EE 119 Homework 1 Solution

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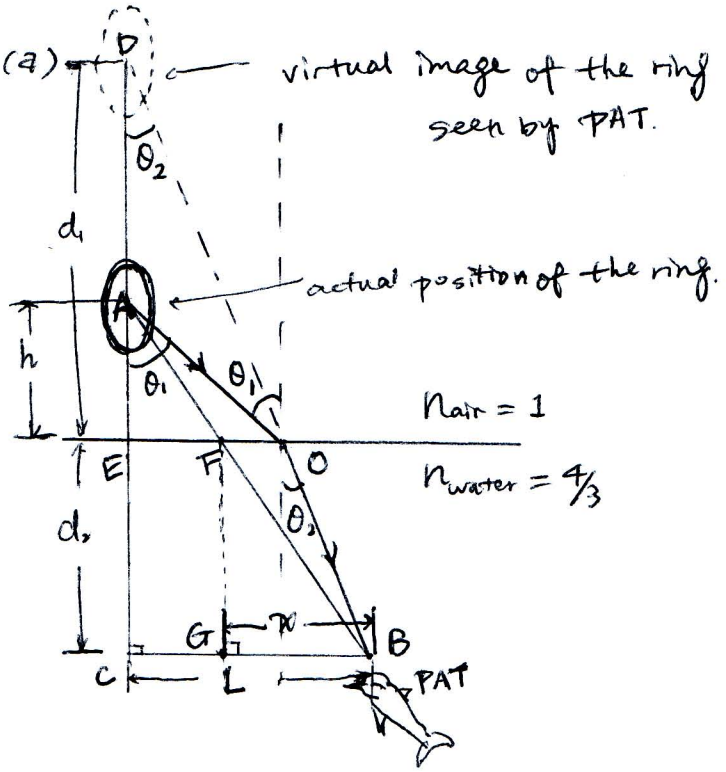
1. (a)



According to Snell's Law, $n_{air} \sin\theta_1 = n_{water} \sin\theta_2$

Under small angle approximation, $\sin\theta \sim \theta \sim \tan\theta$

$$\Rightarrow \quad n_{air} \cdot \theta_1 = n_{water} \cdot \theta_2$$

In $\triangle AEO$, $h = \overline{AE} = \overline{EO} / \tan\theta_1 \approx \overline{EO} / \theta_1$

In $\triangle DEO$, $d_1 = \overline{DE} = \overline{EO} / \tan\theta_2 \approx \overline{EO} / \theta_2$

$$\Rightarrow \quad \frac{h}{d_1} = \frac{\theta_2}{\theta_1} = \frac{n_{air}}{n_{water}} = \frac{3}{4}, \text{ i.e., } h = \frac{3}{4} d_1$$

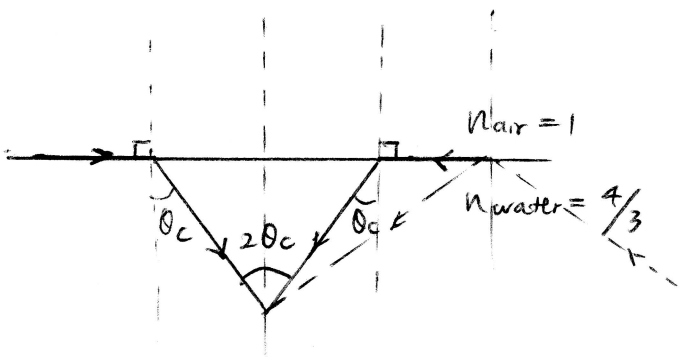
As $\triangle ABC$ and $\triangle BFG$ are similar triangles,

$$\frac{\overline{GB}}{\overline{BC}} = \frac{\overline{FG}}{\overline{AC}} \Rightarrow \frac{x}{L} = \frac{d_2}{d_2 + h} = \frac{d_2}{d_2 + \frac{3}{4} d_1}$$

$$\Rightarrow x = \frac{d_2 \cdot L}{d_2 + \frac{3}{4} d_1}$$

Therefore, Pat should aim at point F at the water surface, which is at a distance of $\frac{d_2 \cdot L}{d_2 + \frac{3}{4} d_1}$ horizontally from itself.

