EE 119 Homework 6 Solution

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1. Solution:

(a) The angular magnification of a telescope is $m = f_o / f_e$ so the focal length of the objective lens is $f_o = mf_e = 4 \times 45 = 180cm$.

(b) The clear aperture of the exit pupil is 3.5 mm. The distance between the objective lens and the eyepiece is $-d_1 = f_o + f_e = 184cm$. The magnification at such a distance is $m = f_e /(f_e + d_1) = f_e /(f_e - f_o + f_e) = -f_e / f_o = -1/45$.

So the exit pupil is 45 times smaller than the objective lens, meaning that the objective lens has a diameter of 3.5 x 45 = 157.5mm.

(c) The object field angle is the angle subtended by the chief ray that barely touches the outside of the eyepiece lens. So it is given by

$$\tan\theta = \frac{d_e}{2}\frac{1}{f_o + f_e} = \frac{1}{184} = 0.0054$$

$$\theta_o = 0.3114^\circ$$

(d) The object and image field angles are related to each other by the angular magnification, so the image field angle is $M \times \theta_o = 14^\circ$.

(e) The diffraction limited angle is determined by diffraction at the entrance pupil, which is the objective lens. It has a diameter of 15.75cm = 0.1575 m.

$$\theta \approx \sin\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-7}}{0.1575} = 4.26 \times 10^{-6}$$

The image angle that this corresponds to is $45 \times \theta = 1.917 \times 10^{-4} = 0.19mrad$.

This is smaller than the resolution of the eye, so the limit of resolution on the telescope is the eye, not diffraction.

Or

The maximum magnification of the telescope for its not being diffraction limited satisfies that $M_{\max} = 11CA_o = 11 \times 15.75 / 2.5 = 69.3 > 45$.

Therefore, the limit of resolution on the telescope is not diffraction, but the observer's eye.

(f) Since the limit on the resolution of the microscope is the observer's eye, through the telescope, one can resolve objects separated by an angle of

$$\alpha = 0.3mrad / M = 3\times10^{-4} / 45 = 6.7\times10^{-6} mrad \quad .$$

So the minimum distance in meters between the Eminent British Captain and the mysterious black-flagged ship of potential doom is

$$D = 0.1/\alpha = 0.1/6.7\times10^{-6} = 1.5\times10^{4} m = 15km .$$

2. Solution:
As this is a design problem, there is no exact answer. The following parameters should be included in your answers, and the grading is based on them:
(1) Magnification;
(2) Choices of CA_o and CA_e;
(3) Diagram with ray trace, entrance and exit pupils;
(4) Choices of f_o and f_e.
If your design flow is reasonable, you'll get full credits.

A sample design scheme:
The telescope must be able to resolve a feature that is 74500mile/10 = 7450mile wide and 746×10^6 miles away, which corresponds to an angle of

$$\theta = 7450/(746\times10^6) = 9.9666\times10^{-6} rad$$

The magnification that would be required for the cones of the eye to resolve this is

$$M = \frac{3\times10^{-4}}{9.9666\times10^{-6}} \approx 30$$

To make sure the telescope is not diffraction limited,

$$60 \ge 5.5\frac{M}{CA_o} \quad => \quad CA_o \ge \frac{5.5}{60}M \quad \text{or} \quad M_{\max} \le 11CA_o \qquad (*)$$

Since the iris of the eye is 4mm, the size of the exit pupil should match, or slightly larger than that, i.e. $CA_e =\sim 4mm$. From solution of prob. 1(b), we know that the transverse magnification of the objective lens (entrance pupil) and its image (exit pupil) through the eyepiece is

$$m = -f_e / f_o = -1/30 .$$

The diameter of the objective lens must be at least $4\times30 = 120mm = 12cm$. Plugging this value of $CA_o = 12cm = 4.8inch$ into the resolution criterion Eq(*), we see that $CA_o \ge \frac{5.5}{60}M$ is valid. So with this choice of M, CA_o and CA_e, the telescope is not diffraction-limited.

