EE 119 Homework 2

Professor: Jeff Bokor TA: Xi Luo
Due Tuesday, Feb 9th 2010
(Please submit your answers in EE119 homework box located in 240 Cory Hall)

1. (a) In class, the angle of deviation (δ) has been formulated for a prism with an apex angle (α). Plot the angle of deviation (δ) versus incident angle for n = 1.5 and apex angle of 60°.

   (b) The angle of deviation should have a minimum. Derive a general expression for the incidence angle at which the deviation δ is minimized. Your answer should be given as a function of the apex angle α and refractive index of the prism. Show that the ray for which the deviation is a minimum traverses the prism symmetrically. This means that the incident angle and the exit angle are the same.
   (Hint: Show $\cos(\theta_1)/\cos(\theta_2^{'}) = \cos(\theta_1^{'})/\cos(\theta_2)$, then make use of Snell's law once again. There are other ways to do this problem, so you don't need to follow the hint.)

   (c) In the case when the angle of deviation is a minimum, find an expression for a refractive index of the prism. Express the refraction index in terms of the minimum angle of deviation δ_m and the apex angle α. This equation forms the basis of one of the most accurate techniques for determining the refractive index of a transparent substance.

   (d) Now a prism (n = 1.5 at λ_o) with an apex angle of 4° is immersed in a liquid (n = 1.3 at λ_o). If a laser beam at wavelength of λ_o passes through this prism and is then reflected by a mirror (Fig. 1), how much do you need to rotate the mirror so that the reflected beam is parallel to the incoming laser beam? Note that here you have to modify the expression of δ given in the lecture note, since that expression is derived for the case when the prism is in the air (n = 1).

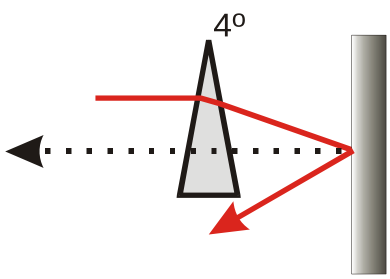


Fig. 1 the incident laser beam is parallel to the base of the prism, and normal to the mirror before it is rotated

2. In birefringent crystals such as quartz, there exists a direction called the **optical axis**. For light propagating in the crystal along a direction other than the optical axis, its polarization can be decomposed into the **ordinary component** that is perpendicular to the optical axis, and the **extraordinary component** that is not. The ordinary component has constant index of refraction n_o. But the index of refraction for the extraordinary component, n_e, varies between n_o and n_{e0} as its propagation direction varies; and the value of n_e equals to n_{e0} only when its propagation direction is perpendicular to the optical axis.

A quartz plate is cut and polished so that its optic axis is parallel to the front and back surfaces (along x-axis), as shown in Fig.2, and its thickness is 1.618×10^{-2}mm. A linearly-polarized yellow light (λ = 589.3nm) is normally incident on the plate and passing through it. The refractive indices of the quartz plate at this wavelength are $n_o = 1.54424$, and $n_{e0} = 1.55335$. What is the polarization state of the exiting light if the incident beam is linearly polarized along

(a) the direction that makes an angle of 45° with the x-axis;
(b) the direction that makes an angle of -45° with the x-axis;
(c) the direction that makes an angle of 30° with the x-axis;
(d) the x-axis.

(Hint: first decompose the polarization of incident light into its ordinary component and its extraordinary component; then find the phase delay difference between the two components as the light exits the plate; phase delay is $\varphi = kn\Delta z$, where k is the wave-vector, n is the refractive index of the medium and Δz is the distance that light has traveled in the medium.)

