EE 119 Homework 10

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1. ***Diode Lasers***

Consider a InGaAsP-InP laser diode which has an optical cavity of length 250 microns. The peak radiation is at 1550 nm and the refractive index of InGaAsP is 4. The optical gain bandwidth (as measured between half intensity points) will normally depend on the pumping current (diode current) but for this problem assume that it is 2 nm.

(a) What is the mode integer *m* of the peak radiation?

(b) What is the separation between the modes of the cavity? Please express your answer as $\Delta\lambda$.

(c) How many modes are within the gain band of the laser?

(d) What is the reflection coefficient and reflectance at the ends of the optical cavity (faces of the InGaAsP crystal)?

(e) The beam divergence full angles are 20° in y-direction and 5° in x-direction respectively. Estimate the x and y dimensions of the laser cavity. (Assume the beam is a Gaussian beam with the waist located at the output. And the beam waist size is approximately the x-y dimensions of the cavity.)

Solution:

(a) The wavelength λ of a cavity mode and length L are related by

$L = m\frac{\lambda}{2n}$, where m is the mode number, and n is the refractive index.

So the mode integer of the peak radiation is

$$m = \frac{2nL}{\lambda} = \frac{2\times 4\times 250\times 10^{-6}}{1.55\times 10^{-6}} = 1290\,.$$

(b) The mode spacing is given by $\Delta f = \frac{c}{2nL}$. As $f = \frac{c}{\lambda}$, $\Delta f = -\frac{c}{\lambda^2}\Delta\lambda$.

Therefore, we have $|\Delta\lambda| = \frac{\lambda^2}{c}\Delta f = \frac{\lambda^2}{2nL} = \frac{(1.55\times 10^{-6})^2}{2\times 4\times(250\times 10^{-6})} = 1.20nm$.

(c) Since the optical gain bandwidth is 2nm and the mode spacing is 1.2nm, the bandwidth could fit in two possible modes.

For mode integer of 1290, $\lambda = \frac{2nL}{m} = \frac{2\times 4\times 250\times 10^{-6}}{1290} = 1550.39nm$

Take m = 1291, $\lambda = \frac{2nL}{m} = \frac{2\times 4\times 250\times 10^{-6}}{1291} = 1549.18nm$

Or take m = 1289, $\lambda = \frac{2nL}{m} = \frac{2\times 4\times 250\times 10^{-6}}{1289} = 1551.59nm$.

So there are two modes in the cavity. They could be

$\lambda = 1549.18nm$ and $\lambda = 1550.39nm$

or $\lambda = 1550.39nm$ and $\lambda = 1551.59nm$. But as said in the problem that the peak radiation is at 1550nm, the two modes should be

$\lambda = 1549.18nm$ and $\lambda = 1550.39nm$.

(d) At normal incidence, the reflection coefficient at the ends of the optical cavity is $r = \frac{n_{InGaAsP} - n_{air}}{n_{InGaAsP} + n_{air}} = \frac{4-1}{4+1} = 0.6$;

And the reflectance is $R = r^2 = 0.36$.

(e) The beam divergence half angle at y direction is

$$\theta_{dy} = \frac{\lambda}{\pi w_{0y}} = 10^{\circ} = \frac{\pi}{18} = 0.1745 rad ,$$

so $w_{0y} = 11.3\mu m$.

Similarly, the beam divergence half angle at x direction is

$$\theta_{dy} = \frac{\lambda}{\pi w_{0y}} = 2.5^{\circ} = \frac{\pi}{72} = 0.0436 rad ,$$

so $w_{0x} = 45.2\mu m$.

Therefore, the laser cavity has x and y dimensions of roughly 90µm × 23µm. This is a rough estimation, but it nevertheless gives approximate dimension of laser diodes.

2. ***Characteristics of common lasers***

We have discussed some of very common lasers in class. There are many other lasers which we have not covered in the lecture. In this problem, you have to do some library researches on the following lasers. You have to describe their special characteristics, how these lasers are operated, and at what wavelengths it lases typically. Include diagrams and state some common applications to get more points.

(a) Liquid lasers (dye lasers).

