EE119 Homework 5

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1. The radius of one of Saturn's rings is approximately 1.2×10^5m. Saturn itself has a radius of 6×10^4 m. Saturn is 1.5 $\times 10^9$ m away from the Earth. You can assume that the light coming from saturn is approximately 500 nm. How big a lens would you need for your telescope to be able to resolve Saturn's rings?

   Solution:

   In order to see Saturn's rings, The angle we need to be able to resolve is

   $$\theta \approx \tan(\theta) = \frac{1.2 \times 10^5 - 6 \times 10^4}{1.5 \times 10^9} = \frac{6 \times 10^4}{1.5 \times 10^9} = 4 \times 10^{-5}$$

   Plugging into the formula for the Rayleigh Criterion on p. 38, we get

   $$D = 1.22\frac{\lambda}{\theta} = 1.22\frac{5 \times 10^{-7}}{4 \times 10^{-5}} = 1.52 \times 10^{-2}\text{m} = 1.5\text{cm}$$

2. A microscope has an entrance pupil with a radius of 2 cm. The specimen is located 1 cm from the objective lens.

   (a) what is the numerical aperture of the microscope?

   Solution:

   The numerical aperture is the sin of the half-angle subtended by the entrance pupil.

   $$NA = \sin(\tan^{-1}\frac{2}{1}) = 0.8944$$

   (b) Two points in the sample are placed at the center of the objective. How close together can these points be and still be resolvable by the Rayleigh criterion?

   Solution:

   The resolution is given by

   $$h = \frac{0.61\lambda}{NA_{\text{entrance}}} = \frac{0.61 \times 500\text{nm}}{0.8944} = 341 \text{ nm}$$

   (c) Repeat parts (a) and (b) for a microscope with entrance pupil that has a radius of 5 cm. By how much does the resolution improve?

   Solution:

   now the resolution is

   $$h = \frac{0.61 \times 500\text{nm}}{\sin(\tan^{-1}(5/1))} = 314\text{nm}$$

3. A nearsighted person cannot focus clearly on an object that is more than 15 cm away.

   (a) What power of corrective lens is needed be used to correct his vision?

   Solution:

   The person needs correction of 1/0.15=-6.66 Diopters

   (b) Assuming the eyeball is 2.5 cm in length, without corrective lenses, how far away from the retina is the image for an object at an infinite distance coming to a focus ?

   Solution:

   The power of the eye is 58.6+6.66=65.25 Diopters, so the focal length of this lens is 1.336/P=2.04 cm. So the image is formed 46 mm away from the retina.

4. A farsighted person has near point at 75 cm. Assume the eye is 2.0 cm long.

   (a) How much power does the eye have when focused on an object at infinity?

   Solution:

   The power of the eye is 1/0.02=50 Diopters

   (b) How much power does the eye have when focused at 75 cm?

   Solution:

   Now the Power of the eye is

$$\frac{1}{f} = \frac{1}{0.02} - \frac{1}{-0.75} = 51.3\text{Diopters}$$

   (c) How much accommodation is required to focus on an object at 75 cm?

   Solution:

   1.3 diopters

   (d) What power corrective lens should be used to enable the patient to focus at the comfortable reading distance of 25 cm?

   Solution:

   at 25 cm, the power required to form an image on the retina is

$$\frac{1}{f} = \frac{1}{0.02} - \frac{1}{-0.25} = 54\text{Diopters}$$

   The eye can provide 51.3 Diopters of power, so reading glasses with a power of +2.7 diopters are necessary.

5. An optical process called lithography is used for making integrated circuits. Assume the optical system has a reduction factor of 4X (makes the image 4X smaller than object).

   (a) Assuming the optical system is diffraction limited, what is the image resolution of a system that uses a DUV laser at 193nm, and NAimage = 0.85.

   Solution:

   The resolution is equal to 0.61λ/NA=138.5 nm.