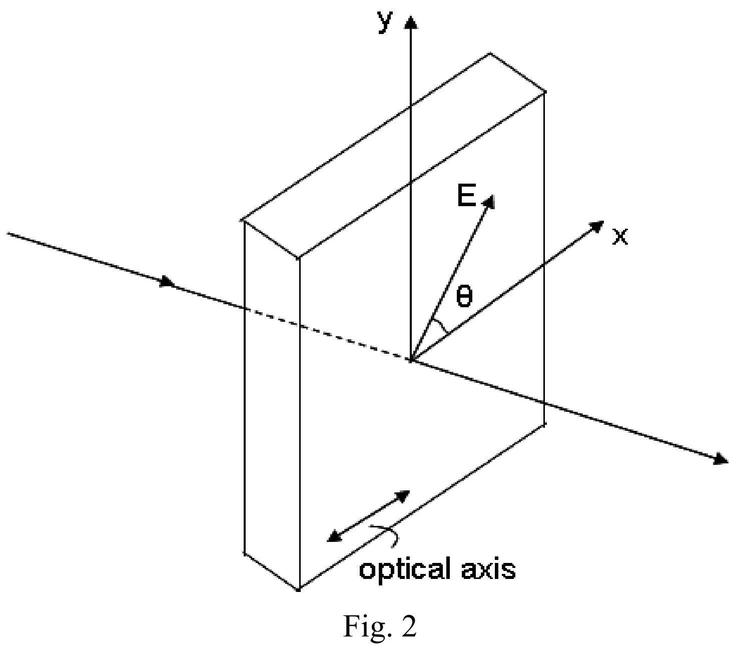


Fig. 2

3. Detective Conan was a famous high-school boy detective before he was forced by this evil guy Gin to swallow a "poisonous" pill which later made him shrink to the size of a primary school kid. Conan loves his job of being a detective helping people solve different cases and decides that he should never stop. His plan is to use a lens to restore his original appearance onto a white wall from an illuminated transparency of his picture, while he hides behind the wall and addresses to the crowd disclosing the mystery of the case and announcing the criminal. The height of his figure in the transparency is 8cm, whereas his real height was 1.6 meters.

   a) What must the transverse magnification of this imaging system have for Conan to restore his appearance to his original height? Should he put the transparency with his head up or down?
   b) Conan wants to place his lens 10 meters from where his image appears. What focal length is required of the lens so that it can achieve the magnification needed in part (a)?
   c) Conan's girlfriend, Lan, has no idea of what has happened to him. She has been told that Conan is on a trip somewhere investigating complicated cases. But as Valentine's Day is approaching, Conan really wants to please Lan and meet her on that day. He then asks Doctor Alee to make a special lens for him so that Lan can see him through it without realizing anything wrong. The lens works as a normal lens except that Lan would not notice its existence. Please draw a diagram of this imaging system, in which you are asked to indicate the possible position of Conan (the object), Lan (the viewer) and the Conan that Lan sees (the image) relative to the lens and its focal points, and to draw the three main rays that help to trace the image. Will Lan looking through the lens see a real image or a virtual image of Conan?

4. One of the two main functions of prisms is "to effect a change in the orientation of an image or in the direction of a propagation of a beam"(From **Hecht** p187).

   For example, while a right-angle prism deviates rays normal to the incident face by 90, it also flips the top and bottom of the image, but the right and left sides remains the same (see Fig. 3(a)). But a right-angle prism with a roof section added

