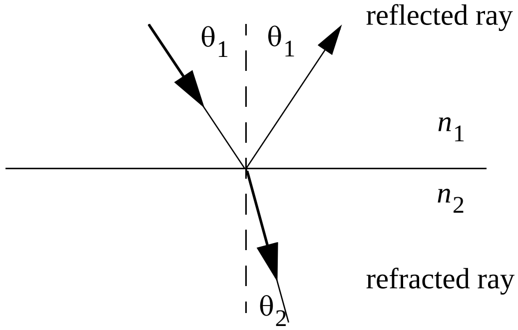
Lecture 2

Reflection and Refraction, Snell's Law

[Reading assignment: Hecht 4.3, 4.4, 4.7]

An important element of optics is the interface between 2 materials with different index of refraction

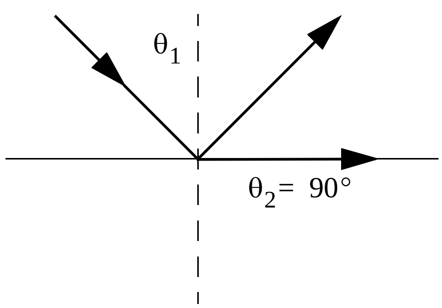


$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \rightarrow \begin{array}{ll} \text{if } n_2 > n_1 & \theta_2 < \theta_1 \\ \text{if } n_1 > n_2 & \theta_2 > \theta_1 \end{array}$$

Total Reflection

If $n_1 > n_2$, then we can have

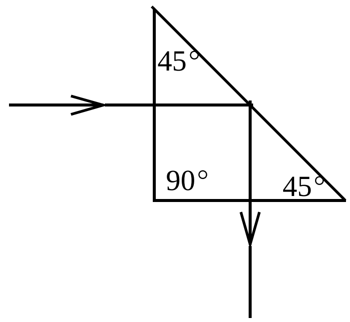


$$\theta_2 = 90°$$
$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\theta_c = \sin^{-1}(n_2/n_1)$$

The refracted ray disappears! The light is totally reflected. This usually occurs inside a prism , and is called total internal reflection. θ_c = "critical angle". For a typical glass with $n = 1.5$, the critical angle is:

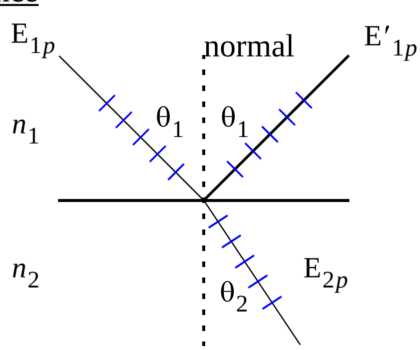
$$\boxed{\sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°}.$$

So for $\theta = 45^\circ$, the light is reflected. A very common prism is the right angle prism

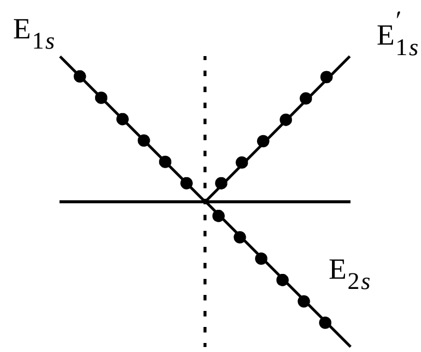


Light Impinging at a Surface

The plane containing the light ray propagation vector and the surface normal is called the "<u>plane of incidence</u>"



p-polarization
(parallel to plane of incidence)



s-polarization

- For a general polarization state incident on a surface, we choose s and p directions to decompose the polarization effects.

Fresnel Reflection Coefficients

[Optional reading: Hecht 4.6]

The magnitude of reflection and transmission at an interface between n_1 and n_2 are given by

$$R_p = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} \qquad T_p = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1} \frac{4\sin^2\theta_2 \cos^2\theta_1}{\sin^2(\theta_1 + \theta_2)\cos^2(\theta_1 - \theta_2)}$$

$$R_s = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} \qquad T_s = \frac{n_2 \cos\theta_2}{n_1 \cos\theta_1} \frac{4\sin^2\theta_2 \cos^2\theta_1}{\sin^2(\theta_1 + \theta_2)}$$

near $\theta_1 = 0$, $R_s = R_p = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$. $\boxed{\text{For } n = 1.5 \;\; R \cong 4\%}$ Notice also, if

$$\theta_2 + \theta_1 = \frac{\pi}{2}\text{, then } \tan \rightarrow \infty\text{, and } R_p \rightarrow 0 \;.$$

θ_1 can be shown to be given by $\boxed{\tan\theta_1 = \frac{n_2}{n_1}}$. This angle is known as “Brewster’s angle” or the “Polarizing angle”.

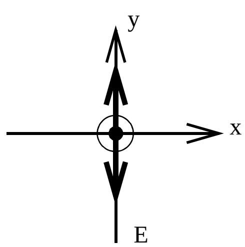
Polarization

Reading Assignment: Hecht 8.1, 8.2]

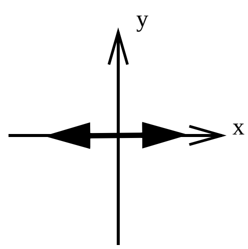
- Light waves have <u>transverse</u> polarization
- The electric field vector points in a direction perpendicular to the propagation direction (ray direction).
- The magnetic field vector is orthogonal to propagation direction. Generally, we can ignore the magnetic field.
- The e-field vector can lie <u>anywhere</u> in transverse plane

Polarization State

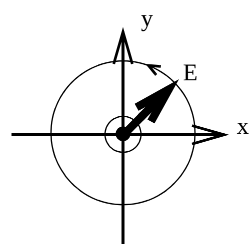
- The e-field oscillates in time at a given point in space
- For light wave propagating in z-direction, let’s look in the x-y plane.