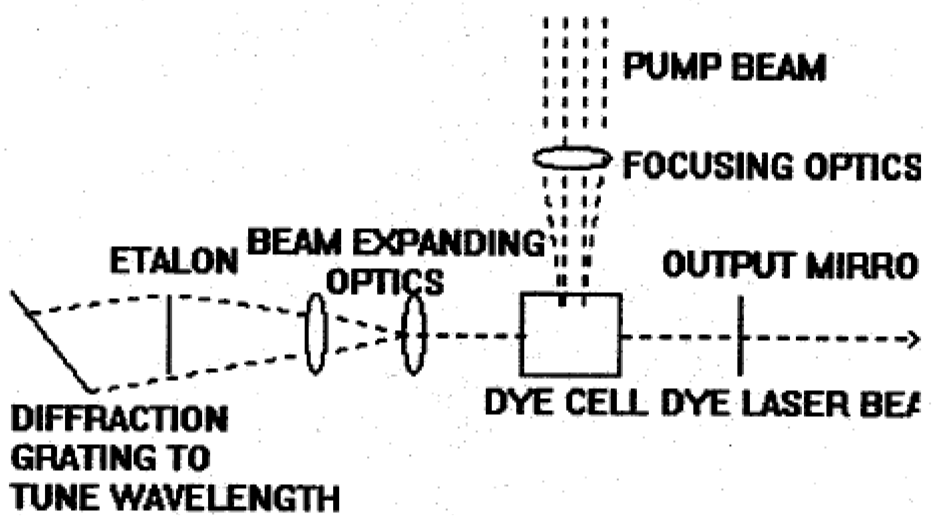
Dye lasers use a liquid solution of a highly emissive dye molecule as the active medium. Dye lasers are usually optically pumped, and rely on vibrational relaxation (i.e. phonons) to create the top two levels of a three-level (or 4-level) system. A great many synthetic dyes have been developed for this application, and the colors of the light emitted by dye lasers can range all over the visible spectrum. A key property of these dyes is that they must have a long excited state lifetime -

the molecule must remain in an excited state for long enough that it can contribute to gain, which means that non-radiative decay pathways such as vibrations must be relatively slow. The spontaneous emission lifetime of typical dye lasers is a few nanoseconds. The emission spectrum of dyes sometimes depends on the solvent the dye is in, so this adds another variable that enables tuning of the gain spectrum to a specific application.

Before the discovery and development of nonlinear semiconducting crystals (such as Ti:Sapphire) dye lasers were a common gain medium for pulsed lasers. Getting broad bandwidth out of lasers made it possible to produce mode-locked pulses of light that were shorter than what could be controlled by electronics. Dye lasers were also frequently pumped by flash lamps, which required large voltages stored in capacitors.

A common dye used as the gain medium is Rhodamine 6G. When dissolved in ethanol, Rhodamine 6G absorbs light between 450 nm and 550 nm, with its absorption maximum at 510 nm. Its emission is maximized in the range of 550 nm in ethanol.

Dye lasers have medical applications such as the treatment of prophylaxis of cerebral vasospasm and treatment of cutaneous vascular lesions.

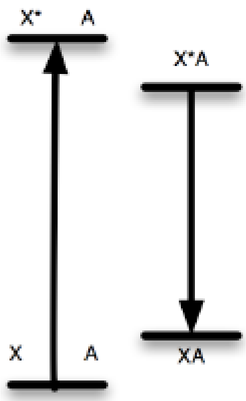


Common Dye Laser Diagram

(b) Excimer lasers.

Excimer lasers have gain due to chemical reactions between two or more atoms.

Excimer is short for "excited dimer". Excimer lasers are electrically pumped. The source of gain in Excimer lasers is the dissociation of the excited dimer into two separate atoms. The dimer gives off energy by releasing light, and then the two atoms return to the ground state and are ready to be recharged again. When the excimer relaxes, the two atoms are close together because they were a dimer in the excited state; however, when they are not excited the atoms repel each other, so the lower state of the lasing transition, which is the state of the non-excited dimer, has a very short lifetime. This makes it possible to achieve very high population inversion if you pump the gas of atoms hard enough.



A qualitative energy level diagram of excimer lasers - An excimer laser is 4-level system where each level is a different configuration of atoms/molecules.

More generally, there is no reason why only two atoms can participate in this reaction; you can have an "excited complex" of atoms, and this is sometimes referred to as an "exciplex". Excimer/Exciplex lasers are used to produce ultraviolet laser light. Because UV light can effectively break chemical bonds in biological tissues, common applications of Excimer lasers are in surgery such as LASIK that involve destroying biological tissue in a controlled fashion (this is called "ablation").

