha}{2})$$

inverting this to get the index of refraction gives

$$n = \frac{\sin(\theta_1)}{\sin(\alpha/2)}$$

where θ_1 is the angle of least deviation. or alternately, since $\delta_{min} = 2 \times \theta_1 - \alpha$,

$$n = \frac{\sin(0.5(\delta_{min} + \alpha))}{\sin \alpha/2}$$

(e) We want to design a prism spectrometer. For a better prism spectrometer, we want to have the angular dispersion of the prism as large as possible. Show that the angular dispersion of different colors $\frac{\partial \delta}{\partial \lambda}$ has a maximum value in the position of minimum deviation. (The position of minimum deviation is the position at which the deviation δ is minimized.)

Solution:

This problem is wrong. Sorry about that. THe position of minimum deviation is not, in fact, the position at which dispersion is maximized. As you saw in problem 3, the dispersion is maximized as the prism approaches the critical angle. However, post prism spectrometers are used at this angle because it's the angle that is the best compromise for reflection, and because it minimizes the astigmatism of the colors (i.e. the colors emerge most symmetrically).

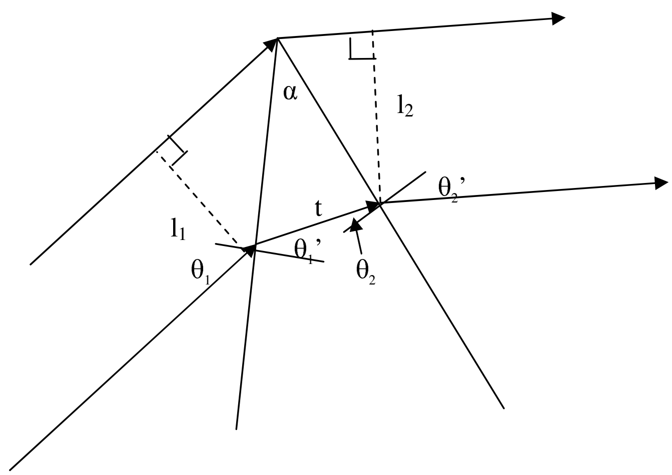

Figure 16: In problem **??**, solve for the incident angle θ_1 at which the dispersion is maximized.

3. Measuring the index of refraction of a transparent material

(a) Plot the Fresnel Coefficients of transmission and reflection for both s- and p-polarized as a function of incident angle for four different values of the refractive index between 1 and 2. Specify what those values are in your plot (which you should turn in). Which incident angle gives you the greatest sensitivity to changes in refractive index? Which polarization is more sensitive to changes in refractive index? If you wanted to determine the refractive index of a material and could only make one measurement,

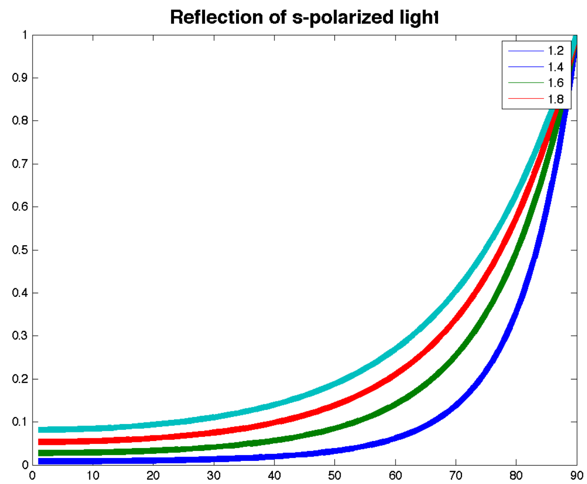

Figure 17: In problem **??**, solve for the incident angle θ_1 at which the dispersion is maximized.

what angle and polarization would you measure at? Would you measure transmission or reflection?

Solution:

For reflection and transmission of p-polarized light, the greatest sensitivity to changes in refractive index happens at normal incidence. For s-polarized light, the greatest sensitivity happens at Brewster's angle. So if I were doing a measurement of a transparent material, I would measure the reflection or transmission of s-polarized light near brewster's angle. I would measure reflection because the change in the reflection coefficient with changing refractive index would be a greater fraction of the total measured intensity.

