Application of Ray Tracing to Telescopes

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Astronomical Telescope

Ray tracing matrix for telescope

An elegant way to calculate a lot of different properties of a telescope is to find its transfer matrix. In figure 1 we have four vectors. $(r_1 \quad \theta_1)$ is the vector of light coming in to the objective lens, and $(r_3 \quad \theta_3)$ is the vector of light coming out of the eyepiece. The transfer matrix for a thin lens is

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

In going from the objective through the eyepiece, light undergoes three transformations. Each of these transformations has a matrix that corresponds to it

1. refraction at the objective:

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_o & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

2. propagation through the telescope a distance $f_o + f_e$

$$\begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 1 & f_e + f_o \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix}$$

3. refraction at the eyepiece

$$\begin{bmatrix} r_4 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_e & 1 \end{bmatrix} \begin{bmatrix} r_3 \\ \theta_3 \end{bmatrix}$$

Multiplying all these matrices together gives

$$\begin{bmatrix} r_4 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_e & 1 \end{bmatrix} \begin{bmatrix} 1 & f_o + f_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f_e & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} r_4 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_e & 1 \end{bmatrix} \begin{bmatrix} -f_e/f_o & f_o + f_e \\ -1/f_o & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} r_4 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} -f_e/f_o & f_o + f_e \\ 0 & -f_o/f_e \end{bmatrix} \begin{bmatrix} r_1 \\ \theta_1 \end{bmatrix}$$

This final matrix tells us that the angular magnification is $-f_o/f_e$, so a ray coming in at the optic axis (r=0) at angle θ will emerge at an angle of $-f_e\theta/f_o$ and at a distance of $(f_e+f_0)\theta$ from the optic axis. It also tells us that the radial magnification is $-f_e/f_o$, so if you want to change the diameter of a beam of collimated light (such as laser light) you can pass it through an afocal system such as a telescope. Notice that the angular magnification and the radial magnification are inverses of each other–if the angles get bigger, the then heights of objects immediately at the lens will get smaller, and vice versa. So the telescope is effectively shrinking the image but maximizing its angular spread.

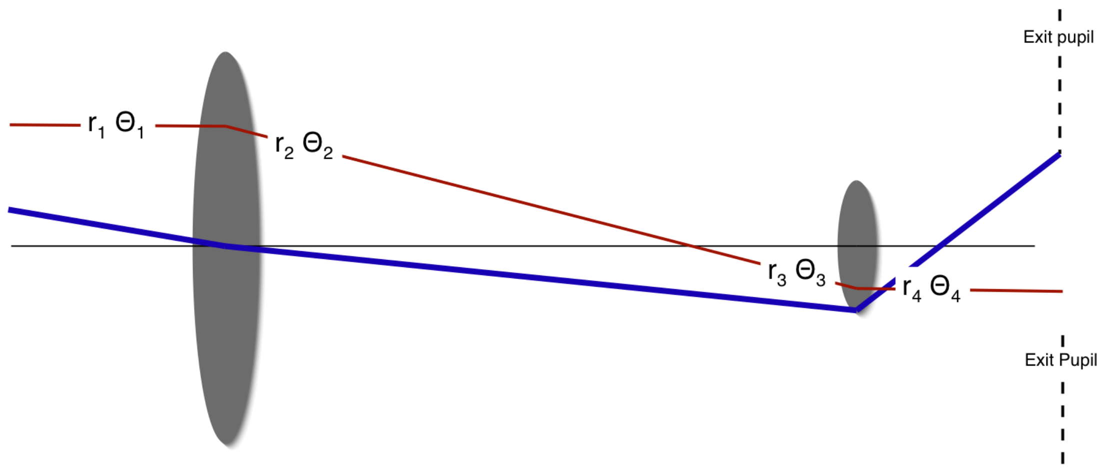

Figure 1:

Telescopes as beam expanders

If you have collimated light (from a "laser", for example) and you want to expand the diameter of the beam but keep it collimated, you can use a pair of lenses that are configured in an afocal configuration like a telescope. If the light is collimated (and if you align the lenses properly) then the angle θ will be zero, and you'll just be looking at the radial magnification of the beam. For two lenses with f_1 and f_2, where the light first goes through f_1 (so f_1 is the objective focal length) the radius of the outgoing beam will be

$$\begin{bmatrix} r_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -f_2/f_1 & f_1+f_2 \\ 0 & -f_1/f_2 \end{bmatrix} \begin{bmatrix} r_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -f_2 r_1/f_1 \\ 0 \end{bmatrix}$$

So the radius of the outgoing beam will be f_2/f_1 times the radius of the incoming beam. This sort of setup is also useful when you have a beam that isn't quite collimated and you want to recollimate it–you can adjust the distances slightly until the outgoing beam has collimated light. It's also useful if you want to focus light down to a small spot and maximize intensity to observe a nonlinear optical effect of a material.