Discussion 12 EE119

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- Interference : "superposition of two or more waves that results in a new wave pattern".

at P. $\vec{E}_1$, $\vec{E}_2$

$$I_p = \langle(\vec{E}_1+\vec{E}_2)\cdot(\vec{E}_1+\vec{E}_2)\rangle = \langle\vec{E}_1\cdot\vec{E}_1\rangle + \langle\vec{E}_2\cdot\vec{E}_2\rangle + 2\langle\vec{E}_1\cdot\vec{E}_2\rangle$$

$$(= I_1 + I_2 + I_{12}$$

Intensity at P is the time average of the square of the field amplitude.

$$I = \langle\vec{E}\cdot\vec{E}\rangle$$

I_{12} — interference term.

$$\vec{E}_1 = \vec{A}_1\cos(\vec{k}_1\cdot\vec{r}-\omega_1 t+\delta_1) \qquad \vec{E}_2 = \vec{A}_2\cos(\vec{k}_2\cdot\vec{r}-\omega_2 t+\delta_2)$$

$$\Rightarrow I = I_1+I_2+I_{12} = I_1+I_2+2\langle\vec{E}_1\cdot\vec{E}_2\rangle = I_1+I_2+2\vec{A}_1\cdot\vec{A}_2\cos\delta$$

$$\delta = [(\vec{k}_1-\vec{k}_2)\cdot\vec{r}+(\delta_1-\delta_2)-(\omega_1-\omega_2)t]$$

The condition for interference —

(1) $\omega_1=\omega_2$ if $\omega_1\neq\omega_2$, $(\omega_1-\omega_2)\neq 0$ $\langle\cos((\omega_1-\omega_2)t+\alpha)\rangle=0$

(2) $\vec{A}_1\cdot\vec{A}_2\neq 0$, $\vec{E}_1 \not\perp \vec{E}_2$ Polarization of the two waves not perpendicular.

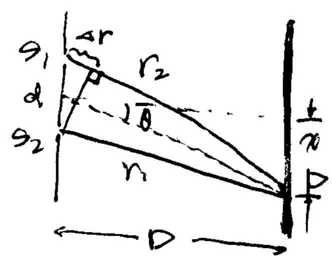
(3) $\delta_1-\delta_2=$ constant, δ_i cannot change randomly with time.

— correlated or coherent.

Phase difference

$$\delta = [(\vec{k}_1-\vec{k}_2)\cdot\vec{r}+(\delta_1-\delta_2)] \qquad I\propto|\cos\delta|^2$$

- Young's two-slit interference experiment



Phase difference

$$\delta = k(r_2-r_1) = \frac{2\pi}{\lambda}d\cdot\sin\theta$$

$$= \frac{2\pi}{\lambda}\cdot d\cdot\frac{x}{D} \qquad I = 2I_0+2I_0\cos\delta$$

$\delta=2m\pi$, $\cos\delta=1$ max $I_{max}=4I_0$, $\frac{2\pi}{\lambda}d\cdot\frac{x}{D}=2m\pi$

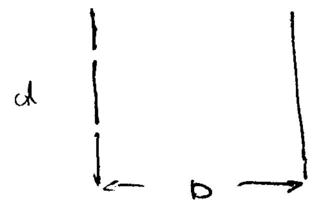
$$\Rightarrow x = \frac{m\lambda D}{d} \quad (m=0,\pm1,\pm2,\ldots)$$, bright fringe m th order

$\delta=2m\pi+\pi$ $\cos\delta=-1$ min $I_{min}=0$, $\frac{2\pi}{\lambda}d\frac{x}{D}=(2m+1)\pi$

$$\Rightarrow x = \frac{(m+\frac{1}{2})\lambda D}{d} \quad (m=0,\pm1,\pm2,\ldots)$$, dark fringe

Example: ① Na light $\lambda_1 = 589.0\,nm$, $\lambda_2 = 589.6\,nm$.

$d = 1\,mm$, $D = 1\,m$

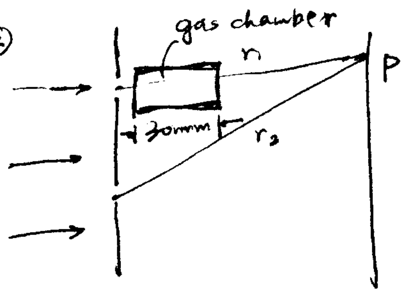


What is the separation between the 10th order bright fringes of these two wavelengths?

$x_1 = \frac{10\lambda_1 D}{d}$, $x_2 = \frac{10\lambda_2 D}{d}$

$\Delta x = |x_1 - x_2| = \frac{10D}{d}|\lambda_1 - \lambda_2| = 6\times10^{-6}\,m$

②



before: air in the gas chamber → stable interference pattern.

then: replace air with another gas of n.

→ pattern moved by 25 fringes.

$\lambda = 656.28\,nm$ $\quad n_0 = 1.000276$

n ?

$\delta \to \delta + \Delta\delta$ $\quad 0 = \delta_0 \to$ zeroth order bright fringe

$\Delta\delta = k(n-n_0)\cdot l$ $\quad 2\cdot 25\pi = \delta_0 + \Delta\delta \to$ 25th order bright fringe.

