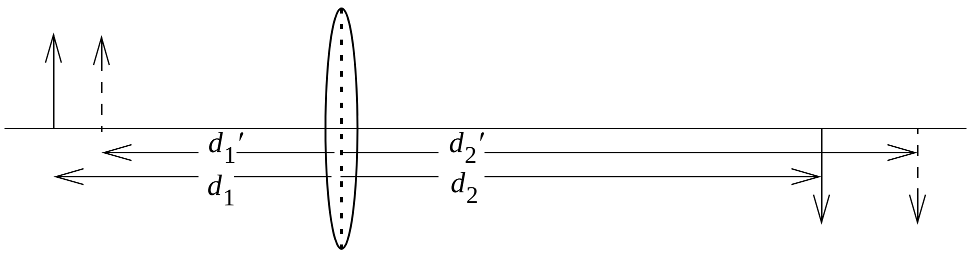
Lecture 4

The <u>longitudinal magnification</u> is of interest.

For a given small shift of an object along the optic axis, how much does the image shift?

We define the longitudinal magnification as:



$$\overline{m} \equiv \frac{\partial d_2}{\partial d_1} = \frac{(d_1+f)f - d_1 f}{(d_1+f)^2} = \frac{f^2}{(d_1+f)^2} = m^2$$

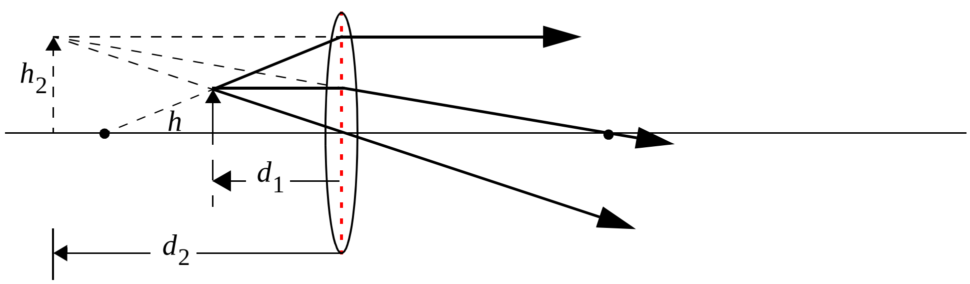
The longitudinal magnification is <u>positive</u> and the <u>square</u> of the transverse magnification

Virtual Image

For an object to the left of the lens, d_1 is <u>negative</u>. Since

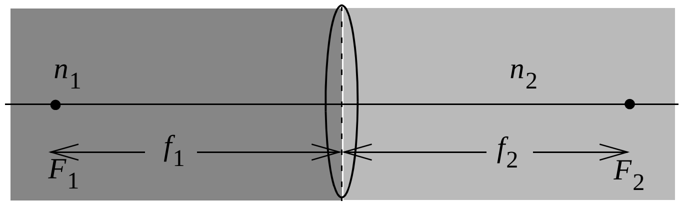
$$\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1}$$

Then if $|d_1| < f$, then d_2 is <u>also</u> negative.



The light emerging from the lens <u>appears</u> to be coming from the object with height h_1 at distance d_2 behind the lens.

For an optical system <u>not</u> immersed in air



The front and back focal lengths are not the same in this case

front focal length: f_1

back focal length: f_2

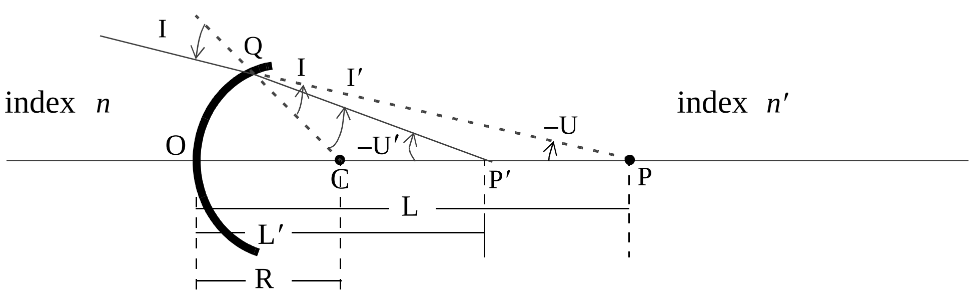
$$\boxed{\frac{f_1}{n_1} = \frac{f_2}{n_2}}$$

lens law becomes: $\quad \frac{n_2}{d_2} = \frac{n_1}{d_1} + \frac{n_1}{f_1} = \frac{n_1}{d_1} + \frac{n_2}{f_2}$

$z_1 z_2 = -f_1 f_2$

$m = \frac{h_2}{h_1} = \frac{n_1 d_2}{n_2 d_1} \qquad \bar{m} = \frac{f_1 f_2}{z_1^2} \qquad \bar{m} \neq m^2$ (show $\bar{m} = \frac{n_2}{n_1} m^2$)

Refraction of light by a spherical surface (following Smith 2.4)



Sign Conventions

1. Radius is positive when center of curvature is to the right of the surface
2. Distance to the right of surface is positive; left negative.
3. For I, I', counterclockwise from the surface normal is positive.
4. For U, U', the angle is positive if the ray slope is positive.
5. Rays travel left to right

In the diagram above all the quantities are positive.