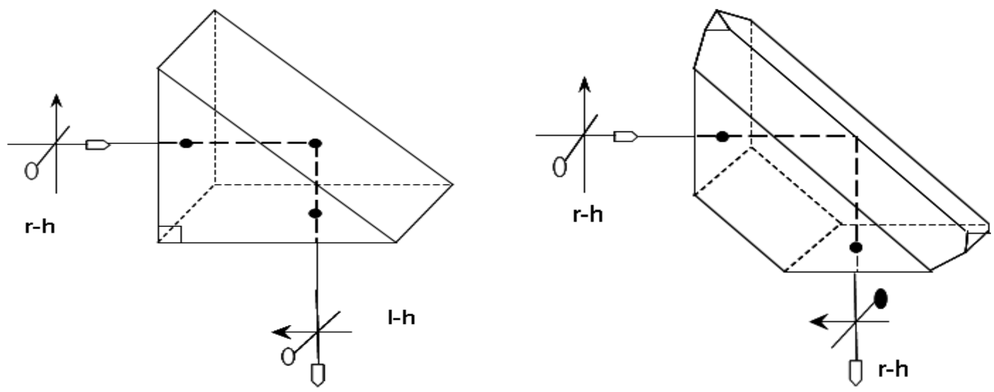


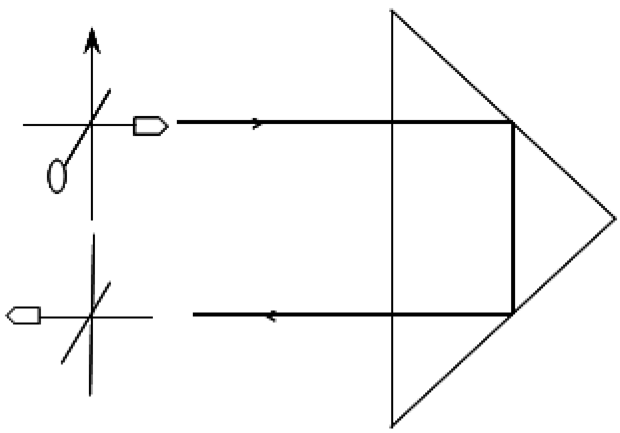
Fig. 3 (a) right-angle prism (**Hecht** Fig 5.61) (b) the Amici prism (**Hecht** Fig 5.64) on its hypotenuse face (named as the **Amici prism**) also flips the right and left of the image (see Fig. 3(b)).

One can always explain the “flipping” of image coordinates by examining the orientation of the virtual mirror image formed with each reflecting surface. Nonetheless, a general conclusion has been summarized to determine whether “flipping” of one coordinate occurs or not, which goes as the following: if we call the plane that is perpendicular to all working surfaces of a prism as its principal cross-section, then at each reflecting surface, the coordinate perpendicular to the principle cross-section remains the same after reflection, but the coordinate that parallel to the principal cross-section flips. Please note that this conclusion does not apply to prisms with a roof section, which one cannot find a single plane perpendicular to all working surfaces.

Referencing this rule and the examples, please determine the direction of the arrows $\longrightarrow$ and the lollypops ——○ (the transverse coordinates) of the image after exiting the follow prisms.

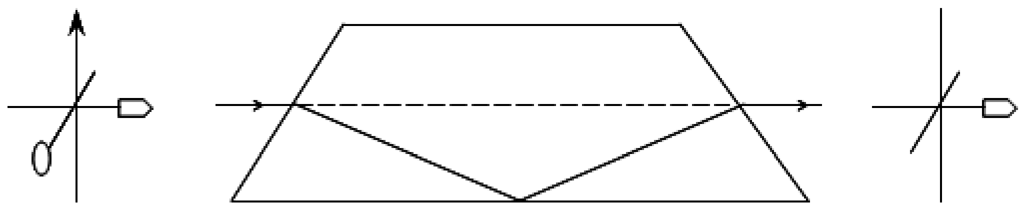
To avoid confusion, use the void circle ○ for lollypops pointing outside the paper plane, and the solid ● for lollypops pointing inside.

