EE 119 Section February 22, 2009

February 23, 2009

Diffraction Limits the Resolution of Optical Systems

Any image captured through a finite aperture contains some fraction of the information of the entire image, because a finite aperture only captures a finite fraction of the rays leaving an image. The intensity of the light from a point source exiting an aperture of radius a at angle θ is given by Hecht 10.56, and also in the notes, to be

$$I(q,R) = I(0)\frac{2J_1(kaq/R)}{kaq/R}$$

Here k is the wavenumber of the light, R is the distance from the aperture to the screen, and q is the position on the screen. These are related to the exit angle by $\sin(\theta) = q/R$, so an equivalent formulation is

$$I(\theta) = I(0)\frac{2J_1(ka\sin(\theta))}{ka\sin(\theta)}$$

Which expression you choose to use depends on what kind of optical instrument you're analyzing. The Bessel Function $J_1(x)$ is a function that is similar to a a sine function, except that it decreases in amplitude with increasing values of its argument-see p. 469 of Hecht for a picture. Its first zero is located at x=3.83, which is something you have to solve for numerically. From that, we get the relationship that the radius of an image is

$$q = 1.22\frac{R\lambda}{2a}$$

Or, using angles

$$\sin(\theta) = 1.22\frac{\lambda}{2a} = 1.22\frac{\lambda}{D}$$

where D=2a is the diameter. We can use this to compute the diffraction limit of an optical system such as our eye

Diffraction limit of the eye

In average light intensity, the pupil of the eye is 4 mm in diameter, and the average wavelength is 500 nm. This means that the smallest angle our eye can resolve due to diffraction is

$$\sin(\theta) \approx \theta = 1.22\frac{550\times10^{-9}}{4\times10^{-3}} = 167\times10^{-6} = 1.7\times10^{-4}$$

At a distance of 25 cm, this gives a resolution limit of $1.4\times10^{-4}\times25\times10^{-3} = 42\times10^{-6}$=42 microns. This is about half of the resolution limit due to spacing between cones in the fovea. However, this resolution varies due to changes in pupil size and in wavelength, so the two limits of resolution in the eye are quite well-matched.

The Eye

1. If the index of refraction of the eye were 1 (the same as air), and we modeled the eye as a single lens, what would be the length of the eye? We know that the power of a normal eye is 58.6 Diopters.

$$\frac{1}{d_2} = 58.6 + \frac{1}{\infty}$$
$$d_2 = 0.017 = 1.17\text{cm}$$

2. The refractive index of the humors in the eye is 1.336. Using this value, what is a more accurate estimate for the length of the eye?

   Solution:

$$\frac{n_{\text{eye}}}{d_2} = 58.6$$
$$l_{\text{eye}} = \frac{1.336}{58.6} = 0.0228 = 2.28\text{cm}$$

3. The separation of cones in the fovea is 1'=1/60°.=0.3 milliradians. The refractive index of the humors in the eye is 1.336. What is the distance between cones in the fovea?

   Solution:

   We can use Snell's law to find the angular separation of the cones of the fovea inside the eye. The refractive index of air is 1:

$$\sin(0.3 \times 10^{-3}) \approx 0.3 \times 10^{-3} = 1.336 \sin(\theta)$$
$$\theta = 3 \times 10^{-4}/1.336 = 2.24 \times 10^{-4}$$

   We found that the length of the eye is about 2.28 cm, so the spacing between the cones in the fovea is $2.24 \times 10^{-4} \times 0.0228\text{m} = 5.2 \times 10^{-6} m \approx 5$ microns.

4. I am nearsighted, and the power of my glasses is -3 diopters. What Is the farthest distance at which I can clearly focus on an object?

   Solution:

   Since my eyes require -3 diopters of correction, the minimum power of my eyes is 58.6-(-3)=61.6 Diopters. However, the image distance from my eye (the distance from the eyeball to the retina) is unchanged, so it's still 2.28 cm. So now we solve for the maximum image distance:

$$\frac{n_{\text{eye}}}{0.0228} = 58.6 = 61.6 + \frac{1}{d_1}$$
$$\frac{1}{d_1} = 61.6 - 58.6 = 3$$

   so the maximum distance that my eyes can focus without glasses is 1/3 meter, or about 30 cm.