EE 119 Homework 4 Solution

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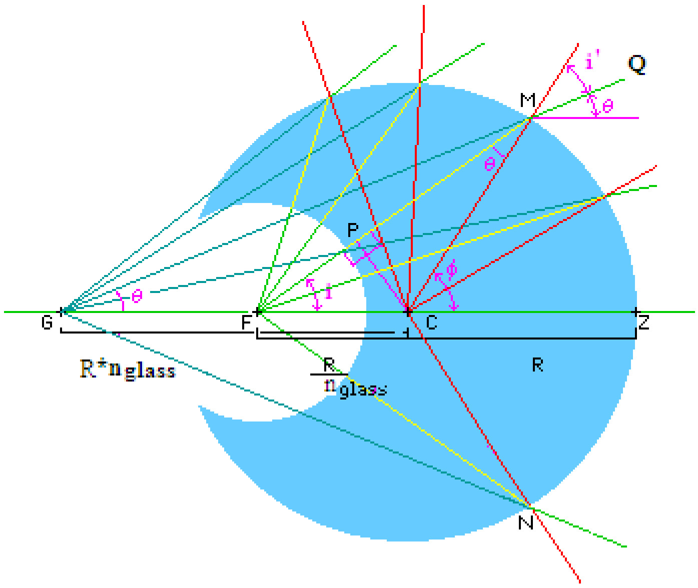
1. Solution(taken from *solution manual* for Hecht):
   Because (a) is symmetrical and looks like a somewhat altered Airy pattern; this is spherical aberration. (b) This pattern is asymmetrical as if the Airy pattern were pulled off to the side, so it corresponds to a little coma. (c) This pattern is asymmetrical along two axes and must be due to astigmatism

2. Prove:



Let's consider an arbitrary ray FP that originates from F making an angle of i with the axis.

Since F is the center of the front spherical surface, light originating from it crosses the front surface without being bent (i.e., F, P, M are on the same straight line). At the back surface, using Snell's Law,

$$n\sin\theta = \sin i' .$$

The triangle FMC can be thought of as consisting two right angle triangles FPC and MPC, sharing the same side PC. Therefore,

$$PC = FC\sin i = MC\sin\theta \quad => \quad \sin i = \frac{MC}{FC}\sin\theta = n\sin\theta .$$

So, $i' = i$.

As G is the point where the refracted ray MQ, when it is back extended, intersects the axis, we then have

$$\angle GMC = i' = i = \angle MFC .$$

We also know that $\angle GCM = \angle FCM$ as they are just two names of the same angle. Thus the triangle GCM is similar to the triangle FCM.

Therefore, $\frac{GC}{MC} = \frac{MC}{FC} \quad \Rightarrow \quad GC = \frac{MC}{FC} MC = nR$

Since FP is an arbitrary ray emerging from F, one can conclude that all rays originating from F is refracted and focused at point G with no spherical aberration, with distance GC = nR.

3. Solution:

(a) The power of the lens in diopters is

$$P = \frac{1}{f} = \frac{1}{0.0125} = 8 \text{ Diopters}$$

(b) The dispersion power v is given by

$$v = \frac{n_F - 1}{n_{G'} - n_D}.$$

Plugging the corresponding values from Table 1, one gets,

for SPC-1, $v_{crown} = \frac{1.52933 - 1}{1.53435 - 1.52300} = 46.6370$

for DF-4, $v_{flint} = \frac{1.66270 - 1}{1.67456 - 1.64900} = 25.9272$.

(c) To design an achromatic lens that minimizes chromatic aberration, one wants to achieve that $P_{G'} = P_D$. Following the same derivation as that described in the lecture notes, the design equations that one can arrive from there are

$$P_{F\,crown} = P_F \frac{v_{crown}}{v_{crown} - v_{flint}} \qquad P_{F\,flint} = -P_F \frac{v_{flint}}{v_{crown} - v_{flint}},$$

in which P_F is the total power of the lens for the F-line

$$P_F = \frac{1}{f} = 8D,$$

$P_{F\,crown}$ is the power of the crown lens at F-line and $P_{F\,flint}$ is the power of the flint lens at F-line.

