1. You will explore some of the differences between real and paraxial rays in this problem. For each part below, trace the specified ray and determine where it crosses the optical axis. Show all calculations and include a diagram. Report your answers to 4 decimal places.

(a) Find L when $\theta = 5^\circ$ with real ray (no paraxial approximation).
(b) Find L when $\theta = 0.5^\circ$ with real ray (no paraxial approximation).
(c) Report part (a) with the paraxial ray (paraxial approximation).
(d) Report part (b) with the paraxial ray (paraxial approximation).
(e) Is there a difference between your answers in (a) and (b)? Is there any difference between your answers in (c) and (d)?
(f) Now compare your answers in (b) and either (c) or (d). Why are they so similar?

![Fig. 1 diagram for problem 1](image)

2. You are asked to design a double convex lens ($R_1 > 0$, $R_2 < 0$) for Batman. He would like to use the lens to project his bat logo onto low hanging clouds at night to obtain a 1000x magnification image. You know that low hanging clouds are generally about 1km overhead, and you know that the available materials have $n = 1.5$. You are also limited by the restriction that one surface must be 4 times the radius of curvature of the other side.

(a) What is the focal length of his desired lens?
(b) What are your design parameters that will make the desired lens ($n$, $R_1$, $R_2$)?

$R_1$ is the radius of curvature of the first spherical surface and $R_2$ is that of the second spherical surface. Please follow the sign conventions. List all the possible answers you can get.
3. A spherical wave can be mathematically described by:

\[ E = E_0 \frac{\text{Re}(e^{-ikr + \phi})}{r} = E_0 \frac{\cos(-kr + \phi)}{r} \]

where \( r \) is the distance of the wave front from the source in 3D space, \( k \) is the wave vector (the amplitude of \( k \) is given by \( \frac{2\pi}{\lambda} \)), and \( \phi \) is a constant phase. In this problem, the optic axis is on the \( z \)-axis.

(a) Show that at a large distance from the point source, close to the optic axis, at constant \( z \), the wave can be approximated by

\[ E = E_0 \frac{\cos[ -k \frac{(x^2 + y^2)}{2z} + \phi ]}{z} \]

In this approximation, \( z \gg (x, y) \). Hint: the taylor expansion of \( \sqrt{1+x} \) for \( x \ll 1 \) is \( \sqrt{1+x} \approx 1 + \frac{x}{2} \).

(b) Show that a spherical wave emerging from a given object point will be converted into a spherical wave converging to the image point given by the usual Gaussian lens law, by using the thin-lens-phase shift.

4. Two thin lenses with focal lengths of + 6.0 cm and + 8.0 cm and apertures of 8.0 cm and 9.0 cm, respectively, are located 4 cm apart. An aperture stop (3 cm in diameter) is located between the two lenses, 3 cm from the first lens. An object 3 cm high is located with its center 12 cm in front of the first lens.

(a) Draw a diagram that shows the entire imaging system.
(b) Find the position and size of the entrance and exit pupils and draw them in the diagram in part (a).
(c) Find the position and size of the final image and draw it in the diagram from part (a).
(d) Draw the two marginal rays and the chief ray from the top end of the object and trace them all the way to the image.

5. The rays incident on the outer edge of a lens (outside of the paraxial regime) suffer from spherical aberration. This is because the nonparaxial rays are too strongly bent. Consider the plano-convex lens as we see in Fig. 2-(a) and (b). Depending on which surface faces the incident rays, the amount of spherical aberration can be reduced. In this problem, you will decide which lens configuration is better in terms of spherical aberration.

a) Find the focal length of the two lenses in the paraxial regime.
b) Let’s assume we have a ray parallel to the optical axis incident on the lens as shown in Fig. 2(a) and (b) with the ray height of 10cm from the optical axis. Calculate where the ray crosses the optical axis (L) for both lenses. Neglect the thickness of the lens. (Hint: you will use Snell’s Law twice for convex-plano lens and once for the plano-convex lens.)

c) Which lens configuration of the two is better in terms of spherical aberration?