EE119 Homework 4

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Due Friday, February 20, 2009

1. Hecht 6.28. Make sure to justify your answer.

   Solution:

   (a) Spherical Aberration: The image is centered, but the wave fronts from the monochromatic point source constructively interfere at places outside of the central focal point because the wavefronts don't all converge to once point.

   (b) Coma: The figure is lopsided and not symmetric about the center point, and still shows signs of spherical aberration.

   (c) Astigmatism: The figure shows two images corresponding two different axes of least confusion

2. Hecht 6.29. Make sure to justify your answer.

   Solution:

   (a) Astigmatism: it is biaxially symmetric

   (b) coma: elongated along one axis.

3. You have a shiny broadband blue laser whose center wavelength is 486 nm (this is 'F' color, see p. 34 of notes). You want to focus this laser to a point with a 12.5 cm converging lens that will minimize chromatic aberration in the region between G' and D light. So you decide to make an achromat from SPC-1 and DF-4 glasses. Refer to table 1 for the refractive indices of the materials. The lens made out of DF-4 glass should have its outer face flat, and the two lenses should be cemented, which means that the distance between them is zero and their inner radii are the same.

   (a) Find the power of the lens in diopters.

   Solution:

   $$P = \frac{1}{f} = \frac{1}{0.0125} = 8 \text{ Diopters}$$

   A diopter is an inverse meter.

   (b) Find the dispersive power of the two lenses.

   Solution:

   The dispersive power is a material property that depends on the three wavelengths of interest. In general, it's given by

   $$\nu = \frac{n_{\mathrm{F}} - 1}{n_{\mathrm{G'}} - n_{\mathrm{D}}}$$

Table : REFRACTIVE INDICES OF TYPICAL OPTICAL MEDIA FOR FOUR COLORS

Medium	Designation	ICT type	ν	n_C	n_D	n_F	$n_{G'}$
Borosilicate crown	BSC	500/664	66.4	1.49776	1.50000	1.50529	1.50937
Borosilicate crown	BSC-2	517/645	64.5	1.51462	1.51700	1.52264	1.52708
Spectacle crown	SPC-1	523/587	58.7	1.52042	1.52300	1.52933	1.53435
Light barium crown	LBC-1	541/599	59.7	1.53828	1.54100	1.54735	1.55249
Telescope flint	TF	530/516	51.6	1.52762	1.53050	1.53790	1.54379
Dense barium flint	DBF	670/475	47.5	1.66650	1.67050	1.68059	1.68882
Light flint	LF	576/412	41.2	1.57208	1.57600	1.58606	1.59441
Dense flint	DF-2	617/366	36.6	1.61216	1.61700	1.62901	1.63923
Dense flint	DF-4	649/338	33.9	1.64357	1.64900	1.66270	1.67456
Extra dense flint	EDF-3	720/291	29.1	1.71303	1.72000	1.73780	1.75324
Fused quartz	SiO_2		67.9		1.4585		
Crystal quartz (*O* ray)	SiO_2		70.0		1.5443		
Fluorite	CaF_2		95.4		1.4338		

Figure 1: diagram for problem 3

in this case, the dispersive power is

$$\nu = \frac{n_{\text{center}} - 1}{n_{\text{blue}} - n_{\text{red}}}$$

And pluging numbers in, I get

$$\nu_{\text{crown}} = 46.6370 \qquad \nu_{\text{flint}} = 25.9272$$

(c) Find the powers of the two lenses.

Solution:

An achromatic lens is one where the power at the bluer wavelength is the same as the power at the redder wavelength, i..e

$$P_{G'} = P_D$$

and the total power at the center wavelength must sum to our desired wavelength:

$$P_{\text{crown},F} + P_{\text{flint, F}} = P_{\text{total}}$$

From p. 35 of the notes (equation 2.18), and plugging in the numbers above, we get the relationships

$$P_{\text{crown}} = P_{\text{total}} \frac{\nu_{\text{crown}}}{\nu_{\text{crown}} - \nu_{\text{flint}}} = 18.0155$$

$$P_{\text{flint}} = -P_{\text{total}} \frac{\nu_{\text{flint}}}{\nu_{\text{crown}} - \nu_{\text{flint}}} = -10.0155$$

(d) Find the radii of the three curved surfaces.