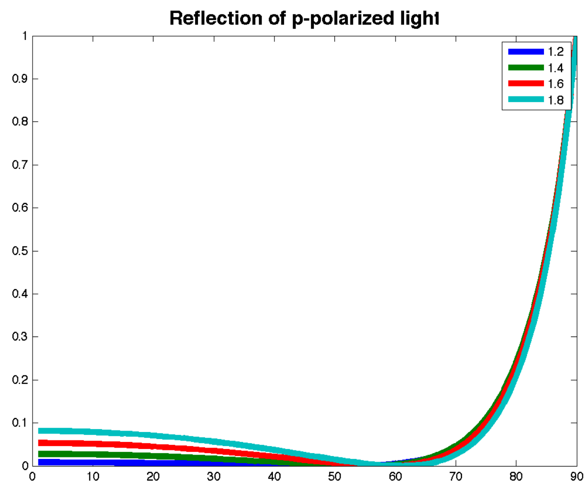

Figure 18: In problem **??**, solve for the incident angle θ_1 at which the dispersion is maximized.

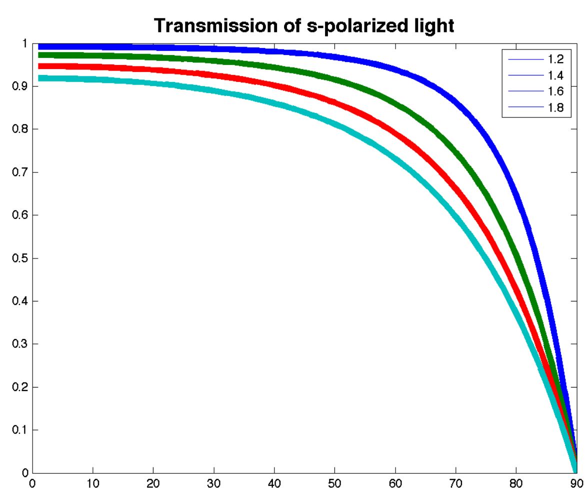

Figure 19: In problem **??**, solve for the incident angle θ_1 at which the dispersion is maximized.

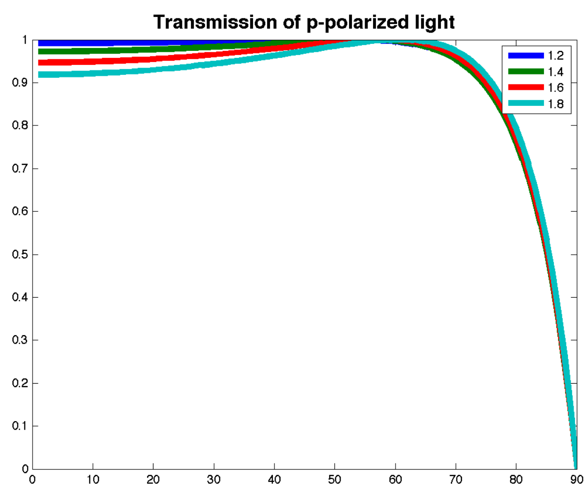

Figure 20: In problem **??**, solve for the incident angle θ_1 at which the dispersion is maximized.

(b) Plot the angle of deviation of a prism with apex angle of 50°as a function of incident angle for the same four values of refractive index you used in part (a). At what incident angle is the sensitivity to changes in refractive index maximized?

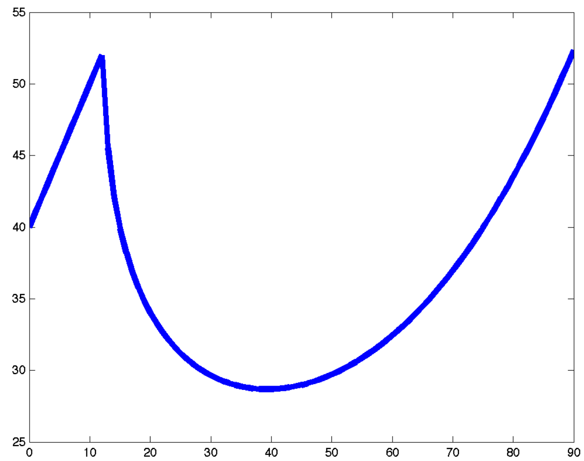

Figure 21: The sensitivity to changes in refractive index is maximized at the critical angle (the smallest angle).