Now we need to choose f_o and f_e. These two numbers can be any, but let's choose those with reasonable arguments.

The object field angle θ is determined by the size of the eyepiece. Let's choose the size of the eyepiece to be 1.2"= 3cm, then $\theta \cong \tan^{-1}(\frac{1.5cm}{s})$. The image fielld angle θ' is simply related to θ by magnification.

We also know that $s = f_o + f_e$ and $m = \frac{f_o}{f_e} = 30$. Let's choose $f_e = 2cm$, then

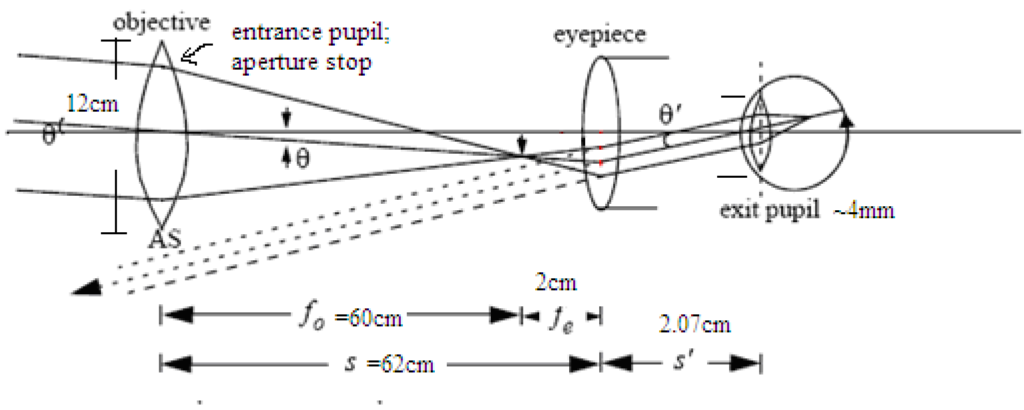
$f_o = 60cm$ and $s = 62cm$.

If s = 62cm, then we have

$$2\theta_o \cong 2\tan^{-1}(\frac{1.5cm}{62}) = 0.048rad$$

and the image field angle $2\theta' \cong 1.44rad$

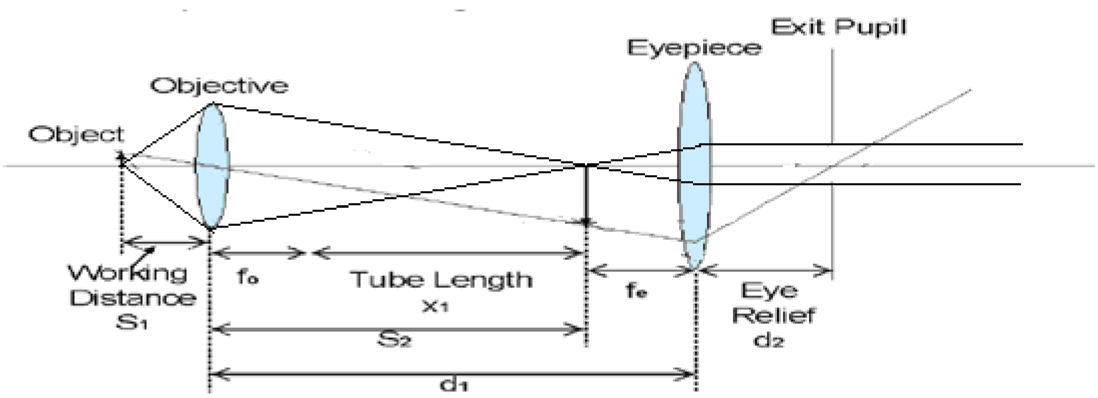
Also $\frac{s}{s'} = |m|$, hence $s' = 2.067cm$



For summary,

$M = 30$, $CA_o = 12cm = 4.8inch$, $CA_e = \sim 4mm$
$f_e = 2cm$, $f_o = 60cm$, $s = 62cm$, $s' = 2.067cm$
eyepiece size = 3cm.

