

EE119 Introduction to Optical Engineering  
Fall 2010  
**Final Exam**

Name: Solutions

SID: \_\_\_\_\_

CLOSED BOOK. THREE 8 1/2" X 11" SHEETS OF NOTES, AND SCIENTIFIC  
POCKET CALCULATOR PERMITTED.

*TIME ALLOTTED: 180 MINUTES*

Fundamental constants you might need:

Planck's constant,  $h = 6.6261 \times 10^{-34}$  J-s

Boltzmann's constant,  $k = 1.3807 \times 10^{-23}$  J/K

Permittivity of free space,  $\epsilon_0 = 8.8542 \times 10^{-12}$  F/m

Permeability of free space,  $\mu_0 = 1.2566 \times 10^{-6}$  H/m

Speed of light in vacuum,  $c = 2.9979 \times 10^8$  m/s

Electron charge,  $e = 1.6022 \times 10^{-19}$  C

Free electron mass,  $m_0 = 9.1094 \times 10^{-31}$  kg

Electron volt,  $1 \text{ eV} = 1.6022 \times 10^{-19}$  J

1 radian = 57.3 degrees

**Total 145 points**

1. True or False. (30points, 2 points for each question)

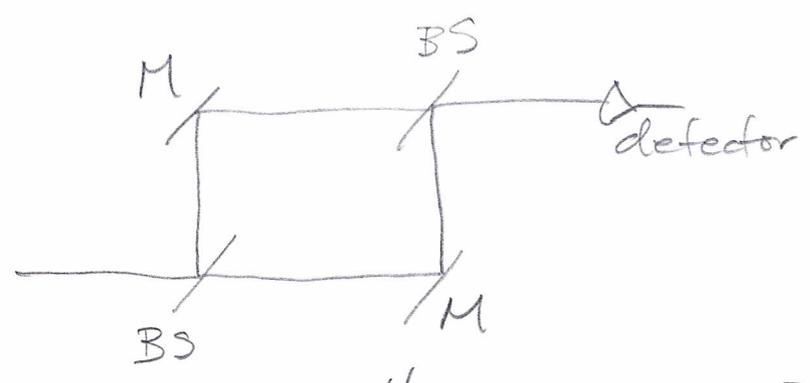
- F A) 1.8 eV photons have a wavelength of 695 nm.
- F B) When a visible light beam passes from water into air, the ray bends towards the surface normal.
- F C) The Brewster's angle only exists when light is refracted from a dense medium (with a larger refractive index) into a rare medium (with a smaller refractive index).
- F D) If the work function of a metal is 2.0 eV, then any photon with a wavelength longer than 200 nm could eject an electron from the surface.
- T E) If right-hand circularly polarized light is incident on a polarizer, 50% of the incident power is transmitted.
- T F) The chief ray always passes through the exact center of the exit pupil of an optical system.
- F G) Of the two matrix addressing modes for LCD, the passive matrix relies on the nonlinearity of the transistor switches and thus experiences less crosstalk than the active matrix scheme.
- F H) The entrance window of an imaging system coincides with the first principle plane.
- T I) Myopia is corrected using a negative lens.
- T J) A camera exposure setting with shutter speed of 1/256 sec and aperture setting f/2 gives an equivalent exposure to 1/64 sec shutter speed and f/4 aperture setting.
- F K) An achromatic doublet lens perfectly corrects chromatic aberration over an entire specified wavelength range.
- F L) For any lens system producing an image in vacuum, the entrance NA cannot be larger than the exit NA.
- T M) For a light beam of given wavelength and average power, the signal/noise ratio due to shot noise from a photomultiplier tube will increase as the quantum efficiency increases.
- F N) For a given wavelength, the maximum core diameter in a step index fiber for single mode operation is independent of the cladding index.
- T O) The effective lens power for two thin lenses in contact is equal to the sum of the individual powers of the two lenses.