4. Photoelectric effect

(a) What wavelength of light must you shine on a material with a 2 eV work function to get it to emit electrons?

Solution

$$\lambda = \frac{c}{\nu} = \frac{c}{\text{Energy(Joules)}/h} = \frac{hc}{\text{Energy(eV)} \times 1.6 \times 10^{-19} J/eV}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\#eV \times 1.6 \times 10^{-19}} = \frac{1.241 \times 10^{-6}}{\#eV} = 0.620 \times 10^{-6} \text{ m} = 620 \text{ nm}$$

Incidentally, the inverse relationship eV· nm =1240 is a very useful one to remember if you go into optics because you need to convert between eV and nanometers a lot.

(b) Marty McFly is teleported back time and space to Nevada in 1875 with the Silver Strike well under way. He has with him a tunable light source, so he decides to strike it big by distinguishing between Silver and Platinum, buying platinum from the miners as thought it were silver, and later selling it to people as platinum. Now he needs to design your instrument. He remember from physics class that the work function of silver is about 4.3 eV and the work function of platinum is about 5.6 eV. What range of colors is your tunable light source going to have to be able to produce in order for your plan to work?

Solution

Using our nifty relationship, we can find that silver will photoemit when illuminated with a color of wavelength 1240/4.3=288 nm, and platinum will photoemit when illuminated with a color of wavelength 1240/5.6= 221.4. So Marty needs an ultraviolet light source.

(c) A year goes by, and Marty amasses loads of platinum. Doc pays him a visit from the future and, impressed with his business, offers to give him a light source is five times more intense, in exchange for half of his amassed platinum. Should Marty accept the offer? Will a more intense light source help him?

Solution

No, because photoemission depends on the energy of the individual photon, not on the intensity of light. Since Marty is already distinguishing between silver ad platinum, he doesn't need more light.

5. As we learned in lecture, a birefringent material is one that has a different index of refraction along axes of the material. The polarization of light that will always have the same refractive index is the "ordinary" direction, and rays through this direction are called "ordinary rays." The rays that see a different refractive index depending on the polarization of the light are the "extraordinary rays".

In this problem, consider the direction of propagation of light to be the z-axis.

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(a) A birefringent plate with $n_e = 1.4$ and $n_o = 1.6$ that is 5 mm thick is placed perpendicular to the z-axis as shown in figure 22. The extraordinary axis, $n_e = 1.4$, is aligned along the y-axis. 500 nm light that is linearly polarized at 20°from the y-axis passes through the plate. What are the phase delays of the x- and y-components of the electric fields? Remember, the phase delay (see p. 24 of the class notes) is $\phi = kn\Delta z$ where k is the wave number, n is the refractive index of the medium, and z is the distance that the light has traveled through the material. Up to a factor of 2π, the phase delay is equal to the difference in the phase of the electric field vector between the start and the end of the material. Is the emerging light linearly polarized or elliptically polarized? If it is linearly polarized, specify the angle from the y-axis; if it is elliptically polarized, specify the direction of rotation and the angle of the semi-major ellipse from from the x-axis (see Hecht p. 329).

Solution:

The phase delay is given by $\phi = kn\Delta z$. Here, $k = \frac{2\pi}{\lambda} = 1.2566^7$ m^{-1}, $z =$ 5mm$=$ 5×10^-3m and n is either n_e or n_o. The x-component of the electric field is delayed by n_o, giving a phase delay of $8.7965{\times}10^4$. The y-component of the electric field has a phase delay of 1.0053×10^{-5}. Since both of these numbers divide evenly into 2π, the phase delay component of the two fields are the same, and the light emerges polarized in the same way that it entered.

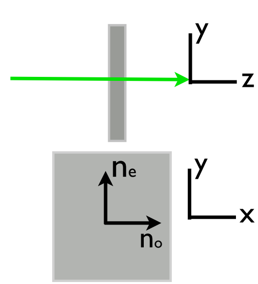

Figure 22: diagram for problem 5a.

(b) Now you cut the birefringent plate in problem 5a differently, so that n_o is parallel to the surface of the plate, and n_e is perpendicular to the surface of the plate, as shown in figure 23. Only one of either s- or p-polarized light ever interacts with n_e. Which polarization is the one that interacts with n_e?

Solution:

p-polarized light interacts with n_e.