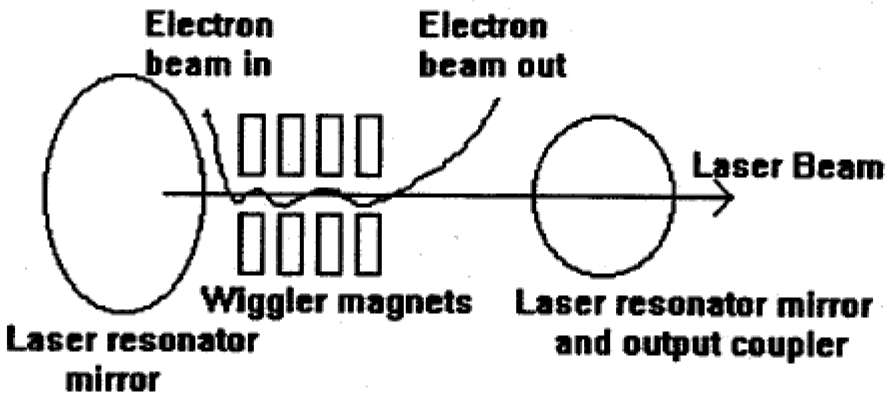
(c) Plasma X-ray lasers.

In plasma x-ray lasers, a pulse of light from another laser strikes a target, stripping its atoms of electrons to form ions and pumping energy into the ions ("exciting" or "amplifying" them). As each exited ion decays from the higher energy state, it emits a photon. Many millions of these photons at the same wavelength, amplified in step, create the x-ray laser beam. The highly ionized material in which excitation occurs is plasma. Wavelengths are in the EUV/soft/hard x-ray region are being actively engineered to become light sources for EUV lithography (13.4nm).

For a short history of x-ray lasers you can read this web page https://www.llnl.gov/str/Dunn.html.

(d) Free electron lasers.

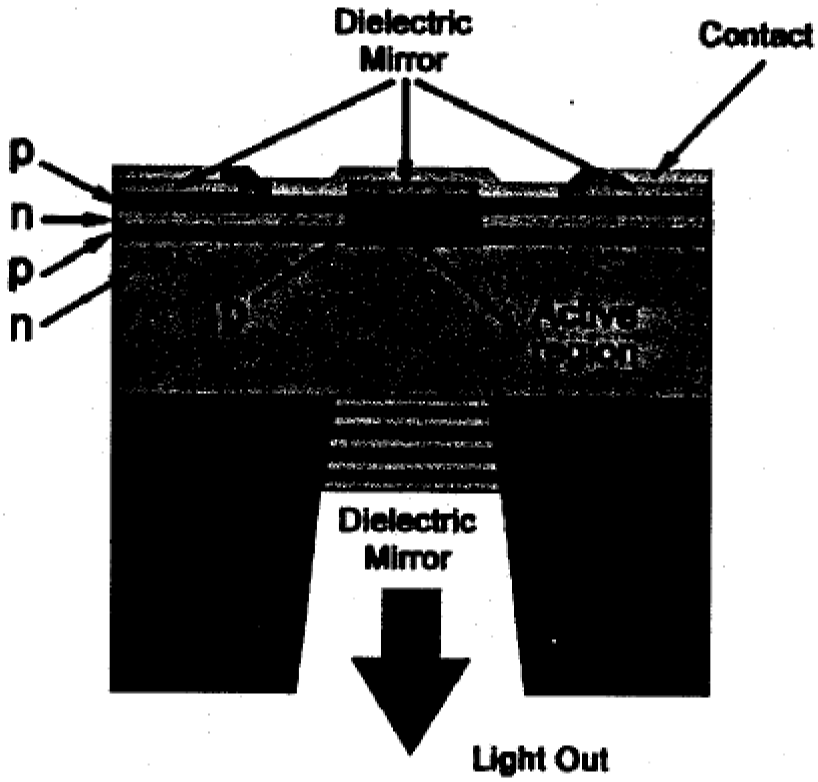
A free electron laser uses free electrons - that is, electrons that aren't bound to atoms - as the gain medium. The electrons are accelerated in an accelerator, and they emit light when their direction of motion rapidly changes. This is done by passing electrons through magnetic structures called undulators - as the electrons decelerate and change direction, they emit light. Since the energy the electron loses is determined by controllable factors such as the electron speed and the strength of the magnetic fields in the undulators, the wavelength of the electron is fully tunable. Free electron lasers such as in the figure below have the ability to generate wavelengths from the millimeter wave to the X-ray region. This is similar to the Advanced Light Source (ALS) here at the Berkeley Lab. These are very large and expensive systems that are primarily used for scientific research. Other applications are laser-power beaming to generate electricity in satellites, laser-rock-fluid interaction, and petroleum well drilling and completion.