2. **Short questions. (40points, 4 points for each question)**

- A) Explain why a p-i-n photodiode is usually operated under reverse bias condition.
- B) Draw a diagram showing a Mach-Zender interferometer. Be sure to clearly label all of the critical components.
- C) A He-Ne laser (633 nm) with a flat output coupler generates an output beam with Gaussian beam radius  $w$  of 0.7 mm at the output. What is the Gaussian beam radius of the beam at a distance of 10 meters?
- D) Xi's uncle has worn glasses for years. As he is getting older he has trouble reading, and he has a habit of peeking under his glasses to read better. What was his original problem (near-sighted or far-sighted), and are his old glasses made with positive or negative lenses?
- E) Light from a laboratory sodium lamp has two strong yellow lines at 589.5923 nm and 588.9953 nm. How far apart in the first order spectrum will these two lines be on a screen 1.0 m from a grating having 6,000 lines per centimeter?
- F) A biconvex lens of 50-mm focal length is made of glass with  $n=1.52$ . If the second radius of curvature (absolute value) is twice as large as the first, what are the two radii?
- G) Assume that the refractive index of the core of a step-index fiber is 1.5, and that the critical angle at the core-cladding boundary is  $10^\circ$  (as measured from the surface, not the surface normal). Calculate the difference in the propagation delay for two kinds of rays: the axial rays and those reflected at the critical angle, for a 1km long fiber..
- H) You are detecting a light beam at 580 nm wavelength with an average power of 2.5 nanoWatts with 60% quantum efficiency. What is the shot-noise limited signal/noise ratio of the detector?
- I) Assume that the length of a laser rod is 150mm and the gain factor of the laser material,  $g$ , is 0.0005/cm. If one of the cavity mirrors reflects all the light that impinges on it, determine the required reflectance of the output cavity mirror.
- J) A HeNe laser is used as light source for a Michelson interferometer. When one of the mirrors moves at a constant speed, the signal detected by a detector at the center of the observation plane changes at 1.58KHz. What is the speed of the moving mirror?

A) Reverse bias has low background and the field separates the electron + hole

B)  $\lambda$



c)  $w(z) = w_0 \left[ 1 + \left( \frac{z}{z_R} \right)^2 \right]^{1/2}$        $z_R = \frac{\pi w_0^2}{\lambda} = 2.43 \text{ m}$   
 $= 3 \text{ mm}$

D) near sighted. negative lens

E) peak occurs at  $f_0 \lambda z$ ,  $f_0 = 6 \times 10^5 \text{ m}^{-1}$

peak 1 is at 0.353755 m  
 peak 2 is at 0.353397 m  
 difference is 0.358 mm

F)  $\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$             $R_2 = -2R_1$

$\frac{1}{f} = n-1 \left( \frac{3}{2R_1} \right)$   
 $R_1 = \frac{3}{2}(n-1)f = \boxed{39 \text{ mm}}$        $R_2 = \boxed{-78 \text{ mm}}$

G)  $n_2 = n_1 \cos \theta = 1.4772$   
 $\Delta T = (n_1 - n_2) \frac{L}{c} = 7.6 \times 10^{-8} \text{ sec}$

H)

$$580 \text{ nm} \rightarrow 2.14 \text{ eV}$$

$$2.5 \times 10^{-9} \text{ W} \rightarrow 1.56 \text{ eV/s} \rightarrow 7.3 \times 10^9 \text{ ph/s}$$

$$60\% \text{ QE gives } 4.3 \times 10^9 \text{ electrons/s} = \text{signal}$$

$$S/N = \sqrt{\text{signal}} = \underline{6.6 \times 10^4 \sqrt{\text{Hz}}}$$

I)

$$R_2 \geq \frac{1}{R_1 e^{2\alpha L}} = \underline{0.985}$$

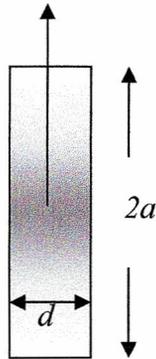
J)

$$\lambda = 633 \text{ nm}$$

$$N/t = 1580 \text{ s}^{-1}$$

$$\text{velocity } V = \frac{l}{t} = \frac{d}{2} \frac{N}{t} = \underline{0.5 \text{ mm/s}}$$