(b) Light entering the water at glancing incidence is refracted and transmitted in the water at the critical angle θ_c from the surface normal. Those lightrays define the cone of light seen by Pat, the dolphin.



$$\sin\theta_c = \frac{n_{air}}{n_{water}} = \frac{3}{4}$$

$$\Rightarrow \theta_c = \arcsin\left(\frac{3}{4}\right)$$

$$= 48.6°$$

Thus the cone angle is

$$2\theta_c = 97.2°$$

When light enters from a medium of a smaller n (air in this case) into a denser medium of a larger n (water here), the refracted rays are bent towards the surface normal. Thus the view from above the water, the image of sky, birds and etc., is compressed into a cone. The angle of this cone is defined by the light rays that are incident at the glancing angle and then refracted at the critical angle θ_c into the water.

Looking up, what Pat sees within the cone (within the critical angle) is the compressed image of what are above the water. And what he sees beyond the critical angle θ_c is light from within the pool, internally reflected at the water/air interface. If the pool is poorly lit, Pat would see the bright cone surrounded by darkness.

2. (a) We first find the energy of a photon at 610nm to be

$$E = h\nu = h\frac{c}{\lambda} = 6.62\times10^{-34}\,J\cdot s \times \frac{3\times10^{8}\,m/s}{610\times10^{-9}\,m}$$

$$= 3.2557\times10^{-19}\,J$$

$$= 2.035\,eV$$

which is smaller than the work function of the material, 2.4 eV. Thus the photon from the nearby laser source cannot provide the minimum energy needed to remove an electron from the material $\Rightarrow$ the detector does NOT detect any electrons.

It won't help if the laser power is doubled to 2mW, because even though we have twice as many photons hitting the material per unit time, none of these photons having enough energy work function and knock electrons from it.

(b) The laser would have to provide photons with an energy at least equal to the work function of the material in order to be capable of knocking electrons from it.

$$E_{photon} = 2.4\,eV$$

$$\Rightarrow \lambda = \frac{c}{\nu} = \frac{c}{\frac{E_{photon}}{h}} = \frac{hc}{E_{photon}} = \frac{6.62\times10^{-34}\,J\cdot s \times 3\times10^{8}\,m/s}{2.4\times1.6\times10^{-19}\,J}$$

$$= 5.19\times10^{-7}\,m = 519\,nm.$$

Thus the laser should have wavelength shorter than (including) 519nm. Notice that shorter wavelength corresponds to larger photon energy.

(c) The laser at the wavelength of 519nm, with an intensity of $1\,nW/m^2$ and a spot diameter of 1mm, produces photon flux

$$\text{photon \#/s} = \left(\frac{1\,nW/m^2}{h\cdot\frac{c}{\lambda}}\right)\cdot\left(\pi\left(\frac{1mm}{2}\right)^2\right) = \frac{1\times10^{-9}\,W/m^2}{6.62\times10^{-34}\,J\cdot s\cdot\frac{3\times10^{8}\,m}{5.19\times10^{-7}\,m}}\times\pi\times(5\times10^{-4}\,m)^2$$

photon flux density $\rightarrow$ photon #/s·m² $\times$ area $= 2052$ photons per second

Since each photon carries enough energy to knock off one electron, 2052 electrons will be ejected from the material per second and then detected by the detector.

3. (a) Superman's speed is $\frac{40{,}000\,km}{2\,s} = 2\times10^7\ m/s$.

