[reading assignment: Hecht, 13.1]

**Light Amplification by Stimulated Emission of Radiation**

Basic laser architecture:
- The key element in any laser is the *gain medium* (light amplification)

\[ I_o = e^{gL}I_i \quad g : \text{gain coefficient} \]

The light intensity is increased as it passes through the gain medium. We can examine what happens in each of our 3 viewpoints of light:
- ray direction is preserved
- photons are added with same \( \lambda \), direction
- wave is reinforced *coherently*

- The next element is feedback by an “optical resonator”
Light bounces back and forth between the mirrors. On each round trip, ~ 4-5% of circulating power leaks out.

This is restored by the round trip gain $e^{2gL}$.

**Laser Threshold Condition**

If the loss is, for example, 4%, and if $e^{2gL} > 1.04$, then the gain exceeds the loss, and the system oscillates. Power grows inside the resonator.

Steady state? High power level saturates the gain. At a certain power level gain = loss.

**Mechanism for gain**

Atomic energy levels

<table>
<thead>
<tr>
<th>Level 1</th>
<th>$E_1$</th>
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<tbody>
<tr>
<td>Level 2</td>
<td>$E_2$</td>
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$hv = E_2 - E_1$

$N_2$ : number of atoms in level 2

$N_1$ : level 1

**Spontaneous emission:** Atoms in level 2 randomly “decay,” emitting a photon with energy $hv$. On average, the atom is in level 2 for time $\tau_2$ before emitting the photon. Then

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_2}$$

Under most conditions $N_2 \ll N_1$.

**Absorption:** Consider an atom in level 1. Now, a photon comes along with energy $hv$. The photon is absorbed, and the atom moves to level 2.

$$\frac{dN_1}{dt} = -B I N_1 = -\frac{dN_2}{dt}$$

**Stimulated emission:** Consider an atom in level 2. Now, a photon comes along with energy $hv$. In this case, the atom emits another photon, with the same $\nu$, the same direction, and moves to level 1. There are now two photons travelling together.

$$\frac{dN_2}{dt} = -B I N_2 = -\frac{dN_1}{dt}$$

Stimulated emission and absorption both occur. The net effect is expressed by the equation:

$$\frac{dN_2}{dt} = -B I (N_2 - N_1)$$
Population inversion

If $N_2 < N_1$, $\frac{dN_2}{dt}$ is positive. Photons are being absorbed, and the excited state population is increasing.

But if somehow $N_2 > N_1$ (i.e., inverted population), $\frac{dN_2}{dt}$ is negative! The excited state population is decreasing. On net, photons are being produced.

This is the origin of gain. To achieve population inversion is not easy. First, energy must be pumped into the system. But this is not enough. We also need favorable energy levels.

4-Level laser

Due to long lifetime in level 3, population stacks up. Any atoms in level 2 rapidly drop back to level 1.

Inversion is reached on $3 \rightarrow 2$ transition.

Saturation: As the circulating laser field builds up, $3 \rightarrow 2$ transitions occur more rapidly. This builds up population in level 2. Now the gain is proportional to $(N_3 - N_2)$. When $N_2 = N_3$, $G \rightarrow 0$. Steady state is reached when $N_3 - N_2$ is just positive enough for gain to be exactly equal to the loss.

Optical resonator

The mirrors impose a boundary condition on electric field, $E = 0$ at the mirror surface. Recall plane wave electric field is $E = \sin(kx - \omega t)$, where $k = \frac{2\pi}{\lambda}$. So, the boundary condition is satisfied if

$$kL = m\pi$$

$$\frac{2\pi}{\lambda}L = m\pi \rightarrow L = \frac{m\lambda}{2} \quad \lambda = \frac{2L}{m}$$
We can visualize this condition as saying that an integer number of half wavelengths fit in the resonator.

Recall $\nu = \frac{c}{\lambda n}$. So, $\nu = \frac{mc}{2nL}$.

This gives the allowed frequencies of oscillation:

$$\frac{c}{2nL}$$

is mode spacing.

**Longitudinal modes** – Typical values:

- **HeNe laser**: $L = 50$ cm $\rightarrow \frac{c}{2L} = 300$ MHz

- **Diode laser**: $L = 0.25$ mm $\rightarrow \frac{c}{2nL}(n \sim 3.5) = 170$ GHz

Laser gain spectrum

Homogeneous vs. inhomogeneous broadening

Saturation behavior can be of two types.

- **homogeneous broadening**: Under saturation, entire gain curve is reduced. Shape is unchanged

- **inhomogeneous broadening**:
  - saturating signal is at $\nu_S$
  - gain only saturates in narrow band around $\nu_S$
The inhomogeneous case is usually due to broadening that results from a collection of atoms with varying resonant center frequencies.

Laser oscillation

Inhomogeneous case: *All* modes above threshold will oscillate.

Homogeneous case: Only the highest gain mode oscillates. Entire gain curve saturates until gain = loss for oscillating mode. Then gain < loss for all other modes.