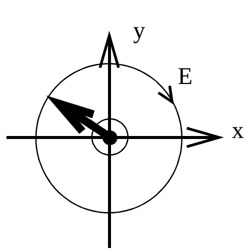
linear polarization
y-direction



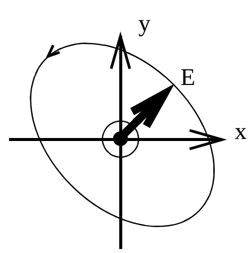
linear polarization
x-direction



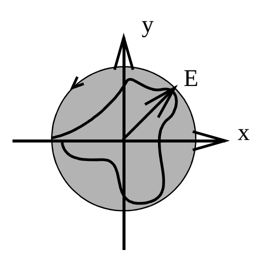
left circular
polarization



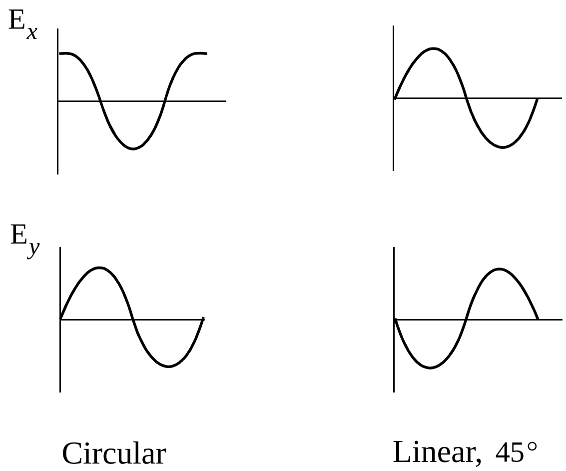
right circular
polarization



elliptical
polarization



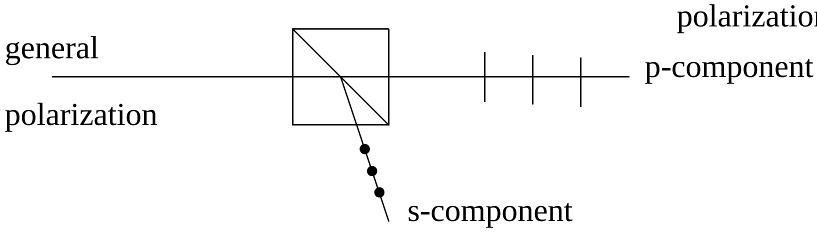
unpolarized
(polarization-vector)
fluctuates randomly



Polarizers are devices which select one polarization

- Polarizing sheet has an allowed direction
  - transmits polarization component of incident light along the allowed direction
  - transmitted light is linearly polarized
- polarizing beamsplitter

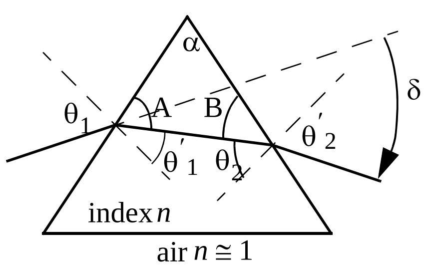
(can be used to analyze input polarization state)



Prisms

[Reading assignment: Hecht, 5.5. See also Smith Ch. 4]

Dispersing prism



Let's calculate the total deviation angle, δ.

- Deviation at first surface is $\theta_1 - \theta'_1$.
- At second surface, deviation is $\theta'_2 - \theta_2$

Total deviation is $\delta = (\theta_1 - \theta'_1) + (\theta'_2 - \theta_2)$

Notice that

$$\boxed{A = \frac{\pi}{2} - \theta_1' \qquad B = \frac{\pi}{2} - \theta_2}$$

but $A + B + \alpha = \pi$, so $\theta_2 = \alpha - \theta'_1$, then

$$\delta = \theta_1 - \theta'_1 + \theta'_2 + -(\alpha - \theta'_1) = \theta_1 + \theta'_2 - \alpha$$

also $\sin\theta'_1 = \frac{1}{n}\sin\theta_1$, and $\sin\theta'_2 = n\sin\theta_2$. Now, writing δ in terms of θ_1, α, and n:

$$\delta = \theta_1 - \alpha + \sin^{-1}(n\sin\theta_2)$$
$$\sin^{-1}[n\sin(\alpha - \theta'_1)]$$

Use $\sin(\alpha - \theta'_1) = \sin\alpha\cos\theta'_1 - \cos\alpha\sin\theta'_1$, $\cos\theta'_1 = \left(1 - \frac{1}{n^2}\sin^2\theta_1\right)^{1/2}$, and $\sin\theta'_1 = \frac{1}{n}\sin\theta_1$. Then

$$\delta = \theta_1 - \alpha + \sin^{-1}\left[n\sin\alpha\left(1 - \frac{1}{n^2}\sin^2\theta_1\right)^{1/2} - \cos\alpha\sin\theta_1\right]$$

$$\boxed{\delta = \theta_1 - \alpha + \sin^{-1}[\sin\alpha(n^2 - \sin^2\theta_1)^{1/2} - \cos\alpha\sin\theta_1]}$$

This formula shows that the deviation increases with increasing index n. For most materials n increases with decreasing λ This is the basis for the splitting of white light into colors by the prism.