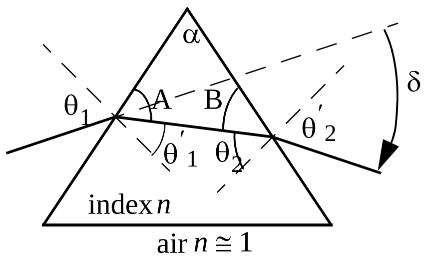
Chapter 2
Prisms, Lenses, and Imaging

Prisms

[Reading assignment: Hecht, 5.5. See also Smith Ch. 4]

Dispersing prism



Let's calculate the total deviation angle, δ.

- Deviation at first surface is $\theta_1 - \theta'_1$.
- At second surface, deviation is $\theta'_2 - \theta_2$

Total deviation is $\delta = (\theta_1 - \theta'_1) + (\theta'_2 - \theta_2)$

Notice that

but $A + B + \alpha = \pi$, so $\theta_2 = \alpha - \theta'_1$, then

$$\delta = \theta_1 - \theta'_1 + \theta'_2 + -(\alpha - \theta'_1) = \theta_1 + \theta'_2 - \alpha$$

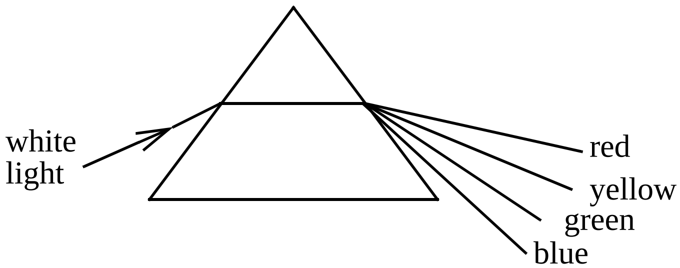
also $\sin\theta'_1 = \frac{1}{n}\sin\theta_1$, and $\sin\theta'_2 = n\sin\theta_2$. Now, writing δ in terms of θ_1, α, and n:

$$\delta = \theta_1 - \alpha + \sin^{-1}(n\sin\theta_2)$$
$$\sin^{-1}[n\sin(\alpha - \theta'_1)]$$

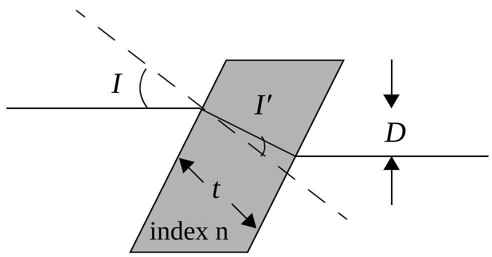
Use $\sin(\alpha - \theta'_1) = \sin\alpha\cos\theta'_1 - \cos\alpha\sin\theta'_1$, $\cos\theta'_1 = \left(1 - \frac{1}{n^2}\sin^2\theta_1\right)^{1/2}$, and $\sin\theta'_1 = \frac{1}{n}\sin\theta_1$. Then

$$\delta = \theta_1 - \alpha + \sin^{-1}\left[n\sin\alpha\left(1 - \frac{1}{n^2}\sin^2\theta_1\right)^{1/2} - \cos\alpha\sin\theta_1\right]$$

This formula shows that the deviation increases with increasing index n. For most materials n increases with decreasing λ This is the basis for the splitting of white light into colors by the prism.



Lateral displacement of a ray passing obliquely through a plane parallel glass plate:



$$D = t\sin I\left[1 - \sqrt{\frac{1 - \sin^2 I}{n^2 - \sin^2 I}}\right]$$

for small incidence angle $I \rightarrow i$

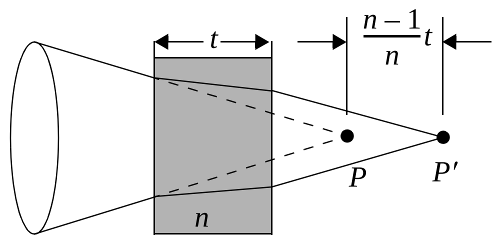
This can be used to laterally displace an image. One application of this very simple device is in a specialized high speed camera. The film has to move so fast that it is driven continuously (rather than actually stopping briefly for each frame as in a conventional camera). A rotating plate is used to make the image track the moving film during exposure of a given frame to prevent blur.

But the plate introduces aberrations.

– Chromatic effect: longitudinal and lateral displacements depend on n which is λ dependent.

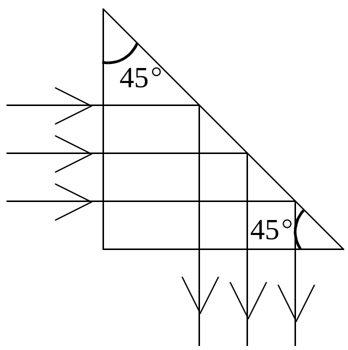
– For a plate used in convergant or divergent light, the amount of displacement is greater for larger angles which gives spherical aberration.

Plane parallel plate placed in between a lens and its focus:



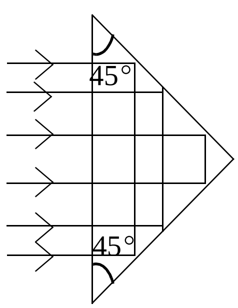
A simple calculation based on the paraxial approximation shows that the focus is displaced by amount . However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration.

Right Angle Prism



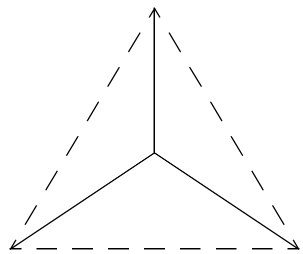
common building block in non-dispersive prism devices

Porro prism



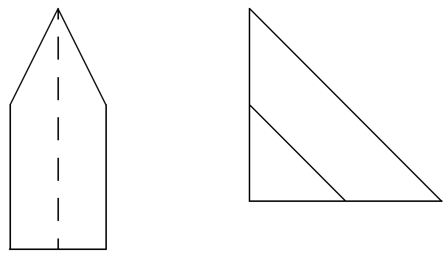
retro reflector (only folds back on itself in one meridian)

Corner cube



(beam reflects back on itself regardless of incident direction)

Roof Prism

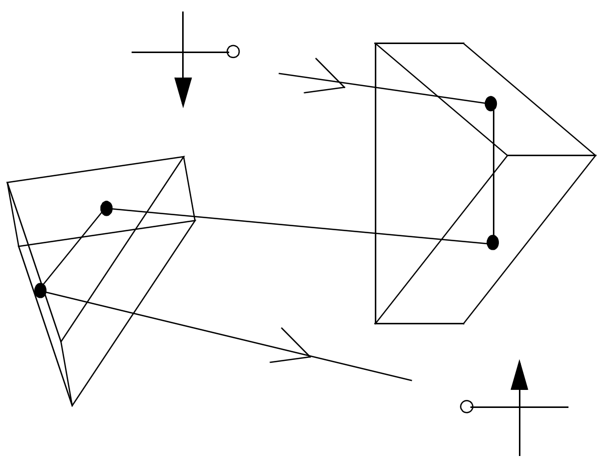


right angle prism with a roof on hypotenuse

provides an inversion of L-R which right angle prism does not.

Erecting Prisms

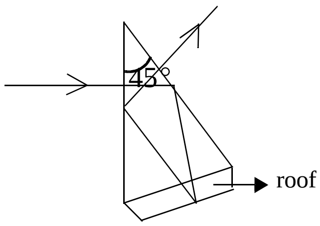
Most telescopes produce an inverted image (both U-D, L-R) to the eye. Erecting prisms re-invert the image to the proper orientation.



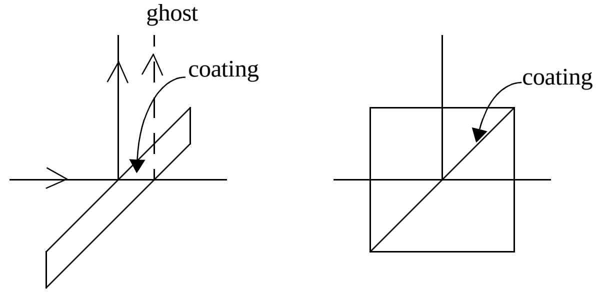
2 porro prisms used together.