In a material with index of refraction n, light travels at the speed

$$v = \frac{c}{n} = \frac{3\times10^8\ m/s}{n}$$

Let $2\times10^7\ m/s > \frac{3\times10^8\ m/s}{n}$

$$\Rightarrow\quad n > \frac{3\times10^8\ m/s}{2\times10^7\ m/s} = 15.$$

Therefore, Superman needs to find a material with $n = 15$, which can then slow light down to his speed, in order to tie the 1km race. Notice that the length of the tube provided in the problem is irrelevant.

(b) We solve for the transmission of light of both polarization using the Fresnel equation given in the lecture note.

For p-polarized light,

$$T_p = \frac{n_2\cos\theta_2}{n_1\cos\theta_1}\,\frac{4\sin^2\theta_2\cos^2\theta_1}{\sin^2(\theta_1+\theta_2)\cos^2(\theta_1-\theta_2)} \qquad \text{— I}$$

θ_2 is calculated from Snell's Law,

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\Rightarrow \theta_2 = \arcsin\left(\frac{\sin 75^\circ}{1.33}\right) = 46.6^\circ$$

For s-polarized light,

$$T_s = \frac{n_2\cos\theta_2}{n_1\cos\theta_1}\,\frac{4\sin^2\theta_2\cos^2\theta_1}{\sin^2(\theta_1+\theta_2)} \qquad \text{— II}$$

$$\frac{\text{I}}{\text{II}} \quad\Rightarrow\quad \frac{T_p}{T_s} = \frac{1}{\cos^2(\theta_1-\theta_2)} = \frac{1}{\cos^2(75^\circ-46.6^\circ)} = 1.293$$

The ratio of p to s polarized light is 1.293.

For p-polarized light fully transmitted, the incident angle should be the Brewster angle

$$\theta_B = \arctan\left(\frac{n_{water}}{n_{air}}\right) = \arctan\left(\frac{4}{3}\right) = 53.1^\circ$$

The optimal angle to heat the water at is 53.1°.

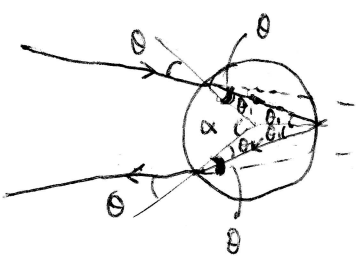
4. (a) As $n_{violet} > n_{red}$, the refracted ray is bent further towards the normal for violet light than red light. Using simple ray-tracing, we know that

1. (violet) rays from high drops pass overhead.
2. (red)
3. (violet)
4. (red) rays from low drops strike below eyes.

After identifying the four rays in Fig. 1, one can readily know that in the primary rainbow, the spectrum goes from top to bottom as red, orange, yellow, green, blue, indigo and violet.

(Top is red.)

(b)



We first express the deviation angle δ in terms of incident angle θ.

After identifying all the reflected and refracted angles in the rain droplet, it becomes obvious that

$$\alpha = 4\theta_1,$$

in which θ_1 can be obtained from Snell's law $n_{air}\sin\theta = n_{water}\sin\theta_1$;

and $\delta = \alpha - 2\theta = 4\theta_1 - 2\theta$.

$$\Rightarrow \delta = 4\arcsin\left(\frac{n_{air}\sin\theta}{n_{water}}\right) - 2\theta.$$

$$\Rightarrow \delta = 4\arcsin\left(\frac{\sin\theta}{1.33}\right) - 2\theta \quad \text{for red light}$$

Let $\frac{d\delta}{d\theta} = 0$

$$\Rightarrow \frac{d\delta}{d\theta} = 4 \cdot \frac{\frac{\cos\theta}{1.33}}{\sqrt{1-\left(\frac{\sin\theta}{1.33}\right)^2}} - 2 = 0$$

$$\Rightarrow \left(\frac{2\cos\theta}{1.33}\right)^2 = 1 - \frac{\sin^2\theta}{1.33^2}$$

$$\Rightarrow \frac{4(1-\sin^2\theta)}{1.33^2} = 1 - \frac{\sin^2\theta}{1.33^2}$$

$$\Rightarrow |\sin\theta| = 0.8624$$

As $0 < \theta < \frac{\pi}{2}$, $\Rightarrow \theta = 59.6°$

At this incident angle of 59.6°, the deviation angle is minimum.

