Lecture 17

Gaussian beams

Plane waves: $E(x, y, z) = E_0 e^{-i(kz - \omega t)}$.

Another solution to Maxwell's equations:

$$\boxed{E(x, y, z) = E_0 \Psi(x, y, z) e^{-i(kz - \omega t)}}$$

transverse beam profile
varies slowly with z

- Paraxial approximation: Ψ variation with z is slow compared to x, y variation.
- Plug this form into Maxwell's equations. Use paraxial approximation. The resulting solution is:

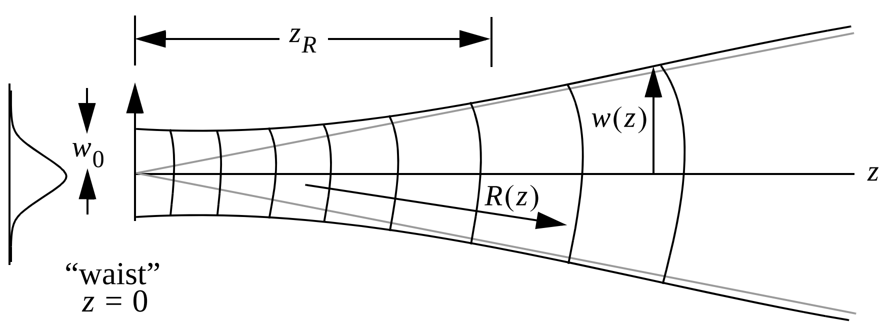
$$E = E_0 \frac{w_0 \exp[-ikz + i\eta(z)]}{w(z)} \exp\left[-\frac{x^2 + y^2}{w^2(z)} - ik\frac{x^2 + y^2}{2R(z)}\right]$$

The transverse amplitude profile of the beam is Gaussian: $\boxed{|E(x, y)| = E_o e^{\frac{-(x^2 + y^2)}{w^2}}}$

$w(z)$: transverse beam radius
$w(z) = w_0[1 + (z/z_R)^2]^{1/2}$

$R(z)$: spherical wavefront radius of beam
$R(z) = z + z_R^2/z$

$\eta(z)$: phase shift of plane wave phase
$\eta(z) = \tan^{-1}(z/z_R)$



$w(z)$: beam spot size. $w(z = 0) \equiv w_0$ (minimum spot)

$R(z)$: wavefront curvature.

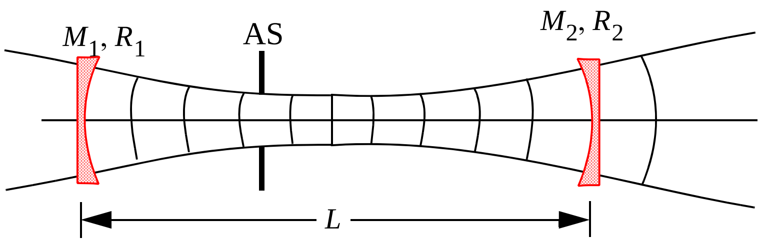
$R(z) = \infty$ at $z = 0$; $R(z) = z$ for $z \gg z_R$

z_R : Rayleigh range – distance where $w = \sqrt{2}\, w_0$

$z_R = \dfrac{\pi w_0^2}{\lambda}$: increases with increasing w_0
: increases with decreasing λ

Lasers can be made to generate this Gaussian beam (in most cases)

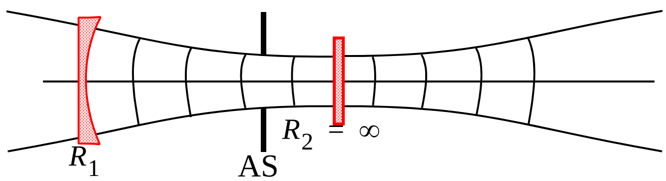
- Use one or two curved mirrors



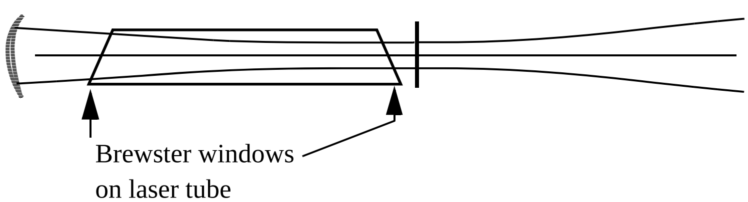
Given R_1, R_2, L, there is one unique Gaussian beam (transverse mode) that fits into the laser resonator. Gaussian beam curvature must *match* mirror curvatures R_1, R_2. Beam waist occurs accordingly.

The output beam is simply the continuation of this Gaussian beam.

To get the beam waist to occur right at the laser output mirror, we use a flat output mirror:



We have to limit the transverse aperture in the laser resonator in order to select the Gaussian mode. We could use a special aperture sized to ~ 3w. Or, the laser gain medium itself could be the aperture:



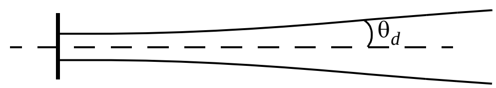
<u>Properties of Gaussian beam</u>

- Long collimation length: $\boxed{z_R = \dfrac{\pi w_0^2}{\lambda}}$

  Take $w_0 = 0.5\text{ mm}, \lambda = 632\text{ nm (HeNe laser)}$. So, $Z_R = 1.2\text{ meters}$.

- Low divergence: $\boxed{\begin{aligned} w(z) &\cong w_0 \frac{z}{z_R} \\ &= \frac{\lambda z}{\pi w_0} \end{aligned}}$ for $z \gg z_R$

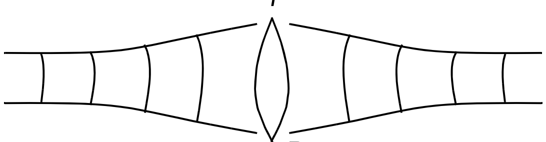
θ_d : divergence half angle



In the paraxial approximation, $\boxed{\theta_d \cong \dfrac{w(z)}{z} = \dfrac{\lambda}{\pi w_0}}$.

Continuing with our example, $w_0 = 0.5\text{ mm}$, $\lambda = 632\text{ nm}, \theta_d = 0.4\text{ mrad}$. So, after 10 m, $w(10\text{ m}) \cong 4\text{ mm}$.

Effect of a lens on a Gaussian beam



- The beam size is unaffected by the lens.
- The beam radius of curvature obeys lens law

$$\boxed{\frac{1}{R_o} = \frac{1}{f} + \frac{1}{R_i}}$$

After the lens, the beam has a radius of curvature R_o. The beam reaches a focus at distance $\approx R_o$ (if $f \gg z_R$).

Suppose the lens has a 4cm diameter, the beam radius at the lens is $w(\text{lens}) = 1\text{ cm}$, and that $R_o = 10\text{ cm}$.

It is easy to show that $\boxed{w_o \cong \dfrac{R_o \lambda}{\pi w(\text{lens})}}$. Then for our case, $w_o = 2\mu\text{m}$.

Note, $w(\text{lens})/R_0 \cong \text{NA}$. So, $w_o = \dfrac{\lambda}{\pi \text{NA}} \cong 0.32\dfrac{\lambda}{\text{NA}}$. This is reminiscent of our finding for imaging systems that resolution = $0.6\dfrac{\lambda}{\text{NA}}$. The spot size for a focused Gaussian beam is very closely related to imaging resolution.