Generally contacted

Schmidt Prism



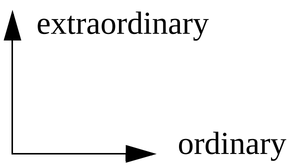
Prism Beamsplitter



Polarizing Prisms

[Reading assignment: Hecht 8.4]

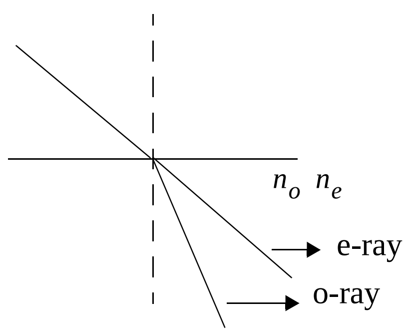
Birefringent crystals: given a propagation direction in the crystal - a set of orthogonal axes can be determined. For the two polarization directions along these axes, the index is different.



The ordinary component has index n_o and the extraordinary component has index n_e. As propagation direction varies the ordinary component always has index n_o but n_e varies between n_o and n_{e0}.

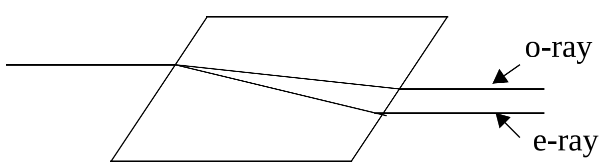
Double Refraction

Light incident on a birefringent crystal



o-ray pol and e-ray pol are not necessarily same as s and p pol.

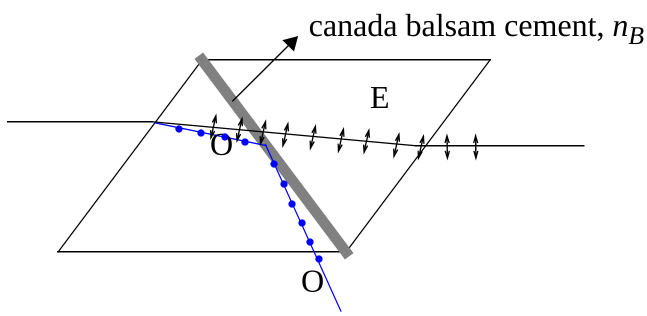
Birefringent plate



Common birefringent crystals-quartz (SiO2), calcite (CaCo3)

Not birefringent: Si, NaCl, GaAs, diamond

Nicol Prism



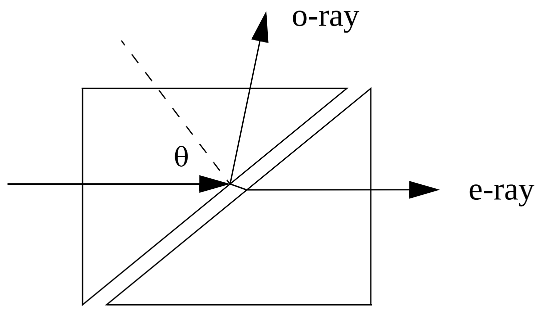
$n_o = 1.658$

$n_e = 1.486$

$n_B = 1.55$

O-ray is internally reflected at the calcite-balsam interface $n_B < n_o$. e-ray is transmitted (angle is cut for Brewster)

Glan Air Prism

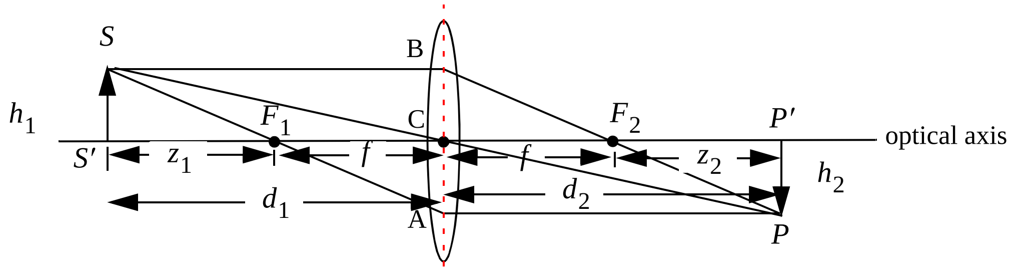


$$n_e < \frac{1}{\sin\theta} < n_o$$

Optical Imaging Systems

[Reading assignment: Hecht 5.2]

Thin lens, focal length f



- Rays entering the lens parallel to the axis, pass through the back focus, F_2
- Rays passing through the front focus, F_1 are "collimated" and emerge parallel to the axis.
- Rays passing through the center of the lens C are not bent.

- For a "thin lens", the rays are all bent at the lens plane, with no translation.
- Sign conventions: Heights above the axis are positive, below the axis are negative; left is negative, right is positive. Focal length for converging lens is positive; diverging lens is negative.

$\boxed{\quad\quad\quad}$ are positive

z_1, d_1, h_2 are negative

Triangles $S'SF_1$ and ACF_1 are similar, so

$$\boxed{\quad\quad\quad}$$

Triangles BCF_2 and $PP'F_2$ are similar, so

$$\boxed{\quad\quad\quad}$$

These two equations give us: $\frac{z_1}{f} = \frac{-f}{z_2}$ or $\boxed{\quad\quad\quad}$ "Newtonian" form of lens law

Now use $z_1 = d_1 + f$, $z_2 = d_2 - f$ (watch signs!)

$$f^2 = -z_1 z_2 = -(d_1 + f)(d_2 - f)$$

$$f^2 = -d_1 d_2 - f d_2 + d_1 f + f^2$$

cancel f^2, divide by $d_1 d_2 f$

$\boxed{\quad\quad\quad}$ "Gaussian" lens law

The lateral or <u>transverse image magnification:</u>

$$m \equiv h_2 / h_1 = f / z_1 = \frac{f}{d_1 + f}$$

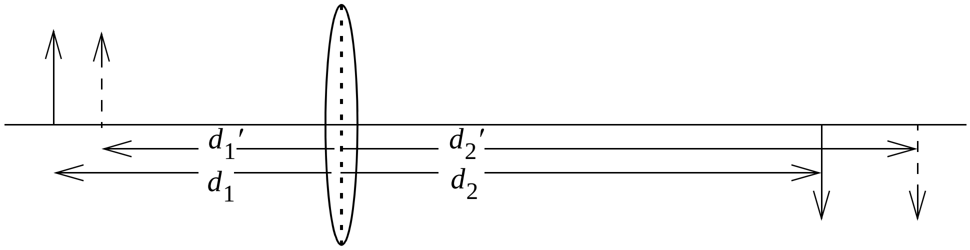
Use the lens law to get $d_2 = \frac{d_1 f}{d_1 + f}$.

so we also find $\boxed{\quad\quad\quad}$

The <u>longitudinal magnification</u> is of interest.

For a given small shift of an object along the optic axis, how much does the image shift?

We define the longitudinal magnification as:



$$\bar{m} \equiv \frac{\partial d_2}{\partial d_1} = \frac{(d_1 + f)f - d_1 f}{(d_1 + f)^2} = \frac{f^2}{(d_1 + f)^2} = m^2$$

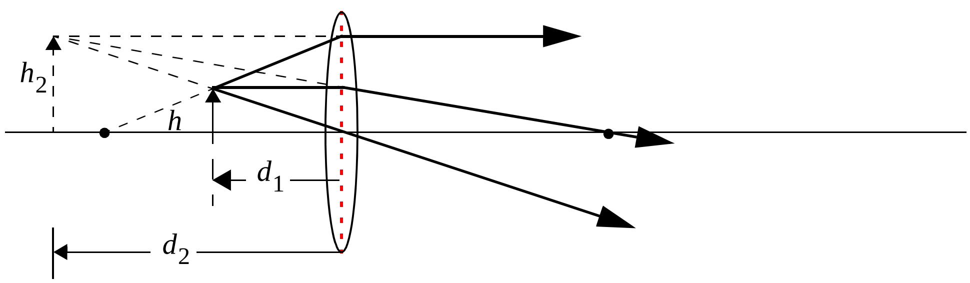
The longitudinal magnification is <u>positive</u> and the <u>square</u> of the transverse magnification

Virtual Image

For an object to the left of the lens, d_1 is <u>negative</u>. Since

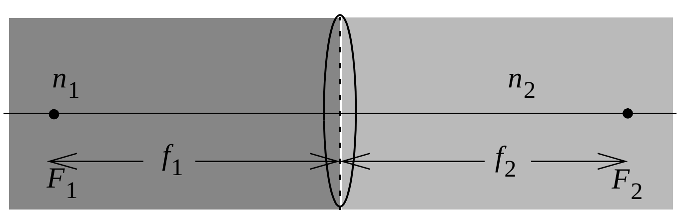
$$\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1}$$

Then if $|d_1| < f$, then d_2 is <u>also</u> negative.