(a) Porro prism

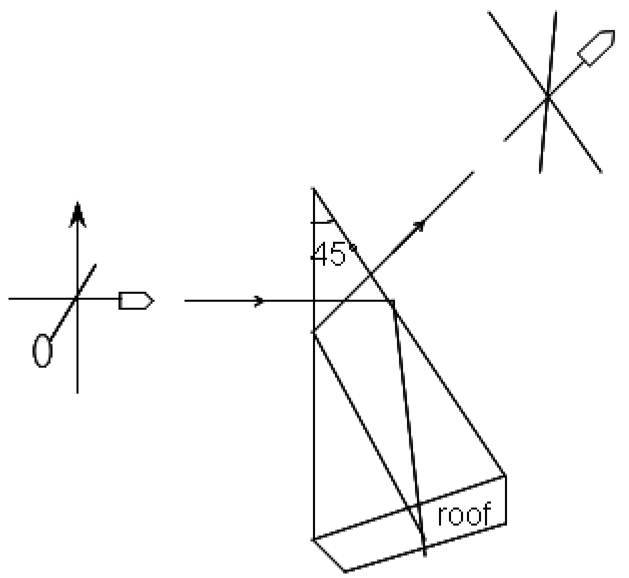


(b) Dove prism

The Dove prism in the picture below (the bottom view of the prism) is that if we rotate the Dove prism in the above picture by 90° around its axis. What can we conclude if we compare the orientation of the exiting images in the upper and lower pictures? One important property of the Dove prism is that if it rotates around its axis by α, its image is rotated with it by 2α.



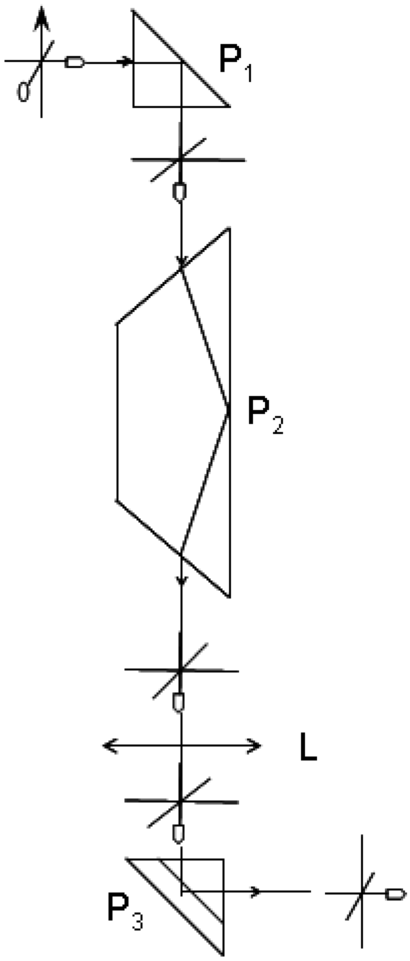
(c) Schmidt Prism



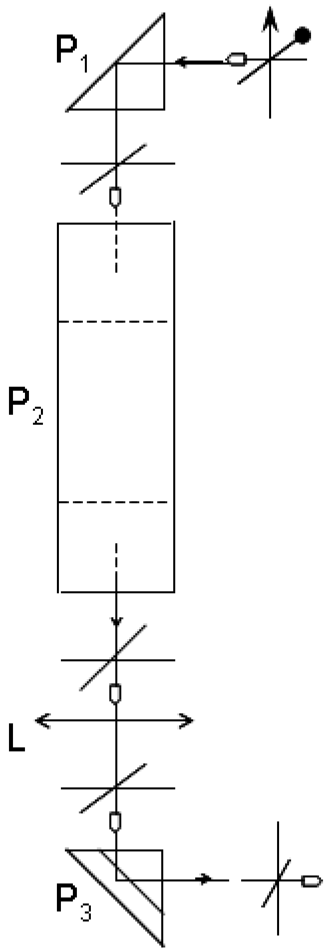
So, can Schmidt prism be used as an erecting prism?

(d) **Panoramic Sight**

A panoramic sight usually consists of a right-angle prism, a Dove prism, a thin-lens, and a right-angle roof prism, shown in the figure below. Please determine the direction of the images after each prism/lens by completing the drawing of arrows and lollipops at each stage. Note that here the lens L inverts the transverse coordinates of the image.



(1)



(2)

One may notice that, in (2) the right-angle prism P_1 is rotated 180° around the longitudinal axis from the one in (1), while the Dove prism P_2 is rotated 90° in the same direction, but the orientation of the images are the same. This reminds us of the interesting property mentioned earlier about the Dove prism, being that it rotates the image twice as fast as it is itself rotated about its axis. Therefore, in the panoramic sight, when the right-angle prism P_1 rotates at an angular velocity of ω, and the Dove prism P_2 rotates with it at twice the angular velocity 2ω, the observer could then get a panoramic view through the eye-piece without turning around himself.