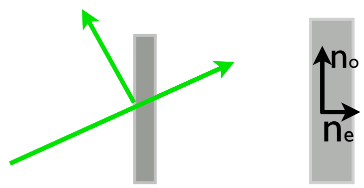

Figure 23: diagram for problem 5b

(c) [Hecht problem 8.34] A ray of light is incident on a calcite plate at 50°. the plate is cut so that the optic axis is parallel to the front face and perpendicular to the plane-of-incidence. Find the angular separation between the two emerging rays. The table in section 8.4.3 (p. 243) of Hecht gives a value of $n_o = 1.6584$ and $n_e = 1.4868$ for calcite.

Solution:

The optic axis of the crystal is the axis of the extraordinary ray (IS THIS RIGHT?). Because the optic axis is perpendicular to the plane of incidence, only s-polarized light will interact with n_e. We use snell's law with $\theta_I = 50°$for both indices of refraction:

$$\theta_o = \sin^{-1}\frac{\sin(50)}{n_o} = 27.5109$$

$$\theta_e = \sin^{-1}\frac{\sin(50)}{n_e} = 31.0129$$

So the angular separation is approximately 3.5°.

6. Lex Luthor obtains a new form of krypotnite that will shrink Superman down to be 10 cm tall. He decides to carry around a lens with him that will restore his appearance to his former 2 meter height.

(a) What transverse magnification must this lens have for superman to look the height he wants?

Solution:

Superman will need a transverse magnification of $m = \frac{h_{image}}{h_{object}} = \frac{2}{0.1} = 20$

(b) Superman wants to be able to stand up to 10 meters behind his lens. What focal length does the lens need to have in order to achieve the magnification required in part 6a?

Solution:

In order for superman to appear upright, he will have to be between the lens and the focal point. Consequently, the lens must have a focal length of at least 10 m. From the lecture notes we have that $m = \frac{f}{d_1+f}$, and using $d_1 = -10$, we can solve for the minimum value of f:

$$\frac{f}{(f+d_1)} = m$$

$$f = mf + md_1$$

$$-md_1 = (m-1)f$$

so

$$f = \frac{-md_1}{m-1}$$

for m=20 and $d_1 = -10$, $f = 200/19 = 10.5$. For very high magnification, where $m - 1 \approx m$, the required focal length will approach $-d_1$.

(c) If Superman wants to appear right-side up when he's walking on the street, on what side of the focal point does he need to be relative to the lens? Draw a diagram. Will people looking through the lens see a real image or a virtual one?

Solution:

He needs to be between the lens and the focal length

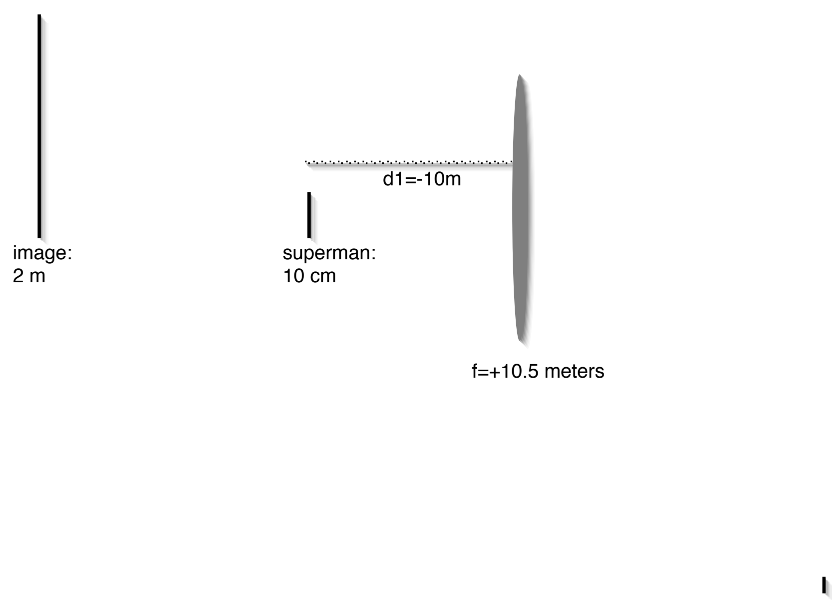

Figure 24: diagram for problem 6c.