Lecture 17

Gaussian beams

Plane waves: \( E(x, y, z) = E_0 e^{-i(kz - \omega t)} \).

Another solution to Maxwell’s equations:

\[
E(x, y, z) = E_0 \Psi(x, y, z)e^{-i(kz - \omega t)}
\]

- Paraxial approximation: \( \Psi \) variation with \( z \) is slow compared to \( x, y \) variation.
- Plug this form into Maxwell’s equations. Use paraxial approximation. The resulting solution is:

\[
E = E_0 \frac{w_0 \exp[-ikz + i\eta(z)]}{w(z)} \exp \left[ -\frac{x^2 + y^2}{w^2(z)} - i\frac{k}{2R(z)} \frac{x^2 + y^2}{w^2(z)} \right]
\]

The transverse amplitude profile of the beam is Gaussian:

\[
|E(x, y)| = E_0 e^{-\frac{(x^2 + y^2)}{w^2}}
\]

\( w(z) \): transverse beam radius
\( w(z) = w_0 \left[ 1 + \left( \frac{z}{z_R} \right)^2 \right]^{1/2} \)

\( R(z) \): spherical wavefront radius of beam
\( R(z) = z + \frac{z^2}{z_R} \)

\( \eta(z) \): phase shift of plane wave phase
\( \eta(z) = \tan^{-1} \left( \frac{z}{z_R} \right) \)

“waist” \( z = 0 \)
Lasers can be made to generate this Gaussian beam (in most cases)

- Use one or two curved mirrors

Given \( R_1, R_2, L \), there is one unique Gaussian beam (transverse mode) that fits into the laser resonator. Gaussian beam curvature must match mirror curvatures \( R_1, R_2 \). Beam waist occurs accordingly.

The output beam is simply the continuation of this Gaussian beam.

To get the beam waist to occur right at the laser output mirror, we use a flat output mirror:

We have to limit the transverse aperture in the laser resonator in order to select the Gaussian mode. We could use a special aperture sized to \(~ 3w\). Or, the laser gain medium itself could be the aperture:

Properties of Gaussian beam
• Long collimation length:
\[
Z_R = \frac{\pi w_0^2}{\lambda}
\]
Take \( w_0 = 0.5 \text{ mm}, \lambda = 632 \text{ nm (HeNe laser)} \). So, \( Z_R = 1.2 \text{ meters} \).

• Low divergence:
\[
w(z) \approx w_0 \frac{z}{Z_R} = \frac{\lambda z}{\pi w_0}
\]
\( \theta_d \): divergence half angle

In the paraxial approximation,
\[
\theta_d \approx \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}
\]
Continuing with our example, \( w_0 = 0.5 \text{ mm}, \lambda = 632 \text{ nm}, \theta_d = 0.4 \text{ mrad} \). So, after 10 m, \( w(10 \text{ m}) \approx 4 \text{ mm} \).

Effect of a lens on a Gaussian beam

• The beam size is unaffected by the lens.
• The beam radius of curvature obeys lens law
\[
\frac{1}{R_o} = \frac{1}{f} + \frac{1}{R_i}
\]
After the lens, the beam has a radius of curvature \( R_o \). The beam reaches a focus at distance \( \approx R_o \) (if \( f \gg Z_R \)).

Suppose the lens has a 4cm diameter, the beam radius at the lens is \( w(\text{lens}) = 1 \text{ cm} \), and that \( R_o = 10 \text{ cm} \).

It is easy to show that \( w_o \approx \frac{R_o \lambda}{\pi w(\text{lens})} \). Then for our case, \( w_o = 2\mu\text{m} \).
Note, \( w_{\text{lens}}/R_0 \approx \text{NA} \). So, \( w_o = \frac{\lambda}{\pi \text{NA}} \approx 0.32 \frac{\lambda}{\text{NA}} \). This is reminiscent of our finding for imaging systems that resolution = \( 0.6 \frac{\lambda}{\text{NA}} \). The spot size for a focused Gaussian beam is very closely related to imaging resolution.