1. Solution:

(a) The angular magnification of a telescope is $m = f_o / f_e$ so the focal length of the objective lens is $f_o = mf_e = 4 \times 45 = 180cm$.

(b) The clear aperture of the exit pupil is 3.5 mm. The distance between the objective lens and the eyepiece is $-d_i = f_o + f_e = 184cm$. The magnification at such a distance is $m = f_e / (f_e + d_i) = f_e / (f_e - f_o + f_e) = -f_e / f_o = -1/45$.

So the exit pupil is 45 times smaller than the objective lens, meaning that the objective lens has a diameter of $3.5 \times 45 = 157.5mm$.

(c) The object field angle is the angle subtended by the chief ray that barely touches the outside of the eyepiece lens. So it is given by

$$\tan \theta = \frac{d_e}{2} \frac{1}{f_o + f_e} = \frac{1}{184} = 0.0054$$

$$\theta_o = 0.3114^\circ$$

(d) The object and image field angles are related to each other by the angular magnification, so the image field angle is $M \times \theta_o = 14^\circ$.

(e) The diffraction limited angle is determined by diffraction at the entrance pupil, which is the objective lens. It has a diameter of $15.75cm = 0.1575$ m.

$$\theta \approx \sin \theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-7}}{0.1575} = 4.26 \times 10^{-6}$$

The image angle that this corresponds to is $45 \times \theta = 1.917 \times 10^{-4} = 0.19 mrad$.

This is smaller than the resolution of the eye, so the limit of resolution on the telescope is the eye, not diffraction.

Or

The maximum magnification of the telescope for its not being diffraction limited satisfies that $M_{max} = 11CA_o = 11 \times 15.75 / 2.5 = 69.3 > 45$.

Therefore, the limit of resolution on the telescope is not diffraction, but the observer’s eye.
(f) Since the limit on the resolution of the microscope is the observer’s eye, through the telescope, one can resolve objects separated by an angle of

\[ \alpha = 0.3 \text{mrad} / M = 3 \times 10^{-4} / 45 = 6.7 \times 10^{-6} \text{mrad} \, . \]

So the minimum distance in meters between the Eminent British Captain and the mysterious black-flagged ship of potential doom is

\[ D = 0.1 / \alpha = 0.1 / 6.7 \times 10^{-6} = 1.5 \times 10^4 \, m = 15 \text{km} \, . \]

2. Solution:

As this is a design problem, there is no exact answer. The following parameters should be included in your answers, and the grading is based on them:

1. Magnification;
2. Choices of CAo and CAe;
3. Diagram with ray trace, entrance and exit pupils;
4. Choices of f_o and f_e.

If your design flow is reasonable, you’ll get full credits.

A sample design scheme:

The telescope must be able to resolve a feature that is 74500 mile/10 = 7450 mile wide and 746 \times 10^6 miles away, which corresponds to an angle of

\[ \theta = 7450 / (746 \times 10^6) = 9.9666 \times 10^{-6} \text{rad} \]

The magnification that would be required for the cones of the eye to resolve this is

\[ M = \frac{3 \times 10^{-4}}{9.9666 \times 10^{-6}} \approx 30 \]

To make sure the telescope is not diffraction limited,

\[ 60 \geq 5.5 \frac{M}{CA_o} \implies CA_o \geq \frac{5.5}{60} M \quad \text{or} \quad M_{\text{max}} \leq 11CA_o \quad (*) \]

Since the iris of the eye is 4mm, the size of the exit pupil should match, or slightly larger than that, i.e. \( CA_e \approx 4 \text{mm} \). From solution of prob. 1(b), we know that the transverse magnification of the objective lens (entrance pupil) and its image (exit pupil) through the eyepiece is

\[ m = -f_e / f_o = -1/30 \, . \]

The diameter of the objective lens must be at least \( 4 \times 30 = 120 \text{mm} = 12 \text{cm} \).

Plugging this value of \( CA_o = 12 \text{cm} = 4.8 \text{inch} \) into the resolution criterion Eq(\(*\)), we see that \( CA_o \geq \frac{5.5}{60} M \) is valid. So with this choice of \( M \), \( CA_o \) and \( CA_e \), the
telescope is not diffraction-limited.

Now we need to choose $f_o$ and $f_e$. These two numbers can be any, but let’s choose those with reasonable arguments.

The object field angle $\theta$ is determined by the size of the eyepiece. Let’s choose the size of the eyepiece to be $1.2'' = 3\text{cm}$, then $\theta \approx \tan^{-1}\left(\frac{1.5\text{cm}}{s}\right)$. The image field angle $\theta'$ is simply related to $\theta$ by magnification.

We also know that $s = f_o + f_e$ and $m = \frac{f_o}{f_e} = 30$. Let’s choose $f_e = 2\text{cm}$, then $f_o = 60\text{cm}$ and $s = 62\text{cm}$.

If $s = 62\text{cm}$, then we have

$$2\theta_o \approx 2 \tan^{-1}\left(\frac{1.5\text{cm}}{62}\right) = 0.048\text{rad}$$

and the image field angle $2\theta' \approx 1.44\text{rad}$

Also $\frac{s}{s'} = |m|$, hence $s' = 2.067\text{cm}$

For summary,

$$M = 30, \ CA_o = 12\text{cm} = 4.8\text{inch}, \ CA_e = \sim 4\text{mm}$$

$$f_e = 2\text{cm}, \ f_o = 60\text{cm}, \ s = 62\text{cm}, \ s' = 2.067\text{cm}$$

eyepiece size = 3cm.