The light emerging from the lens <u>appears</u> to be coming from the object with height h_1 at distance d_2 behind the lens.

For an optical system <u>not</u> immersed in air



The front and back focal lengths are not the same in this case

front focal length: f_1

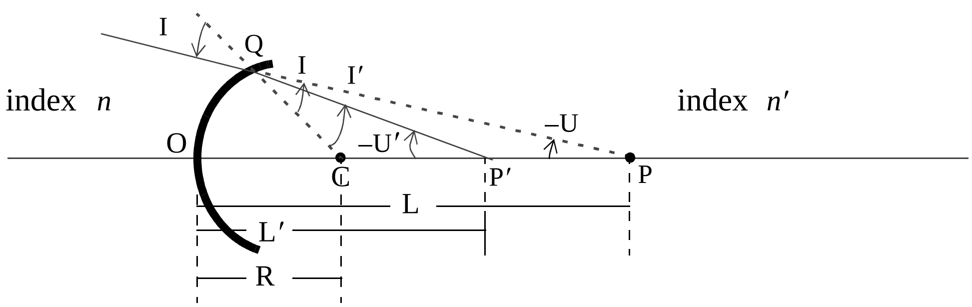
back focal length: f_2

lens law becomes: $$\frac{n_2}{d_2} = \frac{n_1}{d_1} + \frac{n_1}{f_1} = \frac{n_1}{d_1} + \frac{n_2}{f_2}$$

$$z_1 z_2 = -f_1 f_2$$

$$m = \frac{h_2}{h_1} = \frac{n_1 d_2}{n_2 d_1} \qquad \bar{m} = \frac{f_1 f_2}{z_1^2} \qquad \bar{m} \neq m^2 \quad (\text{show } \bar{m} = \frac{n_2}{n_1} m^2)$$

Refraction of light by a spherical surface (following Smith 2.4)



Sign Conventions

1. Radius is positive when center of curvature is to the right of the surface
2. Distance to the right of surface is positive; left negative.
3. For I, I', counterclockwise from the surface normal is positive.
4. For U, U', the angle is positive if the ray slope is positive.
5. Rays travel left to right

In the diagram above all the quantities are positive.

Consider triangle QCP. By law of sines:

(2.1)

Similarly for triangle QCP':

$$\frac{\sin I'}{L' - R} = \frac{-\sin U'}{R} \tag{2.2}$$

Comparing triangles QCP and QCP', we see they share a common angle. Therefore

(2.3)

Finally, by Snell's law

$$n \sin I = n' \sin I' \tag{2.4}$$

We can arrive at the Gaussian lens law (for the single surface) from these equations.

Manipulate Eq. (2.1):

$$\frac{\sin I}{\sin U} = \frac{R-L}{R} = 1 - \frac{L}{R}$$

$$\frac{L}{R} = 1 - \frac{\sin I}{\sin U} = \frac{\sin U - \sin I}{\sin U}$$

$$\frac{R}{L} = \frac{\sin U}{\sin U - \sin I} = 1 - \frac{\sin I}{\sin I - \sin U}$$

Now multiply by $\frac{n}{R}$ to get

(2.5)

Similarly, from Eq.(2.2),

$$\frac{n'}{L'} = \frac{n'}{R} - \frac{n'}{R}\frac{\sin I'}{\sin I' - \sin U'} \tag{2.6}$$

Subtract Eq.(2.5) from Eq.(2.6)

$$\frac{n'}{L'} - \frac{n}{L} = \frac{n'-n}{R} - \left[\frac{n'}{R}\frac{\sin I'}{\sin I' - \sin U'} - \frac{n}{R}\frac{\sin I}{\sin I - \sin U}\right]$$

Using Eq. (2.4),

(2.7)

This has the form of the lens law, except for the dependence on the sin of all the angles.

We shall see that Gaussian Optics applies to spherical surfaces only in the Paraxial Approximation.

Paraxial Approximation

The paraxial approximation refers to the case when all ray angles remain small. (Close to the optic axis.) In this case, for all angles, $\sin x \cong x \quad \tan x \cong x$. By convention, the lower case letter is substituted for the capital in this approximation, so

$$\sin I \to i,\ \sin I' \to i',\ \sin U \to u,\ \sin U' \to u'$$

$L \rightarrow l, L' \rightarrow l'$.

R is unaffected

Eqs. (2.1)-(2.4) become

$$\boxed{} \qquad (2.8)$$

$$\frac{i'}{l'-R} = \frac{-u'}{R} \qquad (2.9)$$

$$i - u = i' - u' \qquad (2.10)$$

$$\boxed{} \qquad (2.11)$$

Then Eq. (2.7) becomes:

$$\boxed{} \qquad (2.12)$$

This is the Gaussian lens law for a single surface.

Consider a ray incident from the left, parallel to the axis. Then $l \rightarrow -\infty$, and we have

$$\frac{n'}{l'} = \frac{n'-n}{R}$$

$$\boxed{}$$

Thus, the back focal length, f_2, is $\frac{n'}{n'-n}R$. Similarly, for an image distance of ∞, we must have $l' \rightarrow \infty$ and

$$\frac{-n}{l} = \frac{n'-n}{R}$$

$$l = -\frac{n}{n'-n}R.$$

This means the front focal length f_1 is $\boxed{}$

(Watch minus sign!)

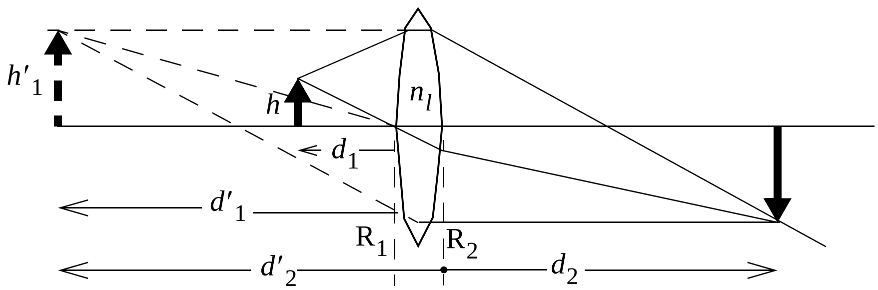
Recall the previous discussion for a lens not immersed in air:

$$\frac{f_1}{n_1} = \frac{f_2}{n_2}\ ;\ \frac{n_1}{d_2} = \frac{n_2}{d_1} + \frac{n_1}{f_1} = \frac{n_1}{d_1} + \frac{n_2}{f_2}$$

These relations clearly hold for the single spherical surface.

Thin Lens Model

We now construct our model for the thin lens in air. Lens index is n_l. We will find the imaging properties of the thin lens by using the previous results for a single spherical surface and applying them twice - once for each of the two surfaces of the lens.



Use Eq. (2.12) for first surface: $n = 1$ $n' = n_l$, $l \rightarrow d_1$, $l' \rightarrow d'_1$

(2.13)

We get a virtual object of height h'_1 at d'_1

Now consider the rays travelling inside the lens from the virtual object. Apply spherical surface law now to the second surface. This time $n' = 1$, $n = n_l$, $l \rightarrow d'_2$, $l' \rightarrow d_2$

(2.14)

The thin lens approximation is that the lens thickness is negligible, so that $d'_2 \cong d'_1$. Using this in Eq (2.13), then substituting in Eq. (2.14),

(2.15)

This is the Gaussian lens law, with the focal length identified as:

(2.16)

This is called the lensmaker's equation.