Plugging the values that we obtained in (a) and (b) into the design equations, we get

$P_{F\,crown}$ = 18.0154

$P_{F\,flint}$ = -10.0154

(d) The radii of the lenses can be found using lens maker's equation. Since we know that the flint glass has an outer flat surface, i.e. $R_2 = \infty$, the radius of the other flint surface is obtained through

$$P_{F\ flint} = \frac{1}{f_{F\ flint}} = (n_{F\ flint} - 1)(\frac{1}{R_1} - \frac{1}{R_2}).$$

Therefore, $R_1 = [\frac{P_{F\ flint}}{(n_F - 1)}]^{-1} = [\frac{-10.0154}{0.6627}]^{-1} = -0.0662m$

As the two lenses are cemented together, the inner radius of the crown lens $R_2' = R_1 = -0.0662m.$ Again using lens maker's equation

$$P_{F\ crown} = \frac{1}{f_{F\ crown}} = (n_{F\ crown} - 1)(\frac{1}{R_1'} - \frac{1}{R_2'}),$$

we get $R_1' = [\frac{P_{F\ flint}}{(n_F - 1)} + \frac{1}{R_2'}]^{-1} = [\frac{18.0154}{0.52933} + \frac{1}{-0.0662}]^{-1} = 0.0528m$.

4. Solution:

(a) From Fig.1, we know that the distance of the object from the lens (its center) is $d_1 = -15cm$, and the height of it is $h_1 = 6cm$. From Gaussian lens law,

$$\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1} = \frac{1}{10} + \frac{1}{-15} = \frac{1}{30},$$

thus the distance of the image from the lens is $d_2 = 30cm$, to the right of the lens. Also, $m = \frac{d_2}{d_1} = -2$, hence the image is inverted, and its height is

$$h_2 = h_1 \cdot m = -12cm$$

(b) The aperture stop itself is the entrance pupil of this imaging system. It is 2cm in front of the lens, and its diameter is 6cm.

(c) The exit pupil of the imaging system is its image of the entrance pupil. We know the position and size of the entrance pupil(aperture stop)

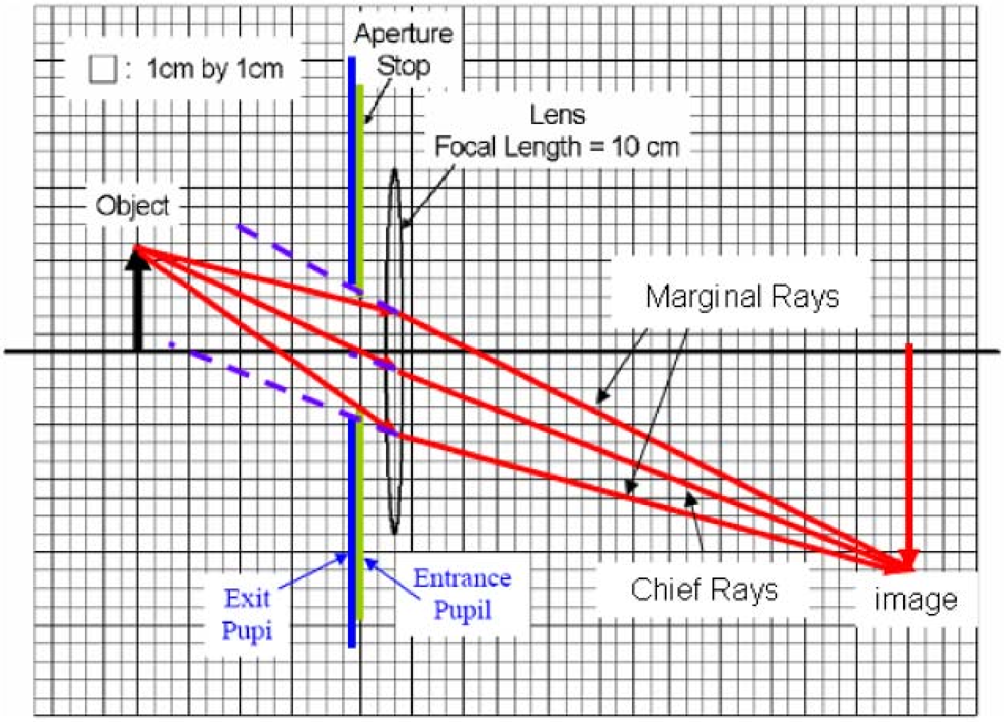
$$d_1' = -2cm, \quad D_1 = 6cm.$$

With Gaussian lens law, $\frac{1}{d_2'} = \frac{1}{f} + \frac{1}{d_1'} = \frac{1}{10} + \frac{1}{-2} = -\frac{1}{2.5}$.