3. Solution: We want to see if the working distance for this microscope is larger than 2mm. If that is the case, then we can put the glass slide between the microscope objective and the sample to be examined.



Given $M_{tot} = 20$, $f_e = 5cm$ and $x' = 160mm$, we know that

$$M_e = \frac{25cm}{f_e} = 5\text{, so } M_o = 4.$$

As $M_o = -\frac{x'}{f_o}$, $f_o = 4cm$.

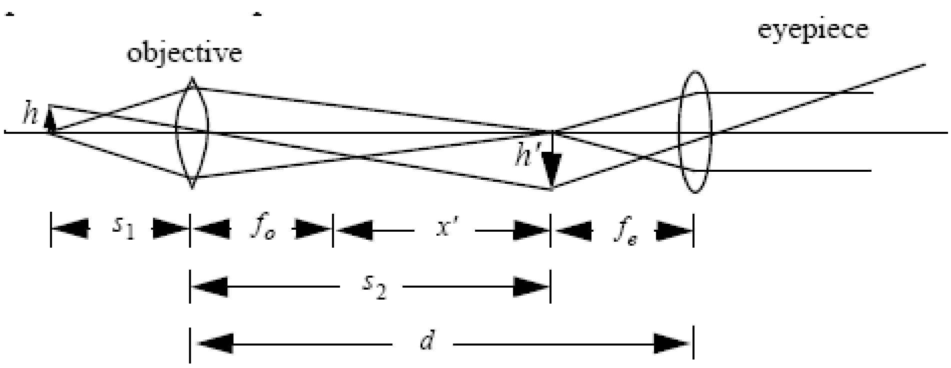
Using Lens' law to find s_1, we have

$$\frac{1}{s_2} = \frac{1}{f_o} + \frac{1}{s_1} \quad \Rightarrow \quad \frac{1}{f_o + x'} = \frac{1}{f_o} + \frac{1}{s_1}$$

Hence s_1 = -5cm > > 2mm.

So it is possible to use this microscope to look at a sample underneath a glass slide of thickness 2mm

4. Solution:
(There can be many answers, as long as you clearly specify selection of magnification; focal length and diameter of the objective; focal length of the eyepiece, and ray trace of the system, you'll get full credit.)



The eye can resolve an object size of ~0.08mm = 80 μm at the distance of 25 cm. To have a resolution of 1μm, the total magnification of the microscope is

$$M_{tot} = \frac{80\mu m}{1\mu m} = 80.$$

Pick a common 20x objective, then the eyepiece should have the magnification of

$$M_e = \frac{80}{20} = 4.$$

Since the standard tube length is 16cm, from $M_o = \left| -\frac{x'}{f_o} \right| = \frac{16cm}{f_o} = 20$,

we get the focal length of the objective lens to be $f_o = 0.8cm = 8mm$.

Also, $M_o = \frac{s_2}{s_1} = \frac{16+0.8}{s_1} = 20$ => $s_1 = 8.4mm > 5mm$ (working distance).

Since $M_o = \frac{25cm}{f_e}$ (25 cm is the standard viewing distance; near point),

$$f_e = \frac{25cm}{4} = 6.25cm.$$

The distance between the objective lens and the eyepiece is then

$$d = f_o + x' + f_e = 23.05cm.$$

In order for the microscope to be not diffraction-limited,

$$M_{\max} \cong 240NA.$$

For $M \geq 80$, $NA \geq 0.33$. Let's choose $NA = 0.4$

n = 1, $\theta \approx \frac{CA_o/2}{s_1} = 0.4$. Thus $CA_o = 0.4 \times 8.4 \times 2 = 6.72mm$.