We conclude that a lens with 2 spherical surfaces satisfies the Gaussian lens law, but only under 2 important approximations

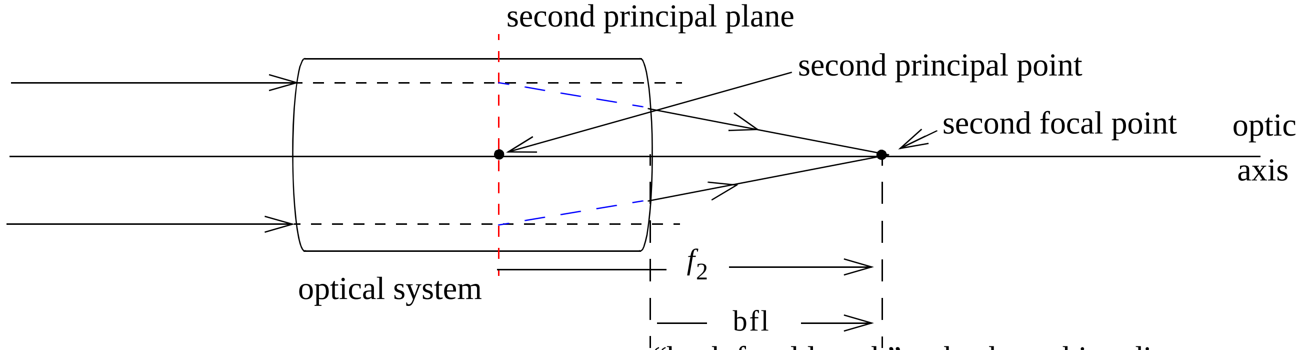
- Paraxial approximation
- Thin-lens approximation

Thick lens or compound lens systems

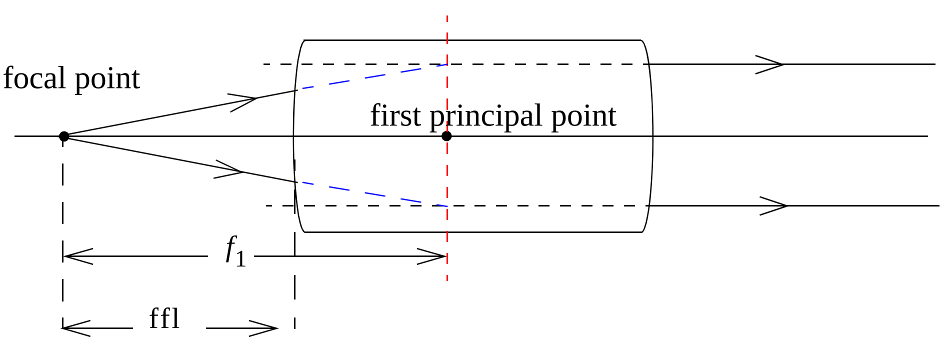
[Reading assignment: Hecht 6.1]

Any symmetric optical system consisting of lenses and spaces can be generalized.

Light rays entering from the left, parallel to the optic axis, come to a focus, at the "second focal point"

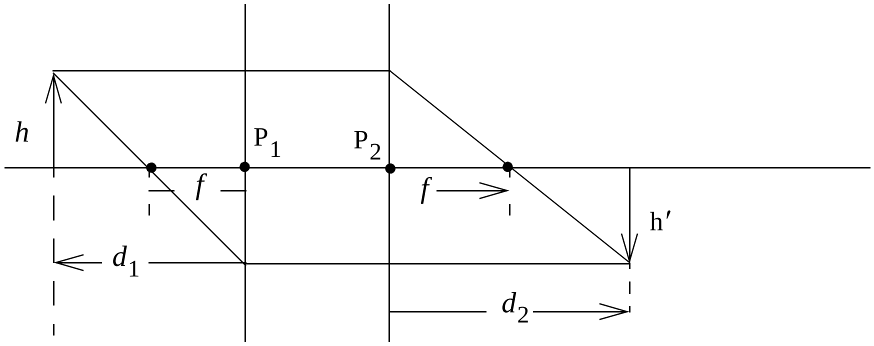


Now, we take the rays entering the system and those emerging from the system and extend them. They intersect on a plane called the "Second principal plane". Similarly, the first focus and "first principal plane" are defined for rays emerging from the system parallel to the axis, which all emanate from a point.



We define f_2 as the distance from the second principal plane to the second focal point. Similarly we define f_1 as the distance from the first principal plane to the first focal point. For a system immersed in air (same index on both sides), $f_1 = f_2$.

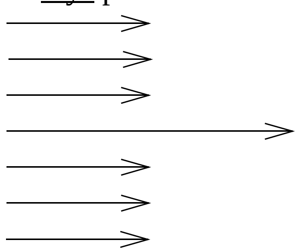
With these definitions, the Gaussian lens law applies as follows:



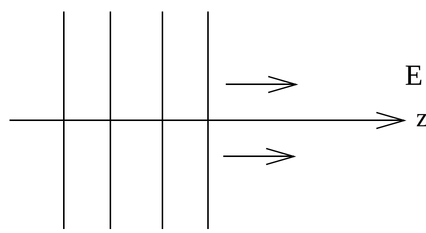
With this geometry, all other relations now apply:

Wave optics of lenses

Set of <u>rays</u> parallel to axis

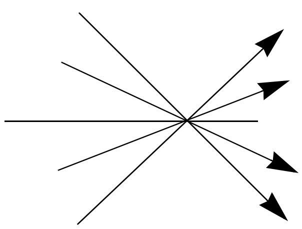


Plane Wave

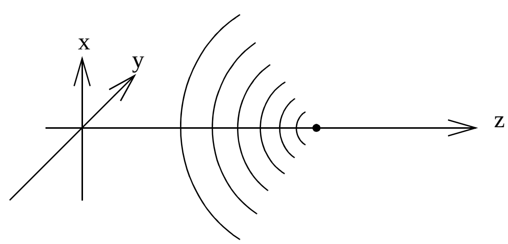


$$E = E_\circ \cos(kz - \omega t)$$
$$k = \frac{2\pi}{\lambda}$$
$$\omega = 2\pi f$$

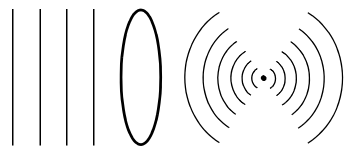
Rays converging to a focus



converging spherical wave



At a given z-plane, the spherical wave has constant phase around circles. The form of the spherical wave is $\cos\left[-\frac{k(x^2+y^2)}{2z_\circ}\right]$ for a spherical wave converting to the point $z_\circ$ on the axis. A lens modifies the wave front, for example from planar to spherical.

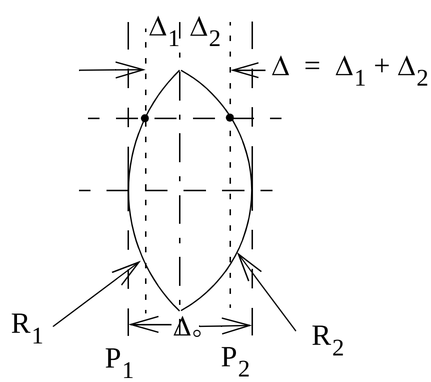


How does this happen?

Optical path length:

Optical waves travel more slowly in the glass since $n > 1$. In glass, the wave is delayed by an amount as if it travelled a distance nl in free space. If $l = l(\mathrm{x,y})$ [or $n = n(\mathrm{x,y})$] then the delay varies with (x,y) so the wavefront gets distorted.

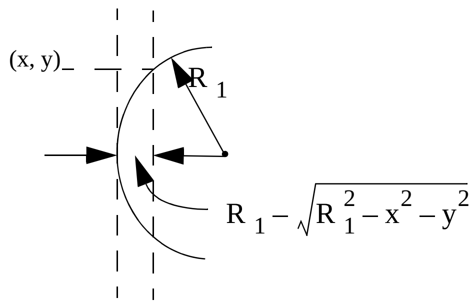
We can analyze the lens in terms of its phase-delay. The light propagates in the glass as $\cos(kn\mathrm{z}) = \cos\phi$, where $\phi = kn\mathrm{z}$ is the phase delay.