Free Electron Laser Diagram

(f) VCSEL (Vertical Cavity Surface Emitting Lasers)

VCSEL, or Vertical Cavity Surface Emitting Laser, is semiconductor microlaser diode that emits light in a cylindrical beam vertically from the surface of a fabricated wafer, and offers significant advantages when compared to the edge-emitting lasers. VCSELs can be fabricated efficiently on wafers. Even more important, the ability to manufacture these lasers using standard microelectronic fabrication method allows integration of VCSELs on-board with other components without requiring pre-packaging. As an enabling technology, VSCELs allow superior new systems and products to be created at a lower cost. VCSELs are promising emitter for fiber data communication in the speed range up to 10Gbs. They enable high performance systems in Gigabit Ethernet, Fiber Channel and ATM markets. Through their integration with original equipment manufacturer's systems design, VCSELs provide enhanced performance benefits to a variety of applications, such as local area networks (LAN), telecommunication switches, optical storage and other optoelectronic systems.



Schematic diagram of an etched well VCSEL

3. ***Fraunhofer diffraction***
   (a) Calculate and sketch (two-dimension) the far field diffraction pattern of the square aperture. The aperture is 0.5mm in size and is located at 10m from the detector plane. Be sure to label x and y-axis. (λ=632.8nm).
   Solution: The far field diffraction pattern is exactly the Fourier transform of the aperture distribution. From the class note,

$$U(x,y,z)=\frac{e^{jkz}e^{j\frac{k}{z}(x^2+y^2)}}{j\lambda z}A\sin c(\frac{2w_x x}{\lambda z})\sin c(\frac{2w_y y}{\lambda z})$$

in which $A\equiv 4w_x w_y$ and w_x and w_y are the half width of the square.

The intensity is then

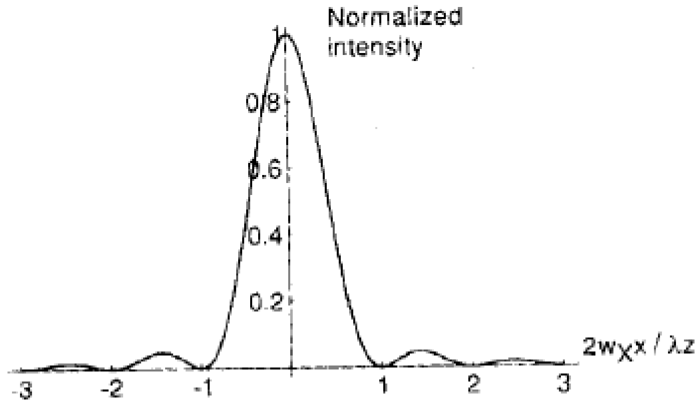
$$I(x,y)=\frac{A^2}{\lambda^2 z^2}\sin c^2(\frac{2w_x x}{\lambda z})\sin c^2(\frac{2w_y y}{\lambda z}).$$