   (b) What is the corresponding resolution on the object side? On the object side, the resolution is four times bigger: 554 nm.

(c) Qualitatively, what does this tell you about how a square object with side dimensions 400nm, and a circular object with diameter 400nm, will be imaged?

Solution:

It will look like a blurry circle

(d) Immersion lithography has been proposed to make smaller features on integrated circuits. Immersion lithography will place water (n=4/3) between the last optical element and the image plane. What is the new resolution of the same system from part (a)?

Solution:

The new numerical aperture is multiplied by the index of refraction so we now have NA=$\frac{4\times0.61}{3}=1.13$. And so our new resolution limit is 104 nm.

6. Compute the approximate size (in mm) of the image of the sun as cast on the retina. Assume the sun has a radius of 695000km and is roughly 150 million kilometers away. The power of the eye is 58.6 diopters.

Solution:

From the lecture notes, we know that the overall power of a human eye is 58.6 Diopters. The distance from the object is 1.5×10^{11} meters. We use the lens law:

$$\frac{1}{d_2} = 58.6 - \frac{1}{1.5e11} \approx 58.6$$

so the magnification will be

$$m = \frac{d_2}{d_1} = \frac{1}{58.6}\frac{1}{1.5\times10^{11}} = 1.14\times10^{-13}$$

and the size of the image of the sun on the retina will be

$$h_2 = 2\times6.95\times10^8\times1.14\times10^{-13} = 1.6\times10^{-4} = 0.16\text{mm}$$

7. After conquering Sartaul (East Turkestan, todays northwestern China and central Asia), Genghis Khan organized and held the Mongolian traditional festivities called, Naadam, which included, Three most manly games. These games were wrestling matches, horse racing, and archery. According to the Genghis Khans script (Mongolian record), Esunge (one of Genghis Khans marksmen) consistently hit targets (coins with 8 cm diameter) that were 535 m away and won the archery competition. In fact, many Mongolian marksmen during the Genghis Khans period were able to hit targets that were 500 m away.

(a) Esunge's eyes have a 3 mm iris diamter. Assuming his eyes are a diffraction -limited optic, what is the smallest separation of yellow-green objects 535 m away that that he can resolve? Use 550 nm for the wavelength. Do not assume that $\sin(\theta) = \tan(\theta) = \theta$. Is it possible for Esunge to see the 8 cm coins with at 535 m? Or is it impossible due to the diffraction limit?

Solution:

The diffraction limit tells us that the smallest resolvable angle through an aperture of size D is

$$\sin(\theta) = 1.22\frac{\lambda}{D}$$

So the resolution limit of Esunge's eyes due to diffraction is

$$\theta = \sin^{-1}(1.22\frac{550 \times 10^{-9}}{3 \times 10^{-3}}) = \sin^{-1}(2.24 \times 10^{-4}) = 2.24 \times 10^{-4}$$

so the size of the smallest resolvable object at 535 meters is $535 \times \tan(2.24 \times 10^{-4}) =$ 0.12meters, so the smallest resolvable object is 12 cm. However, the coins are 8 cm in diameter, so he cannot resolve them.

(b) If Esunge was able to clearly see the targets (coins with D   8 cm) that were 535 m away, what is the maximum separation (in mrad) between the cone cells in his fovea? How does this compare with the average separation found in modern humans, which is 0.3 mrad?

Solution:

The angle subtended by these coins is $\tan^{-1}(0.08/535) = 1.5 \times 10^{-4} = 0.0015$ radians=0.15 mrad, which is the separation between cone cells in Esunge's fovea. This is half that of normal humans.

(c) What is the visual acuity of Esunges eyes?

Solution:

Visual acuity is defined as

$$V.A. = \frac{\text{distance to target}}{\text{Distance at which target subtends 1'}} = \frac{535}{535 \times 0.15/0.3} \approx 2$$

So his visual acuity is twice as good as that of normal, modern humans.