In propagating from plane P_1 to P_2, the light travels a distance $\Delta = \Delta_1 + \Delta_2$ in the glass and a distance $\Delta_\circ - \Delta$ in air, where $\Delta_\circ$ is the thickness at the thickest part of the lens. The phase delay depends on (x, y):

We can calculate Δ, assuming spherical surfaces. Recall the sign convention for the surface radii:

positive radius   negative radius



From this diagram, we can readily obtain

$$\Delta(x, y) = \Delta_\circ - \left[R_1 - \sqrt{R_1^2 - x^2 - y^2}\right] + \left[R_2 - \sqrt{R_2^2 - x^2 - y^2}\right]$$

$$= \Delta_\circ - R_1\left[1 - \sqrt{1 - \left(\frac{x^2 + y^2}{R_1^2}\right)}\right] + R_2\left[1 - \sqrt{1 - \left(\frac{x^2 + y^2}{R_2^2}\right)}\right]$$

In the <u>paraxial approximation</u> $(x^2 + y^2) \ll R_{1,2}^2$, so

$$\sqrt{1 - \left(\frac{x^2 + y^2}{R_{1,2}^2}\right)} \cong 1 - \left(\frac{x^2 + y^2}{2R_{1,2}^2}\right) \qquad \text{, thus}$$

$$\Delta \cong \Delta_\circ - \left(\frac{x^2 + y^2}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This gives a phase delay:

$$\phi(x, y) = k\Delta_\circ + k(n-1)\left[\Delta_\circ - \left(\frac{x^2 + y^2}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]$$

Apart from the constant delay $kn\Delta_\circ$, the phase delay is:

A plane wave incident on the lens has a constant phase. After passing through the lens, the phase is given above. This has the form of a spherical wave, converging to a point at a distance f, where

,

f is the focal length of the lens. This expression is identical to what we found from the ray optics analysis.

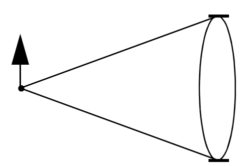
Stop and Apertures

[Reading assignment: Hecht 5.3]

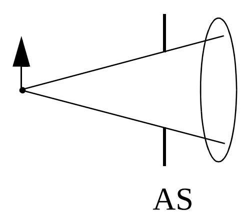
Aperture Stop

Every optical system has some component that limits the light cone that is accepted from an axial object.

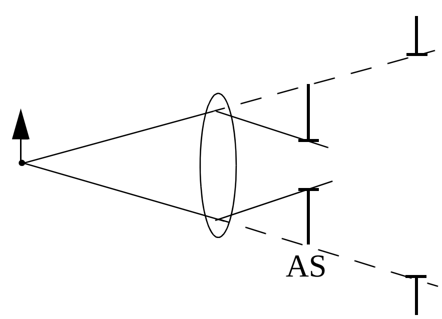
Simple case - single lens



could be the diameter "clear aperture" of the lens.



could be a physical aperture placed somewhere in the optical path.

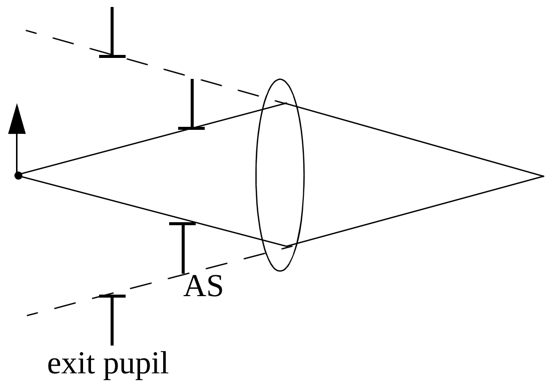


entrance pupil

- Entrance pupil:
  Image of the aperture stop as seen from the object side. Defines the cone of light accepted by the optic.

  The importance of the entrance pupil is that the brightness of the image depends on this cone angle. The larger the acceptance angle, the more light that is collected from each object point, and hence the brighter the image.
- Exit pupil:

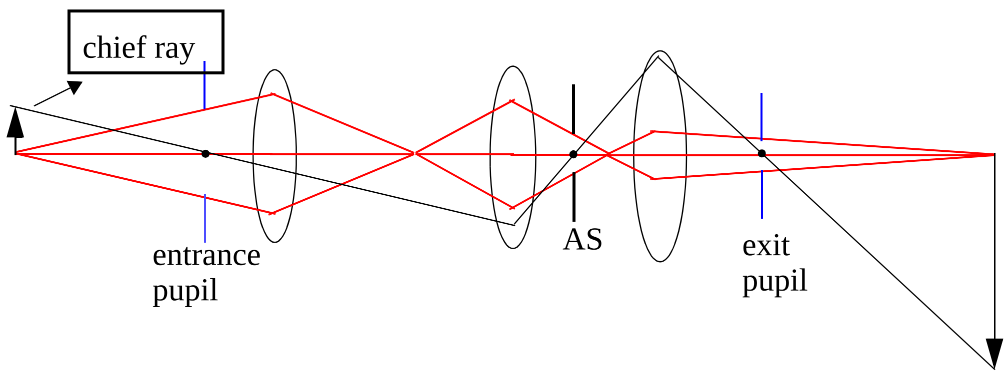
Image of the aperture stop, as seen from the image side of the optic.



The exit pupil defines the cone angle of light converging to the image point. Later, we will see that this is important in determining the image resolution that is set by diffraction.

Entrance and Exit Pupils are Images of each other

The entrance pupil is the image of the stop. The exit pupil is also an image of the stop. So the entrance and exit pupils must also be images of each other. The pupils define the amount of light accepted by and emitted from the optical system.



Chief Ray or Principle Ray

From a given object point, the ray that passess through the center of the pupils.

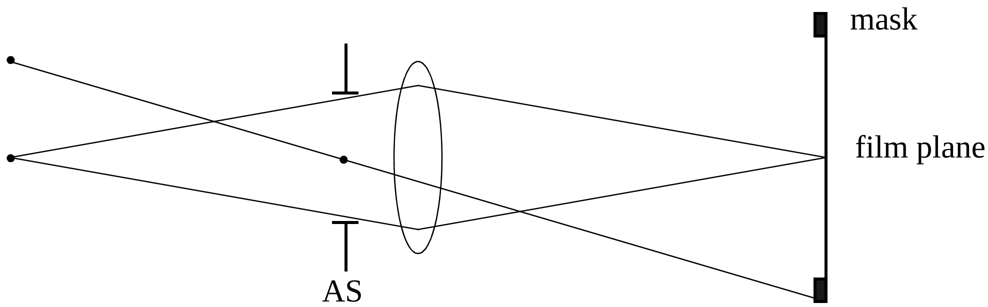
Marginal Ray

From a given object point, a ray that passes at the edge of the pupils.

Field Stop

Another stop in the system limits the extent of the object/image sizes. The <u>chief ray</u> from an object point is blocked by the field stop.

Simple case: a mask at the object or image plane.



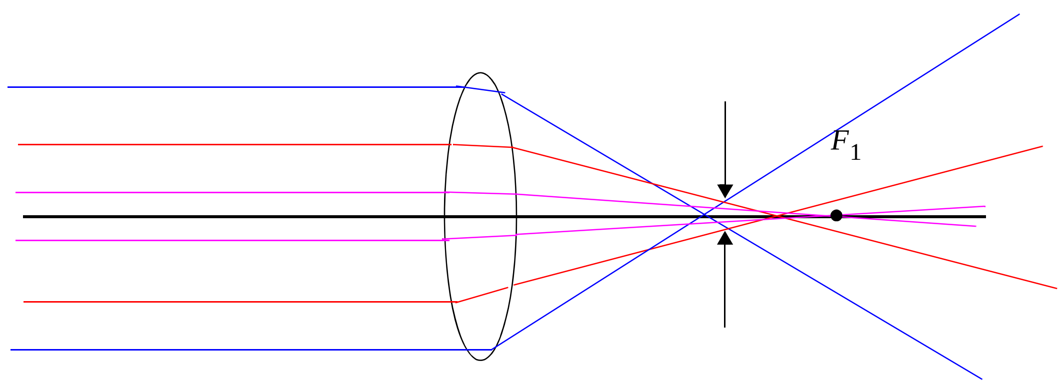
The field stop might also be set by a diaphragm somewhere in the optical path.

- <u>Entrance window</u>: Image of the field stop at the object plane.
- <u>Exit window</u>: Image of the field stop at the image plane.