3. Solution: We want to see if the working distance for this microscope is larger than 2mm. If that is the case, then we can put the glass slide between the microscope objective and the sample to be examined.
Given $M_{tot} = 20$, $f_v = 5cm$ and $x' = 160mm$, we know that

$$M_v = \frac{25cm}{5cm} = 5,$$ so $M_o = 4$.

As $M_o = -\frac{x'}{f_o}$, $f_o = 4cm$.

Using Lens’ law to find $s_1$, we have

$$\frac{1}{s_2} = \frac{1}{f_o} + \frac{1}{s_1} \Rightarrow \frac{1}{f_o + x'} = \frac{1}{f_o} + \frac{1}{s_1}\tag{4}$$

Hence $s_1 = -5cm > > 2mm$.

So it is possible to use this microscope to look at a sample underneath a glass slide of thickness 2mm.

4. Solution:
(There can be many answers, as long as you clearly specify selection of magnification; focal length and diameter of the objective; focal length of the eyepiece, and ray trace of the system, you’ll get full credit.)
The eye can resolve an object size of \( \sim 0.08\text{mm} = 80\ \mu\text{m} \) at the distance of 25 cm. To have a resolution of 1\(\mu\text{m} \), the total magnification of the microscope is

\[ M_{\text{tot}} = \frac{80\mu\text{m}}{1\mu\text{m}} = 80. \]

Pick a common 20x objective, then the eyepiece should have the magnification of

\[ M_e = \frac{80}{20} = 4. \]

Since the standard tube length is 16cm, from

\[ M_o = \left| \frac{x'}{f_o} \right| = \frac{16\text{cm}}{f_o} = 20, \]

we get the focal length of the objective lens to be

\[ f_o = 0.8\text{cm} = 8\text{mm}. \]

Also,

\[ M_o = \frac{s_2}{s_1} = \frac{16 + 0.8}{s_1} = 20 \Rightarrow s_1 = 8.4\text{mm} > 5\text{mm} \quad \text{(working distance)}. \]

Since

\[ M_o = \frac{25\text{cm}}{f_e} \quad \text{(25 cm is the standard viewing distance; near point),} \]

\[ f_e = \frac{25\text{cm}}{4} = 6.25\text{cm}. \]

The distance between the objective lens and the eyepiece is then

\[ d = f_o + x' + f_e = 23.05\text{cm}. \]

In order for the microscope to be not diffraction-limited,

\[ M_{\text{max}} \cong 240NA. \]

For \( M \geq 80, \ NA \geq 0.33. \) Let’s choose \( NA = 0.4 \)

\[ n = 1, \ \theta \approx \frac{CA_o/2}{s_1} = 0.4. \] Thus \( CA_o = 0.4 \times 8.4 \times 2 = 6.72\text{mm}. \)

Summary:

\[
\begin{align*}
M_{\text{tot}} &= 80; \\
\text{objective} &\quad f_o = 8\text{mm}, \quad M_o = 20, \quad NA = 0.4 (CA_o = 6.72\text{mm}) \\
\text{eyepiece} &\quad f_e = 6.25\text{mm}, \quad M_e = 4 \\
x' &= 16\text{cm}, \quad s_1 = 8.4\text{mm} > 5\text{mm} \quad \text{working distance} \\
d &= 23.05\text{cm}
\end{align*}
\]
If the object side were immersed in oil (n=1.5),

\[ s_1' = \frac{s_1}{n} = \frac{8.4}{1.5} = 5.6\,\text{mm} > 5\,\text{mm}. \]

For \( CA_0 = 6.72\,\text{mm} \),

\[ \theta \approx \frac{CA_o/2}{s_1'} = \frac{CA_o/2}{s_1/n} \]

\[ NA = n \sin \theta' \approx n \frac{CA_o/2}{s_1/n} = 0.9 \]

That means we can resolve smaller feature size with the same design. The magnifications will increase by 1.5, which are

\[ M_o' = \frac{s_2}{s_1'} = \frac{16.8\,\text{cm}}{0.56\,\text{cm}} = M_o \times 1.5 = 30 \]

and \( M_{tot}' = 120 \).

For the microscope not to be diffraction-limited, \( M_{max} \approx 240NA \implies NA \geq 0.5 \)

Therefore, with the above design, the working distance becomes 5.6mm when the objective is immersed in oil \( n = 1.5 \). This still meets the requirement \( s_1 > 5\,\text{mm} \). Also, \( NA \) increases, microscope has better resolution and it is not diffraction limited. \( M_{tot} \) increases too, which means that \( f_o \) can be made larger (\( M_o \) can be smaller).

5. Solution:

(a)
(b) The distance between the slide and the projector lens is $s_2$ in the diagram.

As

$$m = \frac{f_p}{d_1 + f_p} = \frac{20}{d_1 + 20} = -100 \quad s_2 = -d_1 = 20.2\text{cm}.$$ 

(c) The distance between the projector lens and the screen is $d_2$.

As

$$m = \frac{d_2}{d_1} = -100 \quad d_2 = -100 \cdot d_1 = 20.2\text{m}.$$ 

(d) The distance between the filament and the condenser is $s_1$ in the diagram.

By Lens’ law, we have

$$\frac{1}{s_2} = \frac{1}{f_o} + \frac{1}{s_1} \Rightarrow \frac{1}{20.2} = \frac{1}{6} + \frac{1}{s_1}$$

Solving for $s_1$, we get $s_1 = -8.54\text{cm}$