Solution:

Now that we know the powers (and by inverting the power, the focal length) we need to invert the lensmaker equation. We are told that the flint glass should have an outer flat face, so we start with finding the other radius of the flint face:

$$\frac{1}{f_{\text{flint}}} = P_{\text{flint}} = (n_{\text{center}} - 1)(\frac{1}{R_1} - \frac{1}{R_2})$$

since R_1 of the flint is infinity, this relationship reduces to

$$\frac{1}{R_{2,\text{flint}}} = -\frac{P_{\text{flint}}}{(n_{\text{center}} - 1)} = -\frac{-10.0155}{0.6627} = \frac{1}{15.1131} = 0.0662\text{m} = 6.622\text{cm}$$

The inner surfaces of the crown and flint glasses are cemented together, so $R_{1,\text{crown}} = R_{2,\text{flint}} = 6.62$ cm. So for the crown glass, we have

$$\frac{1}{P_{\text{crown}}} = (n_{\text{center}} - 1)(\frac{1}{0.062} - \frac{1}{R_2})$$

$$\frac{1}{R_2} = -\frac{1}{P_{\text{crown}}(n_{\text{center}} - 1)} - \frac{1}{0.062} = -\frac{1}{18.0155 \times 0.5233} + \frac{1}{0.0528} = \frac{1}{15} = 5.28\text{cm}$$

(e) Now you figure out a way to make the bandwidth of you laser even broader, and you still want to focus it. Repeat part (d) (and whatever is needed of the previous parts) for a lens that is to be corrected for G' and C light. Is the radius of the combined lens (i.e. the outer radius of the crown) bigger or smaller than in part (d)?s Solution: I use the following matlab code to do this calculation, and with it I got that R

```
%achromat
totalfocallength=0.125;
totalPower=1/totalfocallength;
lenstype='planoconvex'

crownmat='SPC-1';
flintmat='DF-4 ';

materialnames=['BSC  '; 'BSC-2'; 'SPC-1';'LBC-1'; 'TF   '; 'DBF  '; 'LF   '; 'DF-2 ';
mt=cellstr(materialnames);
%nc is reddest
nc=[1.49776 1.51462 1.52042 1.53828 1.52762 1.66650 1.57208 1.61216 1.64357 1.71303 1
nd=[1.50000 1.51700 1.52300 1.54100 1.53050 1.67050 1.57600 1.61700 1.64900 1.72000 1
nf=[1.50529 1.53364 1.52933 1.54735 1.53790 1.68059 1.58606 1.62901 1.66270 1.73780
ng=[1.50937 1.52708 1.53435 1.55249 1.54379 1.68882 1.59441 1.63923 1.67456 1.75324
%ng is bluest
nred=nd;
ncenter=nf;
nblue=ng;
crown=strmatch(crownmat,materialnames);
flint=strmatch( flintmat,materialnames);
```

```
dispersivepowercrown=(ncenter(crown)-1)/(nblue(crown)-nred(crown));
dispersivepowerflint=(ncenter(flint)-1)/(nblue(flint)-nred(flint));
vvaluecrown=1/dispersivepowercrown;
vvalueflint=1/dispersivepowerflint;

Powercrown=totalPower*dispersivepowercrown/(dispersivepowercrown-dispersivepowerflint)
Powerflint=-totalPower*dispersivepowerflint/(dispersivepowercrown-dispersivepowerflin

fcrown=1/Powercrown;
fflint=1/Powerflint;

switch lenstype
    case 'biconvex',
        radiuscrown(1)=(2*(ncenter(crown)-1))/Powercrown
       radiuscrown(2)=-radiuscrown(1)
       radiusflint(1)=radiuscrown(2)
     radiusflint(2)=1/(1/radiusflint(1)-Powerflint/(ncenter(flint)-1))
    case 'planoconvex',
     radiusflint(1)=0;
     radiusflint(2)=-fflint*(ncenter(flint)-1)
    radiuscrown(1)=radiusflint(2);
  radiuscrown(2)=1/(-Powercrown/(ncenter(crown)-1)+1/radiuscrown(1))
end
```

4. The rays incident on the outer edge of a lens (outside of the paraxial regime) suffer from spherical aberration. This is because the nonparaxial rays are too strongly bent. Consider the plano-convex lens as we see in Fig. 1-(a) and (b). Depending on which surface faces the incident rays, the amount of spherical aberration can be reduced. In this problem, you will decide which lens configuration is better in terms of spherical aberration.