Aberrations

[Reading assignment: Hecht 6.3]

As we have seen, spherical lenses only obey Gaussian lens law in the paraxial approximation. Deviations from this ideal are called <u>aberrations.</u>

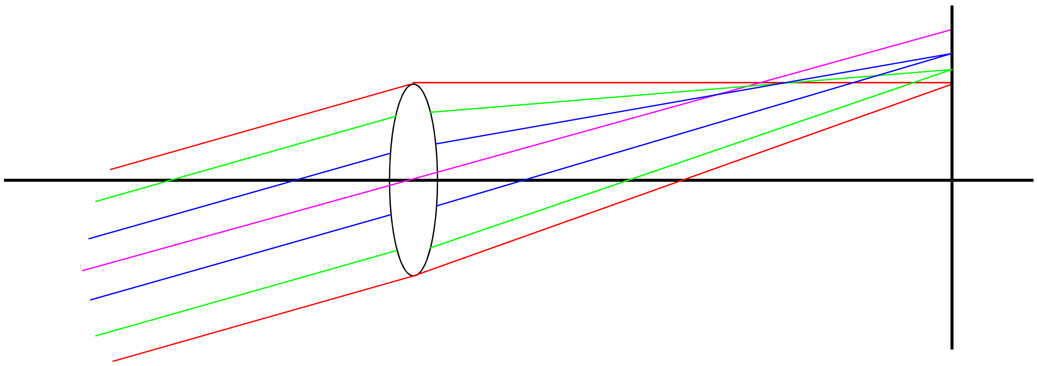


Rays toward the edge of the pupil (even parallel to the axis) violate the paraxial condition on the incidence angle at the first surface. They focus closer (for biconvex lens) than F_1. No truly sharp focus occurs. The least blurred spot (smallest disc) is called circle of least confusion, or best focus. This form of symmetric aberration is <u>spherical aberration.</u>

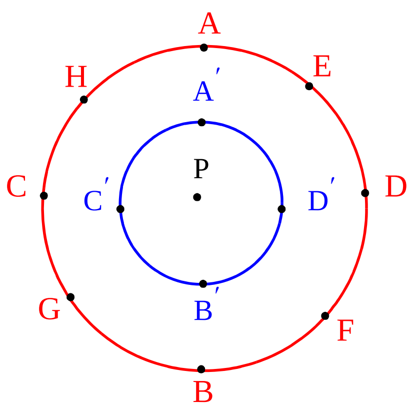
There are many forms of aberration.

<u>Coma:</u> Variation of magnification with aperture.

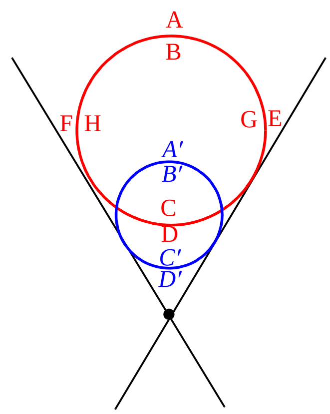
Rays passing through edge portions of the pupil are imaged at a different height than those passing through the center.



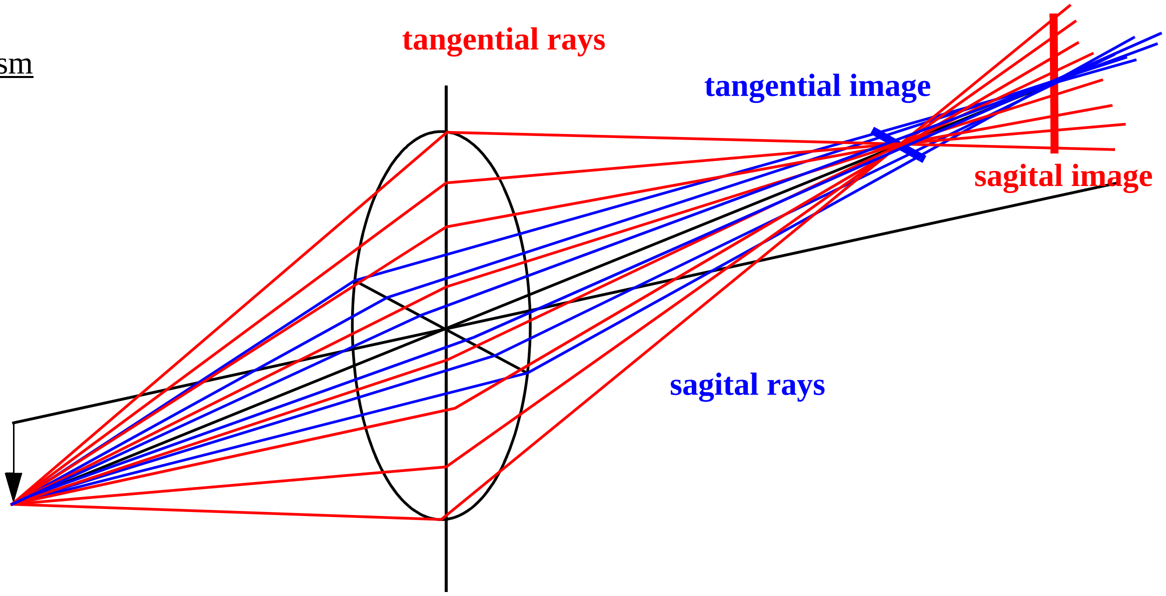
<u>Map of Rays in Pupil</u>



<u>Image Plane</u>

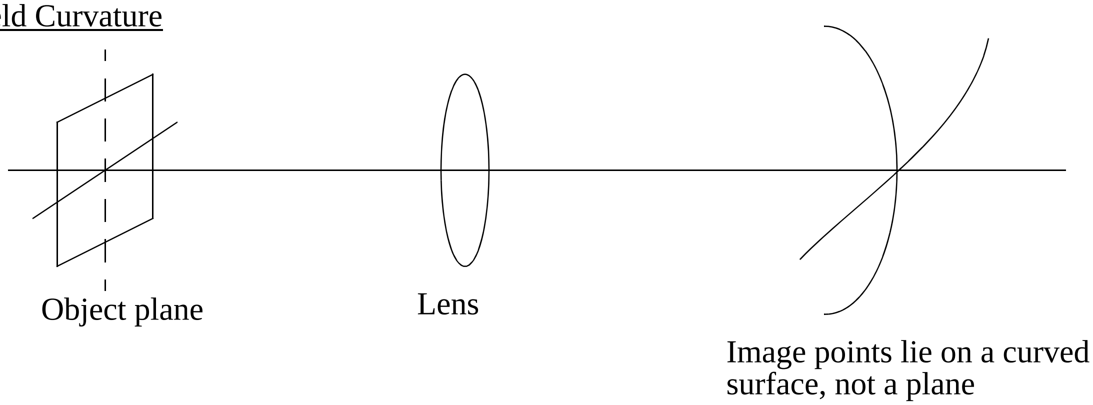


Astigmatism



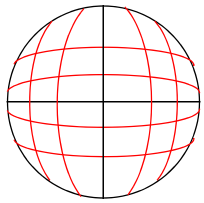
In astigmatism the tangential and sagittal images do not coincide. There are 2 line images with a circle of least confusion in the middle.

Field Curvature

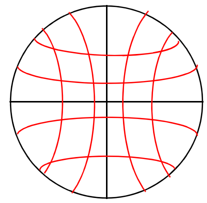


Positive lenses give inward curvature
negative lenses give backward curvature.

Distortion: Field dependent magnification



Barrel distortion



Pincushion distortion

Five Primary Aberrations

Spherical, coma, astigmatism, field curvature, distortion

Wave Front Aberration

In a wave-optics picture, the thin lens is represented by phase delay.

$$\phi(x, y) = -k\frac{x^2 + y^2}{2} = -k\Delta(x, y)$$

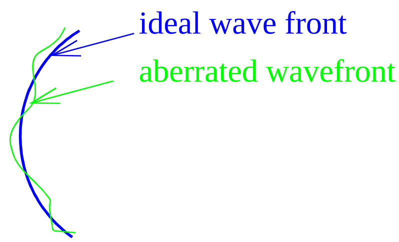
Which gives Gaussian imaging. Aberrations modify ϕ. A spherical lens only gives this ϕ in the paraxial approximation.

- For a complex optical system, we can collect the effects of all the lenses and represent them as a phase delay in the exit pupil. Usually, we subtract the quadratic phase to find the aberration. The residual is called the wave front error; or wfe

$$\Delta(x, y) = -\frac{x^2 + y^2}{2f} + W(x,y)$$

Aberration wfe

$\Delta(x, y)$ usually depends on the field coordinate. In other words, the aberrations can vary depending on where you are in the field of view.