Plugging the values provided in the problem, we have

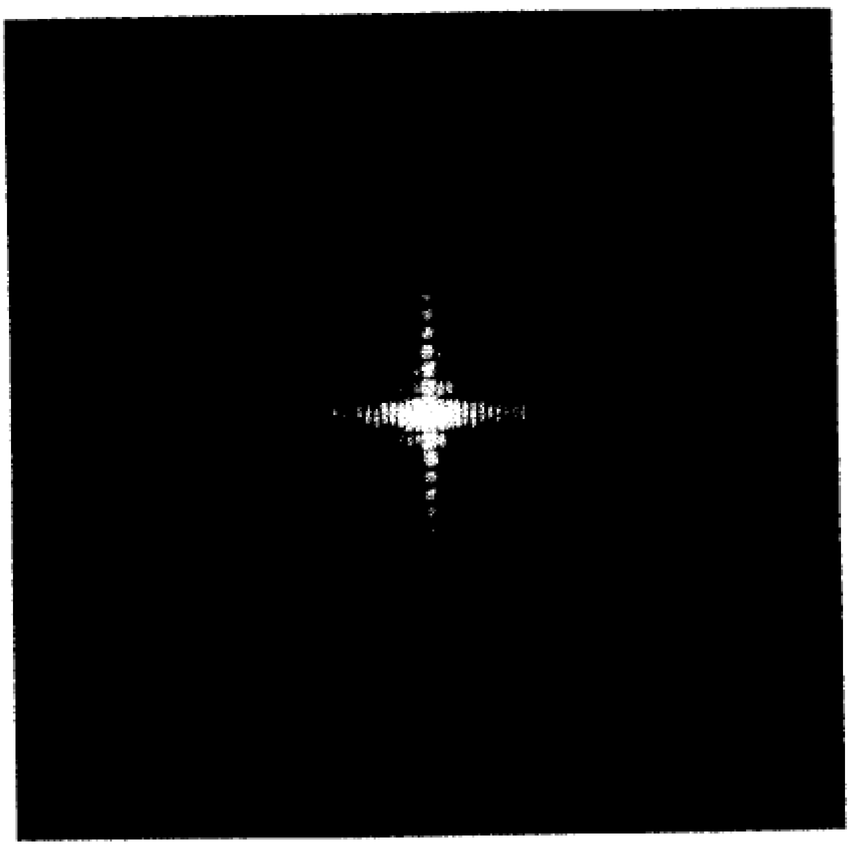
$$I(x,y)=1.56\times 10^{-3}\sin c^2(79.01x)\sin c^2(79.01y).$$

The sketches (both one-dimension and two-dimension) of the far-field diffraction pattern of a square aperture are shown below (**note that the two-dimension sketch is for a rectangular with** $w_x / w_y = 2$**; for a square,**

the x and y pattern sizes should be the same. You can take a step here to think over why the spots in x are half the width of those in y).



Cross section of the Fraunhofer diffraction pattern of a rectangular aperture



The Fraunhofer diffraction pattern of a rectangular aperture ($w_x / w_y = 2$)

(b) Find an expression for the intensity distribution in the Fraunhofer diffraction pattern of the aperture shown in Fig.1. Assume unit-amplitude, normally incident plane-wave illumination. The aperture is circular and has a circular central obstruction. Outer radius is R_1 and inner radius is R_2.

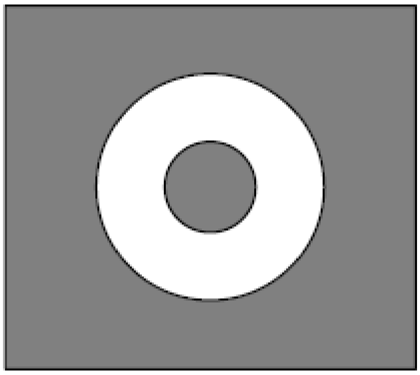


Fig. 1

(Hint: The electric field of the Fraunhofer diffraction pattern from a circular aperture is: $E(r) = e^{jkr} e^{jk\frac{r^2}{2z}} \frac{A}{jz\lambda}\left[2\frac{J_1(kwr/z)}{kwr/z}\right]$

where J1 is the Bessel function of first order, k is a wavevector and w is the size of the circular pupil- same notation as in class; then use superposition.)

Solution: For a circular aperture of radius R_1 that has a circular central obstruction of radius R_2, due to the superposition, the total electric field of its Fraunhofer diffraction pattern is

$$E_{tot}(r) = E_{R_1}(r) - E_{R_2}(r)$$

where E(r) is the general expression for the electric field of the Fraunhofer diffraction pattern from a circular aperture of radius w

$$E(r) = e^{jkr} e^{jk\frac{r^2}{2z}} \frac{A}{jz\lambda}\left[2\frac{J_1(kwr/z)}{kwr/z}\right].$$

So $E_{tot}(r) = e^{jkr} e^{jk\frac{r^2}{2z}} \frac{2A}{j\lambda kr}\left[\frac{J_1(kR_1r/z)}{R_1} - \frac{J_1(kR_2r/z)}{R_2}\right].$

And the field intensity is

$$I(r) = |E_{tot}(r)|^2 = \frac{4A^2}{\lambda^2 k^2 r^2}\left[\frac{J_1(kR_1r/z)}{R_1} - \frac{J_1(kR_2r/z)}{R_2}\right]^2$$

When $R_1 \to \infty$, I(r) just becomes an airy pattern; when $R_2 \to 0$, I(r) is still an airy pattern with a delta-function at its center.