$m = 25$

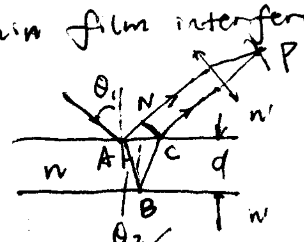
$\Delta\delta = k(n-n_0)l = 2m\pi = 50\pi$

$\frac{2\pi}{\lambda}(n-n_0)l = 2m\pi = 50\pi$

$(n-n_0)l = 25\lambda \Rightarrow n = \frac{25\lambda}{l} + n_0$

$= 1.0008229$

- Thin film interference

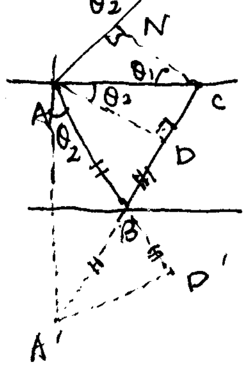


$I(P) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos k\cdot\Delta$

$\Delta = n(AB+BC) - n'AN$

$= 2nd\cos\theta_2$

⇐ Interference of equal inclination



$n(AB+BC) - n'AN$

$n'AN = n'AC\cdot\sin\theta_1 = AC\cdot n\sin\theta_2 = n\cdot DC$

$n(AB+BC) - n'AN = n\cdot AB + n\cdot BD$

$= n\cdot AD' = n\cdot AA'\cdot\cos\theta_2$

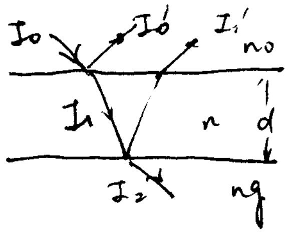
$= 2nd\cos\theta_2$.

$\Delta = 2nd\cos\theta_2 + \left(\frac{\lambda}{2}\right)$ ← additional phase shift due to reflection $(n_2 > n_1)$

$k\cdot\Delta = \frac{2\pi}{\lambda}\cdot\left(2nd\cos\theta_2 + \frac{\lambda}{2}\right) = 2m\pi$, bright fringe

" $= (2m+1)\pi$, dark fringe

— Anti-reflection coating



$R_0 = \left(\frac{n-n_0}{n+n_0}\right)^2 \qquad I_0' = R_0 I_0$

$R_g = \left(\frac{n_g-n}{n_g+n}\right)^2 \qquad I_1' \approx I_0 R_g$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos k\Delta$

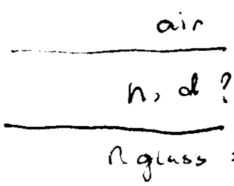
$I = 0$, ① $|I_1| = |I_2|$ ② $k\Delta = (2m+1)\pi$.

① $\left(\frac{n-n_0}{n+n_0}\right)^2 = \left(\frac{n_g-n}{n_g+n}\right)^2 \Rightarrow n = \sqrt{n_0 n_g}$

② $\delta = 2nd \cdot k = \frac{4\pi n d}{\lambda} = (2m+1)\pi \qquad m = 0, 1, 2.$

$nd = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$

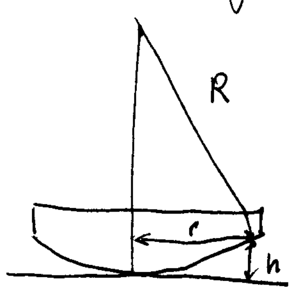
Example: $n_0 = 1 \qquad n_{glass} = 1.6 \qquad \lambda = 500nm$



$n = \sqrt{n_0 \cdot n_{glass}} = 1.26$

$d = \frac{\lambda}{4n} = 98.82nm$

— Newton Ring.



$\delta = k \cdot 2nh + \pi = (2m+1)\pi$ mth dark ring

$\frac{2\pi}{\lambda}(2nh + \frac{\lambda}{2}) = (2m+1)\pi$

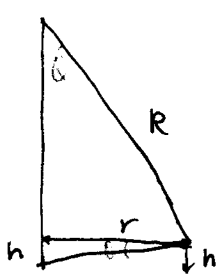
$2nh + \frac{\lambda}{2} = \frac{2m+1}{2}\lambda$

$nh = m\lambda$

$n_{air} = 1$

Example: Prove that $R = \frac{r^2}{N\lambda}$ N is the Nth order dark fringe (ring)

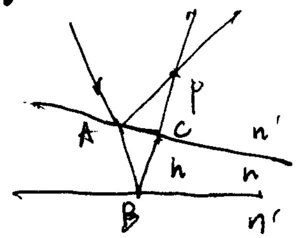
r is the radius of the Nth dark ring to the center.



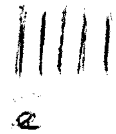
$\frac{r}{h} = \frac{R}{r} \Rightarrow r^2 = Rh$

$R = \frac{r^2}{h} = \frac{r^2}{N\lambda}$.

– Interference of equal thickness



$$\Delta = 2nh\cos\theta_2 + \frac{\lambda}{2}$$



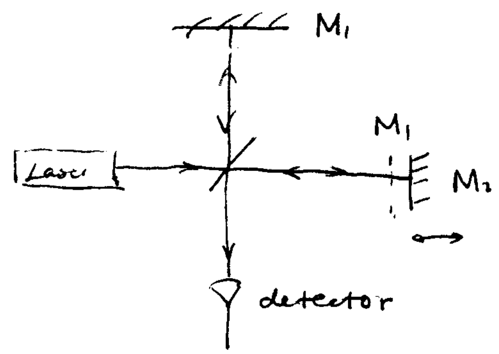
$\Delta = m\lambda$ · bright fringe

$\Delta = (m+\frac{1}{2})\lambda$ dark fringe

from one fringe to its neighbor, $\Delta x = e$ $\Delta h = \lambda/2n$

angle $\alpha = \frac{\Delta h}{e} = \frac{\lambda}{2ne}$.

– Michelson Interferometer



M_2 moves by $\frac{\lambda}{2}$, $I_{max} \to I_{min} \to I_{max}$

① If two mirrors are perfectly flat and are perfectly perpendicular

② If one of the mirrors is tilted

③ If one of the mirror is not flat

④ If the incident light is spherical wave,

what will the interference pattern be?