Expressed in this way, the primary aberrations are written as:

Spherical aberration:

Coma:

Astigmatism:

ρ : is normalized radial coordinate in the pupil

h' : image height

Field Curvature: $A_d \rho^2 h'^2$

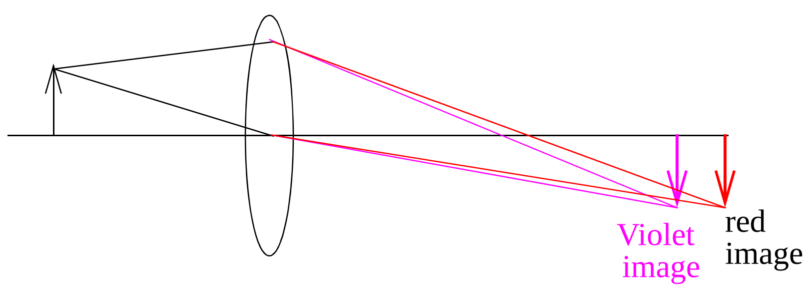
Distortion: $A_t h'^3 \rho \cos\theta$

<u>Monochromatic Aberrations:</u> All of the preceding discussion refers to aberrations that do not depend on wavelength.

<u>Chromatic Aberrations:</u> Dependance of wavefront on wavelength.

Consider the simple thin lens equation:

The index n is generally λ dependent, $n(\lambda)$, so f is λ dependent.



Change in image distance: longitudinal chromatic aberration

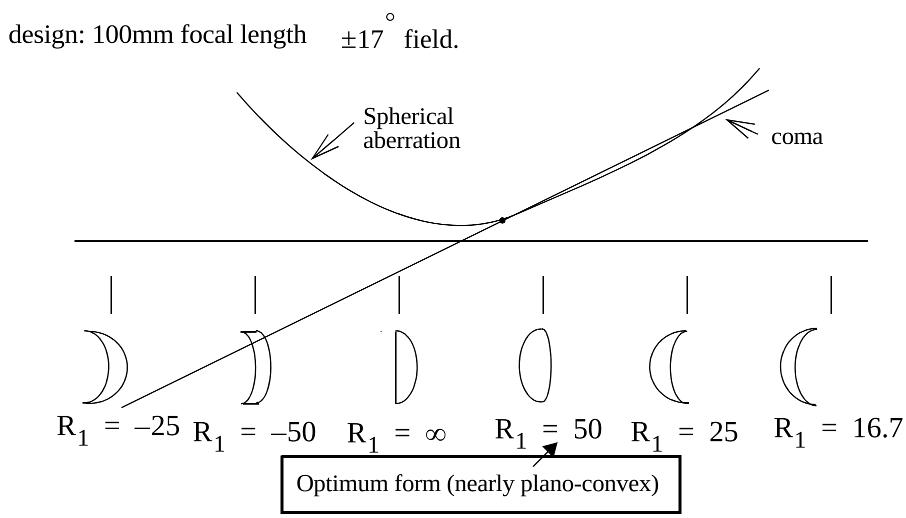
Change in magnification: lateral color. Lateral color is usually more noticeable

<u>Achromat:</u> lens designed to cancel chromatic aberration.

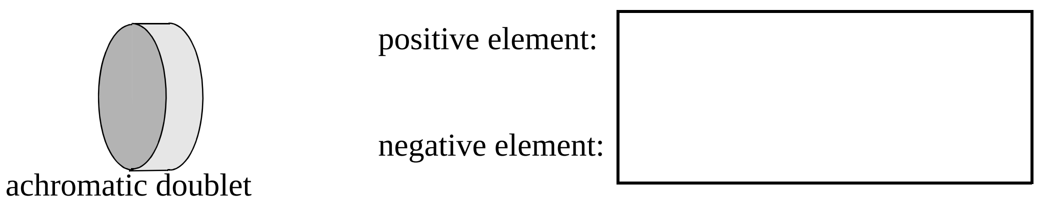
<u>Lens Design:</u>

- The general problem of lens design involves cancelling aberrations
- Aberration depends on the lens index, as well as the surface radii.
- Complex lens systems can minimize aberrations

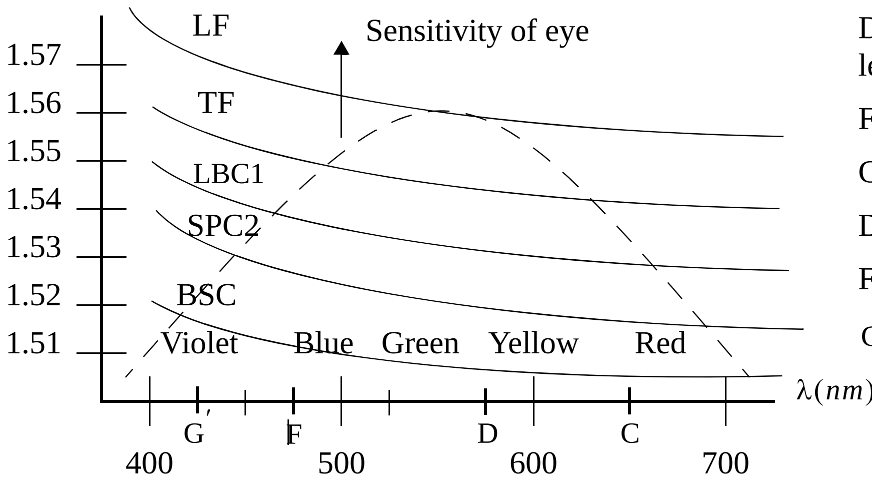
<u>Simple singlet case</u>: For a given desired focal length, there is freedom to choose one of the radii for a singlet The spherical aberration and coma depend on the particular choice, so these aberrations can be minimized by the design form. This is illustrated in the following diagram:



<u>Achromatic doublet.</u> Two elements made from different glass materials



We generally choose design an achromat to minimize chromatic aberration across the visible part of the spectrum.

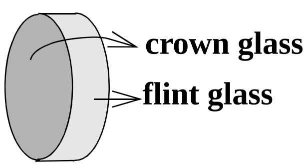


Different glasses for use in lenses.

Fraunhofer designations.

C H	656.3 nm
D Na	589.2
F H	486.1
G' H	434.0

Design of Cemented Doublet Achromat



The 'D' wavelength, near the center of visual brightness curve is chosen as the nominal wavelength for specifying focal length. We then choose 2 indices on either side, for achromatization, for example, 'C', and 'F'.

For 2 thin lenses in contact

$$\frac{1}{f_D} = \frac{1}{f_{D'}} + \frac{1}{f_{D''}}$$

prime: crown glass

double prime: flint glass

define lens power $\boxed{\quad\quad}$ with f in meters,

P units are <u>diopters</u>

$$\boxed{\quad\quad\quad\quad}$$

$$= (n_D' - 1)\left(\frac{1}{r_1'} - \frac{1}{r_2'}\right) + (n_D'' - 1)\left(\frac{1}{r_1''} - \frac{1}{r_2''}\right)$$

Define $K' = \left(\frac{1}{r_1'} - \frac{1}{r_2'}\right)$ $K'' = \left(\frac{1}{r_1''} - \frac{1}{r_2''}\right)$

$$P_D = (n_D' - 1)K' + (n_D'' - 1)K''$$

$$P_F = (n_F' - 1)K' + (n_F'' - 1)K''$$

$$P_C = (n_C' - 1)K' + (n_C'' - 1)K''$$

Achromatic design means we make

$$(n_F' - 1)K' + (n_F'' - 1)K'' = (n_C' - 1)K' + (n_C'' - 1)K''$$

Simplifies to:

For normal dispersion K' has the opposite sign from K''. One lens must be positive one lens must be negative.

For the center of the spectrum (D-line)

$$P_D' = (n_D' - 1)K'$$

$P_D'' = (n_D'' - 1)K''$, so

$$\frac{K'}{K''} = \frac{(n_D'' - 1)P_D'}{(n_D' - 1)P_D''}$$

Combining results, we find:

$$\frac{P_D''}{P_D'} = -\frac{(n_D'' - 1)\ (n_F' - n_C')}{(n_D' - 1)\ (n_F'' - n_C'')} \equiv -\frac{\nu''}{\nu'} \tag{2.17}$$

is a property of a given glass called the "dispersion constant"

ν is called the "dispersive power" or V-number. Glass manufacturers spec these numbers for use by designers. Now, from Eq. (2.17),

$$\tag{2.18}$$

and

$$P_D' = P_D \frac{\nu'}{\nu' - \nu''} \qquad\qquad P_D'' = -P_D \frac{\nu''}{\nu' - \nu''} \tag{2.19}$$

Eqs. (2.18) and (2.19) are the design equations.