3. **GRIN lens. (20 points)** A gradient index lens, shown below, is made from a glass disk of constant thickness in which there is an index of refraction variation in the radial direction,  $n(r)$ . Given a disk of radius  $a$ , and thickness  $d$ , find the radial variation of the index that will produce the equivalent of a conventional lens of focal length,  $f$ . [Hints: Assume a suitable thin-lens approximation. Try to duplicate the thin-lens phase shift derived in class for a conventional lens in the paraxial approximation.]



thin lens phase shift:

$$\phi(r) = \phi_0 - \frac{2\pi}{\lambda} \frac{r^2}{2f}$$

the phase shift passing through the GRIN lens is:

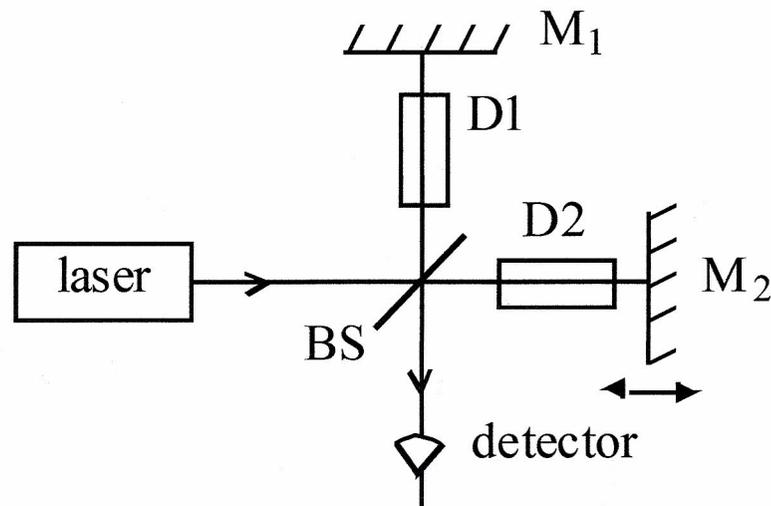
$$\phi(r) = \frac{2\pi}{\lambda} \frac{n(r)d}{d}$$

so we want:

$$n(r) = n_0 - \frac{r^2}{2fd}$$

The value of  $n_0$  can be chosen for convenience.

4. **Michelson interferometer (20 points)** A Michelson interferometer can be used to accurately measure the refractive indices of gases. In the figure below, vacuum chambers D1 and D2 are placed in two arms of the interferometer. The length is 10cm for both chambers. Now we fill in D2 with pure oxygen, we observe that the detector signal varies by 92 fringes (full cycles).
- A) What is the refractive index of pure oxygen?
- B) If the accuracy in measuring the fringe shift on the detector is 1/10 of the fringe, what is then the accuracy of the refractive index of the gas?