   (a) Find the focal length of the lens in the paraxial regime.
   Solution:
   We use the lensmakers equation.

   $$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2}) = 0.5(\frac{1}{20} - \frac{1}{\infty}) = 40\text{cm}$$

   (b) Lets assume we have a ray parallel to the optical axis incident on the lens as shown above. Calculate where the ray crosses the optical axis (L). Neglect the thickness of the lens. (Hint: You will use Snell's law twice)
   Solution:
   The incident angle of the beam is

   $$I_1 = \sin^{-1}\frac{h}{R_1} = \sin^{-1}\frac{10}{20} = 30^\circ$$

   plugging into Snell's law to solve for I_1' (assume $n_{\text{air}} = 1$)

   $$\sin(I_1) = \frac{h}{R_1} = n_l \sin(I_1')$$

so

$$I_1^{'} = \sin^{-1}\frac{h}{n_l R_1} = \sin^{-1}\frac{10}{30} = 19.4712°$$

Since we can assume that $h = h'$,

$$I_2 = 90 - (90 - I_1 + I_2) = I_1 - I_2 = 30 - 19.47 = 10.53°$$

And using Snell's law again, we get

$$I_2^{'} = \sin^{-1}(1.5\sin(10.53°)) = 15.9143°$$

So the distance where the emerging ray intersects the optic axis is

$$L' = \frac{h}{\tan(I_2^{'})} = \frac{10}{\tan(15.91} = 35.07\text{cm}$$

(c) Now flip the lens around so light is incident on the planar side. Repeat parts (a) and (b). (Hint: You will use Snell's law only once)

Solution:

part a: We use the lensmakers equation again, but now the radii are switched, and the sign of the curved radius is flipped.

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2}) = 0.5(\frac{1}{\infty} - \frac{1}{-20}) = 40\text{cm}$$

part b: Because the first incident angle is normal to the surface of the lens, $I_1^{'} = I_1$. The incident angle I_2 is going to be the same as I_1 was in the first configuration, so now the angle of the emerging ray is

$$I_1^{'} = \sin^{-1}\frac{nh}{R_1} = \sin^{-1}3/4 = 48.59°$$

The angle between the emerging ray and the optic axis is $I_1 - I_1^{'} = 19.59°$So the position where the ray intersects the optic axis is

$$L' = \frac{h}{\tan(I_2^{'})} = \frac{10}{\tan(18.59} = 29.73\text{cm}$$

(d) Which lens geometry is better for minimizing spherical aberration?

Solution:

The first configuration (convex-plano) is better because the position where the ray intersects the optic axis is closer to the thin-lens focal point.

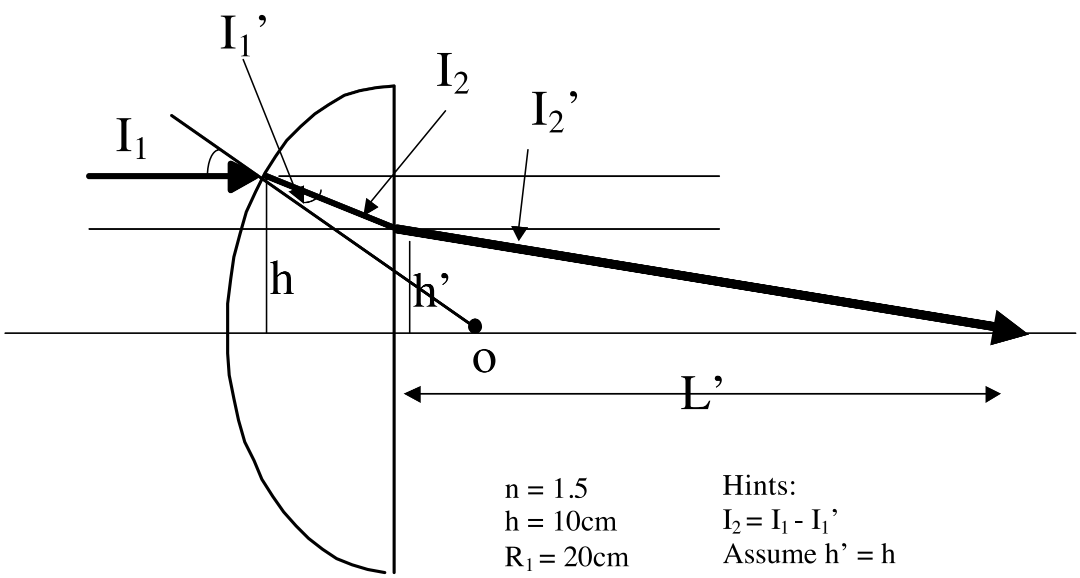

Figure 2: diagram for problem 4