Consider triangle QCP. By law of sines:

$$\boxed{\frac{\sin I}{L-R} = \frac{-\sin U}{R}} \tag{4.1}$$

Similarly for triangle QCP':

$$\frac{\sin I'}{L'-R} = \frac{-\sin U'}{R} \tag{4.2}$$

Comparing triangles QCP and QCP', we see they share a common angle. Therefore

$$\boxed{I-U = I'-U'} \tag{4.3}$$

Finally, by Snell's law

$$n\sin I = n'\sin I' \tag{4.4}$$

We can arrive at the Gaussian lens law (for the single surface) from these equations.

Manipulate Eq. (4.1):

$$\frac{\sin I}{\sin U} = \frac{R-L}{R} = 1-\frac{L}{R}$$

$$\frac{L}{R} = 1-\frac{\sin I}{\sin U} = \frac{\sin U - \sin I}{\sin U}$$

$$\frac{R}{L} = \frac{\sin U}{\sin U - \sin I} = 1-\frac{\sin I}{\sin I - \sin U}$$

Now multiply by $\frac{n}{R}$ to get

$$\boxed{\frac{n}{L} = \frac{n}{R} - \frac{n\sin I}{R(\sin I - \sin U)}} \tag{4.5}$$

Similarly, from Eq.(4.2),

$$\frac{n'}{L'} = \frac{n'}{R} - \frac{n'}{R}\frac{\sin I'}{\sin I' - \sin U'} \tag{4.6}$$

Subtract Eq.(4.5) from Eq.(4.6)

$$\frac{n'}{L'} - \frac{n}{L} = \frac{n'-n}{R} - \left[\frac{n'}{R}\frac{\sin I'}{\sin I' - \sin U'} - \frac{n}{R}\frac{\sin I}{\sin I - \sin U}\right]$$

Using Eq. (4.4),

$$\boxed{\frac{n'}{L'} - \frac{n}{L} = \frac{n'-n}{R} - \frac{n\sin I}{R}\left[\frac{1}{\sin I' - \sin U'} - \frac{1}{\sin I - \sin U}\right]} \tag{4.7}$$

This has the form of the lens law, except for the dependence on the sin of all the angles.

We shall see that <u>Gaussian Optics</u> applies to spherical surfaces only in the <u>Paraxial Approximation</u>.

Paraxial Approximation

The paraxial approximation refers to the case when all ray angles remain small. (Close to the optic axis.) In this case, for all angles, $\sin x \cong x \quad \tan x \cong x$. By convention, the lower case letter is substituted for the capital in this approximation, so

$\sin I \to i,\ \sin I' \to i',\ \sin U \to u,\ \sin U' \to u'$

$L \to l, L' \to l'$.

R is unaffected

Eqs. (4.1)-(4.4) become

$$\boxed{\frac{i}{l-R} = \frac{-u}{R}} \tag{4.8}$$

$$\frac{i'}{l'-R} = \frac{-u'}{R} \tag{4.9}$$

$$i - u = i' - u' \tag{4.10}$$

$$\boxed{ni = n'i'} \tag{4.11}$$

Then Eq. (4.7) becomes:

$$\boxed{\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{R}} \tag{4.12}$$

This is the Gaussian lens law for a single surface.

Consider a ray incident from the left, parallel to the axis. Then $l \to -\infty$, and we have

$$\frac{n'}{l'} = \frac{n'-n}{R}$$

$$\boxed{l' = \frac{n'}{n'-n}R}$$

Thus, the back focal length, f_2, is $\frac{n'}{n'-n}R$. Similarly, for an image distance of ∞, we must have $l' \to \infty$ and

$$\frac{-n}{l} = \frac{n' - n}{R}$$

$$l = -\frac{n}{n' - n}R\,.$$

This means the front focal length f_1 is $\boxed{\frac{n}{n' - n}R}$

(Watch minus sign!)

Recall the previous discussion for a lens not immersed in air:

$$\frac{f_1}{n_1} = \frac{f_2}{n_2} \;\; ; \;\; \frac{n_1}{d_2} = \frac{n_2}{d_1} + \frac{n_1}{f_1} = \frac{n_1}{d_1} + \frac{n_2}{f_2}$$

These relations clearly hold for the single spherical surface.