- Design starts with desired f_D, P_D
- Next choose your glass materials, i.e. ν', ν''
- Find P_D', P_D'' from Eq. (2.19), then get K', K''
- Choose radii (still some freedom left in choice of radii for minimization of monochromatic aberrations). A common, simple choice is to make the crown lens biconvex, and to cement the two lenses together, with no gap. This means:

biconvex

cemented

Then r_2'' is set by the constraint of Eq. (2.19).

For crown glass facing parallel light, this gives a good design to minimize spherical and coma. It can be fine tuned by careful choice of ν', ν''

Example: Design 10cm focal length cemented doublet using the crown and flint glasses. $P_D = 10D$

	$\mathbf{n_C}$	$\mathbf{n_D}$	$\mathbf{n_F}$	$\mathbf{n_{G'}}$
crown	1.50868	1.511	1.51673	1.52121
flint	1.61611	1.6210	1.63327	1.64369

$\nu' = 63.4783 \quad \nu'' = 36.1888$

$P_D' = 23.2611D \quad P_D'' = -13.2611D$

note (checks!)

Using biconvex for positive element $r_1' = -r_2'$

$$K' = \frac{2}{r_1'} = \frac{P_D'}{n_D' - 1} = 45.5207$$

$r_1' = 0.043961\text{m} = 4.3961\text{cm}$

with $r_1'' = -r_1'$, $K'' = -\frac{1}{r_1''} - \frac{1}{r_2''} = \frac{P_D''}{n_D'' - 1} = -21.3544$

So, $\boxed{\qquad\qquad\qquad}$. This completes the design.

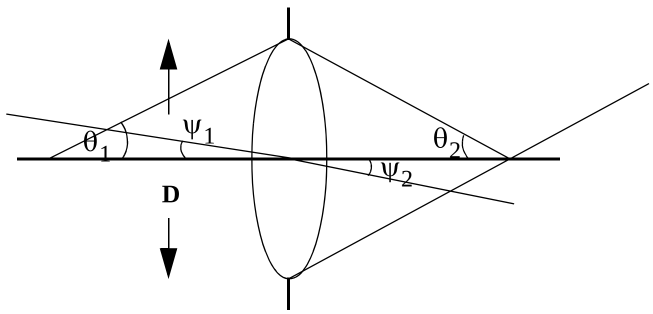
Now check how well it works:

$P_C = (n_C' - 1)K' + (n_C'' - 1)K'' = (0.50868)45.5207 + (0.6161)(-21.3544) = 10.0012D$

$P_F = 9.9988D$

Resolution limit of an optical system

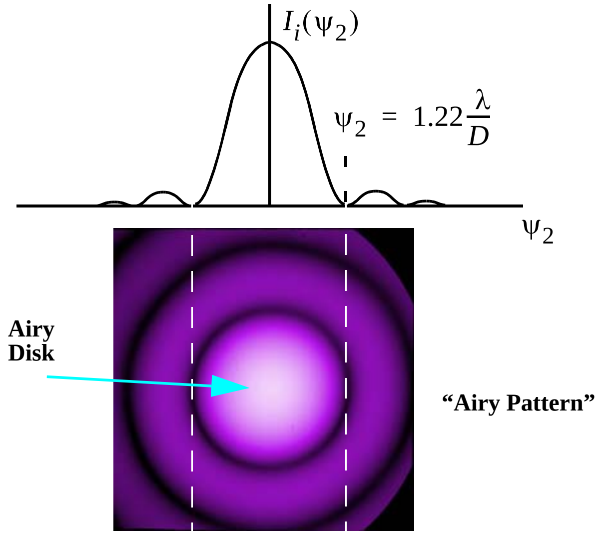
[Reading assignment: Hecht 10.2.6]

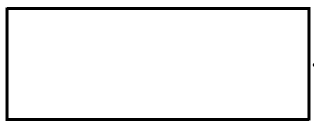


Due to diffraction at the aperture stop, the image of a point is slightly blurred. Diffraction theory tells us that the image depends on the shape of the aperture. For a circular aperture:

$$I_i(\psi_2) = I_o\left[\frac{2J_1\left(\pi\frac{D}{\lambda}\sin\psi_2\right)}{\frac{\pi D}{\lambda}\sin\psi_2}\right]^2$$

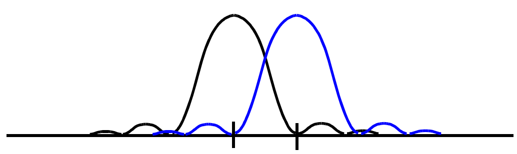
$J_1(x)$ is a special function called the “Bessel Function of the First Kind”.



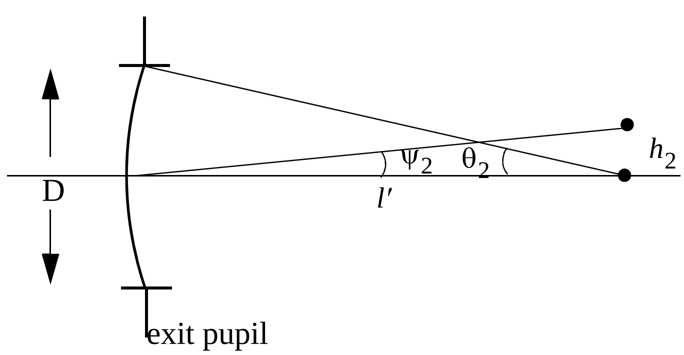
The first zero in the pattern occurs at .

If 2 points lie close together in the object plane, the Airy patterns will overlap. The criterion for whether the 2 points can be resolved depends on the type of imaging application and it is somewhat arbitrary. A very common criterion is Rayleigh's criterion.

According to Rayleigh's criterion, 2 spots are resolved if the maximum of the pattern from one point falls on the first minimum of the other.



We say that the angular resolution in the image plane is $1.22\lambda/D$



with l' the distance to the exit pupil (radius of exit sphere), we have

$$l' = \frac{-D}{2\sin\theta_2}$$

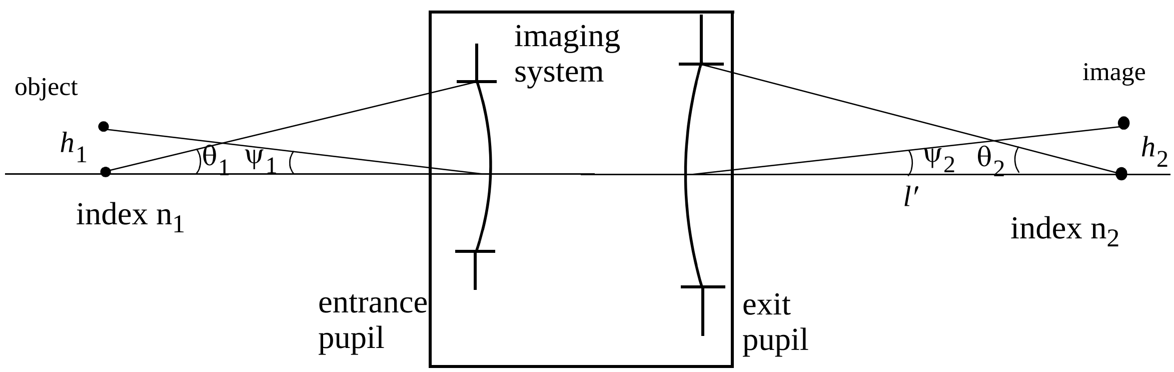
for small ψ_2 (small h_2), $h_2 \cong l'\psi_2$

so

where $NA_2 \equiv \sin\theta_2$.

The “Numerical Aperture” or NA is a very important property of an imaging system. It is simply the sine of the half angle subtended by the pupil. Here, NA_2 is the numerical aperture of the exit pupil.

Somewhat more generally, consider a complete imaging system. The entrance pupil subtends an angle θ_1 with an object of height h_1,



A general property of imaging systems holds that:

$$h_1 n_1 \sin\theta_1 = h_2 n_2 \sin\theta_2 .$$

The numerical aperture is generalized if the object and image spaces are immersed in different index of refraction.

$$n \sin\theta_1 \equiv NA_1 . \quad n_2 \sin\theta_2 \equiv NA_2$$

Here NA_1 is the “entrance” numerical aperture and NA_2 is the “exit numerical aperture.

Once again we have:

For large $NA \sim 0.6$ $\quad h_2 = \lambda$, so the resolution ~ wavelength of light.

Also notice that:

$$\frac{h_2}{h_1} = m = \frac{NA_1}{NA_2}$$

which says that the entrance and exit numerical apertures have a ratio equal to the transverse magnification.