Summary:

$M_{tot} = 80$;
objective $f_o = 8mm$, $M_o = 20$, $NA = 0.4\,(CA_o = 6.72mm)$
eyepiece $f_e = 6.25mm$, $M_e = 4$
$x' = 16cm$, $s_1 = 8.4mm > 5mm$ working distance
$d = 23.05cm$

If the object side were immersed in oil (n=1.5),

$$s_1' = \frac{s_1}{n} = \frac{8.4}{1.5} = 5.6mm \quad > 5\text{mm}.$$

For $CA_o = 6.72$mm, $\theta \approx \frac{CA_o/2}{s_1'} = \frac{CA_o/2}{s_1/n}$

$$NA = n\sin\theta' \approx n\theta' \approx n(\frac{CA_o/2}{s_1/n}) = 0.9$$

That means we can resolve smaller feature size with the same design. The magnifications will increase by 1.5, which are

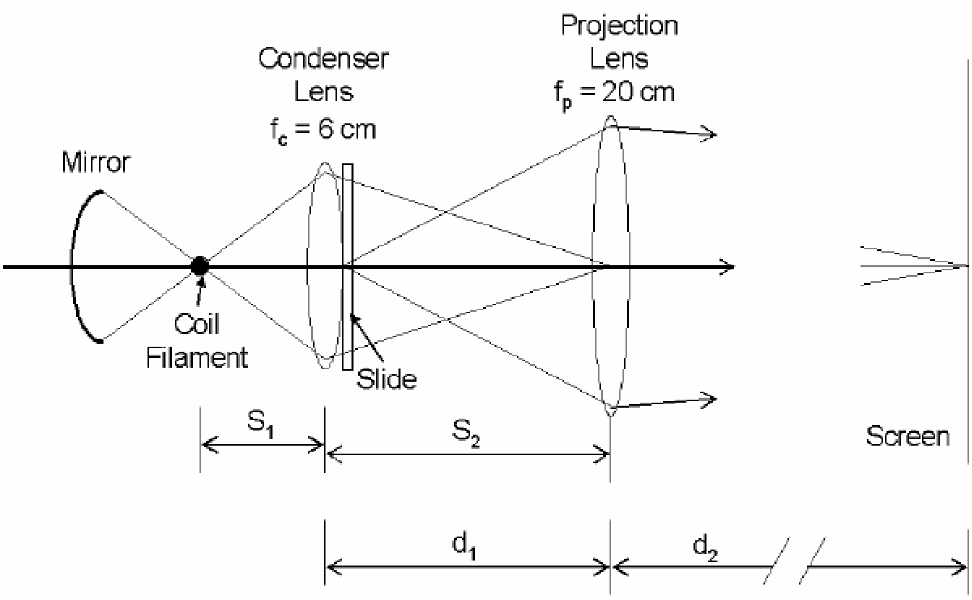
$$M_o' = \frac{s_2}{s_1'} = \frac{16.8cm}{0.56cm} = M_o \times 1.5 = 30$$

and $M_{tot}' = 120$.

For the microscope not to be diffraction-limited, $M_{\max} \cong 240NA$ => $NA \geq 0.5$

Therefore, with the above design, the working distance becomes 5.6mm when the objective is immersed in oil n = 1.5. This still meets the requirement $s_1 > 5mm$. Also, NA increases, microscope has better resolution and it is not diffraction limited. M_{tot} increases too, which means that f_o can be made larger (M_o can be smaller).

5. Solution:
   (a)



(b) The distance between the slide and the projector lens is s_2 in the diagram.

As $m = \dfrac{f_p}{d_1 + f_p} = \dfrac{20}{d_1 + 20} = -100 \quad s_2 = -d_1 = 20.2cm$.

(c) The distance between the projector lens and the screen is d_2.

As $m = \dfrac{d_2}{d_1} = -100$, $d_2 = -100 \cdot d_1 = 20.2m$.

(d) The distance between the filament and the condenser is s_1 in the diagram.
By Lens' law, we have

$\dfrac{1}{s_2} = \dfrac{1}{f_o} + \dfrac{1}{s_1}$ => $\dfrac{1}{20.2} = \dfrac{1}{6} + \dfrac{1}{s_1}$ Solving for s_1, we get $s_1 = -8.54cm$