$$a) \quad 2(n-1)L = 92\lambda$$

$$n = \frac{92\lambda}{2L} + 1$$

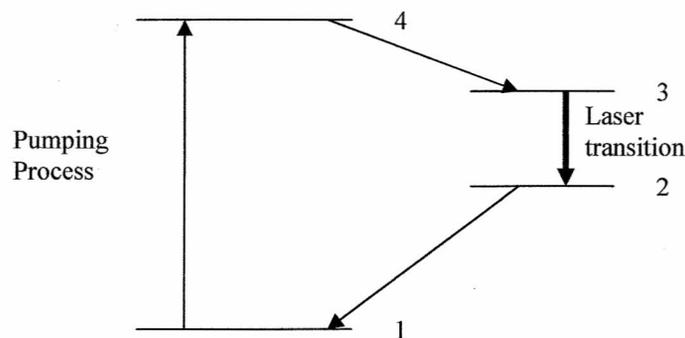
$$n = 1.000291$$

$$b) \quad 2\Delta n L = \lambda/10$$

$$\Delta n = \frac{\lambda}{20L} = 3.17 \times 10^{-7}$$

5. Lasers. (25 points)

- A) Calculate the longitudinal mode spacing for a He-Ne laser operating at 632.8 nm with 35 cm spacing between the mirrors. Assume  $n = 1$  inside the He-Ne gas. [5 points]
- B) Suppose the laser is operating in a single longitudinal mode with a bandwidth of 25 MHz. What is the corresponding linewidth (in wavelength)? Give your answer in nm. [5 points]
- C) Consider the energy level diagram for a 4-level laser shown below. If  $N_1, N_2, N_3, N_4$  represent the number densities of atoms in levels 1, 2, 3, and 4, respectively, state the condition for population inversion on the laser transition. [5 points]
- D) Write a differential equation for the population in level 3 that describes spontaneous emission from level 4 to level 3, and spontaneous and stimulated emission from level 3 to level 2. [5 points]
- E) Discuss gain saturation in the context of these energy levels. [5 points]



A)  $c/2L = 428.6 \text{ MHz}$

B)  $\lambda = c/\nu$   
 $\Delta\lambda = -c/2\nu^2 \Delta\nu$   
 $= -\lambda \frac{\Delta\nu}{\nu} = -\frac{\lambda^2 \Delta\nu}{c}$   
 $= \underline{0.000033 \text{ nm}}$

C)  $N_3 > N_2$

D)  $\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3} - BI(N_3 - N_2)$

E) gain saturates when  $N_3 = N_2$

7. **Laser cavity (20 points).** Design a laser cavity for a laser that is 1m long, operating at  $1.06 \mu\text{m}$  wavelength. Make the beam waist occur exactly in the middle of the laser cavity, with a Gaussian beam radius of  $w_0 = 0.5 \text{ mm}$ . Specify the radius of curvature of the two mirrors.

$$R_1 \left( \begin{array}{c} | \\ z=0 \end{array} \right) R_2$$

$$z = -\frac{L}{2} \qquad z = 0 \qquad z = \frac{L}{2}$$

Need to find  $R(z = L/2)$

$$w_0 = 0.5 \text{ mm}, \quad \lambda = 1.06 \mu\text{m}$$

$$z_R = \frac{\pi w_0^2}{\lambda} = 0.7409 \text{ m}$$

$$R(z = 0.5 \text{ m}) = z + \frac{z_R^2}{z} = 1.597 \text{ m}$$

both mirrors have the same concave shape.

one faces the opposite direction, so  $R_2 = -1.597 \text{ m}$

8. **Optical fibers. (20 points)** An optical fiber has cladding index  $n_1 = 1.40$ , and core index  $n_2 = 1.45$ . It is used with a laser at wavelength  $1.5 \mu\text{m}$ . The core  $d = 7.0 \mu\text{m}$ .

- A) Is this fiber single-mode for the given wavelength? If not, how many modes does it support? [5 points]  
 B) What is the maximum  $d$  for single mode operation at this wavelength? [5 points]  
 C) For the given  $d = 7.0 \mu\text{m}$ , what is the minimum wavelength for single mode operation? [5 points]  
 D) What is the NA of this fiber? [5 points]

$$A) \quad \cos \theta_c = \frac{n_1}{n_2} \rightarrow \theta_c = 15.09^\circ$$

$$\sin \theta_m = (m+1) \frac{\lambda}{2n_2 d} = 0.26$$

$$m = \frac{(0.26) 2n_2 d}{\lambda} - 1$$

$$= 2.52$$

So  $m = 0, 1, 2$  are guided  $\rightarrow$  3 modes

$$B) \quad m < 1$$

$$\frac{(0.26) 2n_2 d}{\lambda} - 1 < 1$$

$$d < \frac{\lambda}{0.26 n_2} = \underline{3.98 \mu\text{m}}$$

$$C) \quad \lambda > 0.26 n_2 d$$

$$\lambda > \underline{2.64 \mu\text{m}}$$

$$D) \quad NA = (n_2^2 - n_1^2)^{1/2} = \underline{0.377}$$