Thus $d_2' = -2.5cm$, $m = \frac{d_2'}{d_1'} = 1.25$, and $D_2 = D_1 \cdot m = 7.5cm$.

The exit pupil is 2.5cm in front of the lens, and its diameter is 7.5cm.

(d) See the sketches in the diagram.



The above diagram should be the right answer since the question asks you to draw marginal rays "from the tip of object". As given by the definition in the lecture note, which is **a more general definition of marginal rays, marginal rays can be "from a given object point", which does not have to be an axial object point.** The "definition" in the Hecht book, page 172, which says a marginal ray "goes from the axial object point", is **just a special case**. However we still give full credits if you draw the marginal rays from the axial object point because the two seemingly different definitions are confusing.

(e) The object-side numerical aperture of the system is then

$$NA_1 = \sin\theta_1,$$

in which $\theta_1 = \tan^{-1}(\frac{3}{15-2}) = 13.0^\circ$. So $NA_1 = \sin 13.0^\circ = 0.2250$.

(f) As $\theta_2 = \tan^{-1}(\frac{7.5/2}{30+2.5}) = 6.58^\circ$, the image-side numerical aperture

$$NA_2 = \sin 6.58^\circ = 0.1146.$$

$$NA_1 / NA_2 = 0.2250 / 0.1146 = 1.96 \approx m$$

Our analysis is based on the paraxial approximation. NA_1/NA_2 is very close to m, but it seems the angles (θ_1 and θ_2) are just slightly larger than the upper limit for the paraxial approximation, causing a minor discrepancy.

(g) According to Rayleigh's criterion, the smallest feature on the object that the imaging system can resolve is

$$\psi_1 = \frac{0.61\lambda}{NA_1} = \frac{0.61 \times 520nm}{0.225} = 1.41\mu m.$$

(h) If the aperture stop is removed from the system, the lens itself then serves as an aperture stop, entrance pupil and exit pupil.

$$\theta_1 = \tan^{-1}(\frac{10}{15}) = 33.69^\circ, \quad NA_1 = \sin\theta_1 = 0.5547$$

$$\theta_2 = \tan^{-1}(\frac{10}{30}) = 18.43^\circ, \quad NA_2 = \sin\theta_2 = 0.3161$$

$$\psi_1 = \frac{0.61\lambda}{NA_1} = \frac{0.61 \times 520nm}{0.5547} = 0.572\mu m$$

$$NA_1 / NA_2 = 0.5547 / 0.3161 = 1.75 \approx m$$

The imaging system without a stop yields a better result as it can resolve smaller features. Larger apertures reduce the diffraction effects.

However, NA_1/NA_2 now deviates considerably from m because the angles (θ_1 and θ_2) are too large for the paraxial approximation to be accurate.

5. Solution:

(a) The image resolution of the system is

$$\psi = \frac{0.61\lambda}{NA} = \frac{0.61 \times 193nm}{0.85} = 138.5nm$$

(b) As $NA_{object} / NA_{image} = m = 1/4$, the numerical aperture of the object side is

$$NA_{object} = 0.85 \times 1/4 = 0.2125$$

$$\psi' = \frac{0.61\lambda}{NA_{object}} = \frac{0.61 \times 193nm}{0.2125} = 554nm$$

Thus the resolution on the object side is 554nm.

(c) Since both shapes are under the resolution limit, the imaging system will not be able to show fine details of the two. They will image very similarly, as two blur spots, so you would not be able to tell their original shapes based on their images.

(d) With immersion lithography, the new resolution of the system is reduced to

$$\psi = \frac{0.61\lambda}{NA} = \frac{0.61 \times 193nm}{n_{water} \times 0.85} = 104nm.$$