$$\delta_{min} = 4\arcsin\left(\frac{\sin\theta}{1.33}\right) - 2\theta$$

$$= 4\arcsin\left(\frac{0.8624}{1.33}\right) - 2\times 59.6°$$

$$= 42.5°$$

(c) For violet light, $n = 1.34$.

$$\delta = 4\arcsin\left(\frac{\sin 59.6°}{1.34}\right) - 2\times 59.6°$$

$$= 41°$$

We use the value of incident angle from (b) as an approximation to simplify the calculation, δ is slightly different from 40°.

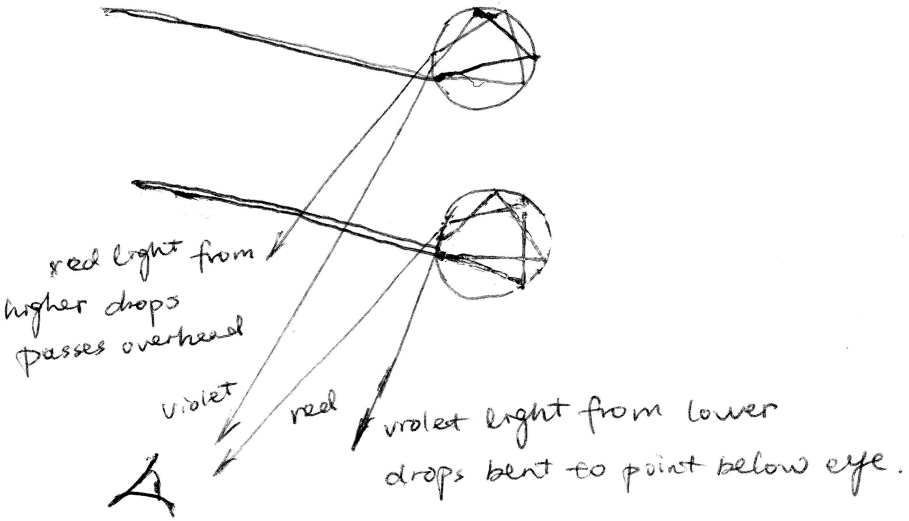
If we go through the same procedure as in (b), we would also get

$$\sin\theta = 0.8572 \Rightarrow \theta = 59°$$

$$\delta = 4\arcsin\left(\frac{\sin 59°}{1.34}\right) - 2\times 59°$$

$$= 41°$$

(d)



The spectrum of the secondary rainbow from top to bottom goes violet, indigo, blue, green, yellow, orange and red

(Top color is violet)

* Online resource referenced in this problem:
http://hyperphysics.phy-astr.gsu.edu/Hbase/atmos/rbowpri.html
http://eo.ucar.edu/rainbows/

5. Answer From Hecht:

In each part the x and y components have the same amplitude E_0

(a) $\vec{E} = (\hat{i} - \hat{j})\, E_0 \cos(kz - \omega t)$ is a $\mathcal{P}$ state at 135° or −45°

(b) $\vec{E} = (\hat{i} - \hat{j})\, E_0 \sin(kz - \omega t)$ is also a $\mathcal{P}$ state at 135° or −45°

(c) E_x leads E_y by $\pi/4$. Therefore it is an $\mathcal{E}$ state and left-handed.

(d) E_y leads E_x by $\pi/2$. Therefore it is an $\mathcal{R}$ state.

Please note <u>state of polarization</u> defined in Hecht book as the following:

$\mathcal{P}$-state: linearly polarized or plane-polarized light

$\mathcal{R}$- or $\mathcal{L}$-state: right- or left-circular light

$\mathcal{E}$-state: elliptical polarization