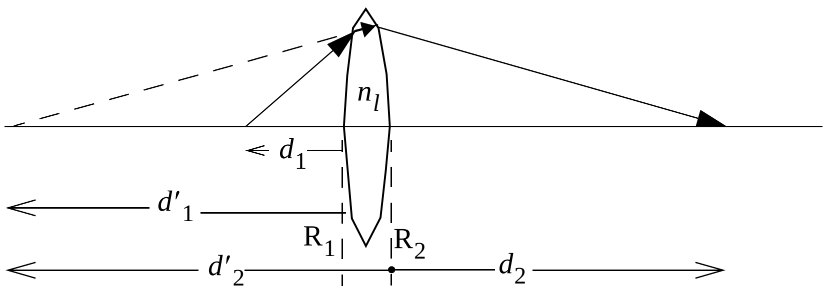
Lecture 5

Thin Lens Model

We now construct our model for the thin lens in air. Lens index is n_l. We will find the imaging properties of the thin lens by using the previous results for a single spherical surface and applying them twice - once for each of the two surfaces of the lens.



Use Eq. (4.12) for first surface: $n = 1 \quad n' = n_l,\ l \to d_1,\ l' \to d'_1$

$$\boxed{\frac{n_l}{d'_1} - \frac{1}{d_1} = \frac{n_l - 1}{R_1}} \tag{5.1}$$

We get a virtual object at d'_1

Now consider the rays travelling inside the lens from the virtual object. Apply spherical surface law now to the second surface. This time $n' = 1,\ n = n_l,\ l \to d'_2\ ,\ l' \to d_2$

$$\boxed{\frac{1}{d_2} - \frac{n_l}{d'_2} = \frac{1 - n_l}{R_2}} \tag{5.2}$$

The <u>thin lens</u> approximation is that the lens thickness is negligible, so that $d'_2 \cong d'_1$. Using this in Eq (5.1), then substituting in Eq. (5.2),

$$\boxed{\frac{1}{d_2} - \frac{1}{d_1} = \frac{1 - n_l}{R_2} + \frac{n_l - 1}{R_1}} \tag{5.3}$$

This is the Gaussian lens law, with the focal length identified as:

$$\boxed{\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \tag{5.4}$$

This is called the <u>lensmaker's equation</u>.

We conclude that a lens with 2 spherical surfaces satisfies the Gaussian lens law, but only under 2 important approximations

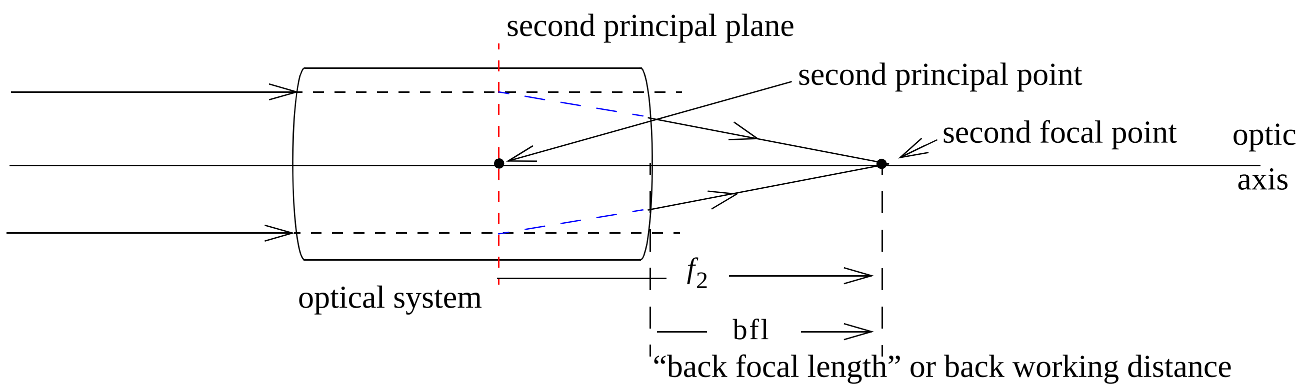
- Paraxial approximation
- Thin-lens approximation

Thick lens or compound lens systems

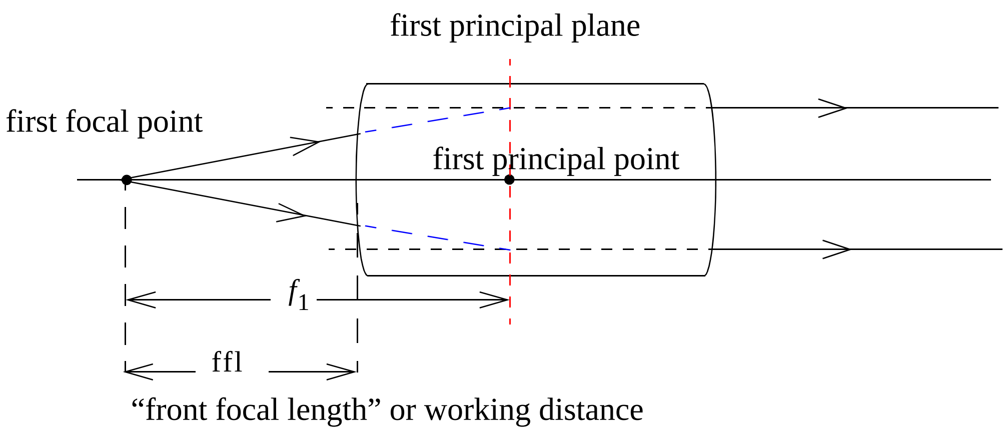
[Reading assignment: Hecht 6.1]

Any symmetric optical system consisting of lenses and spaces can be generalized.

Light rays entering from the left, parallel to the optic axis, come to a focus, at the "second focal point"

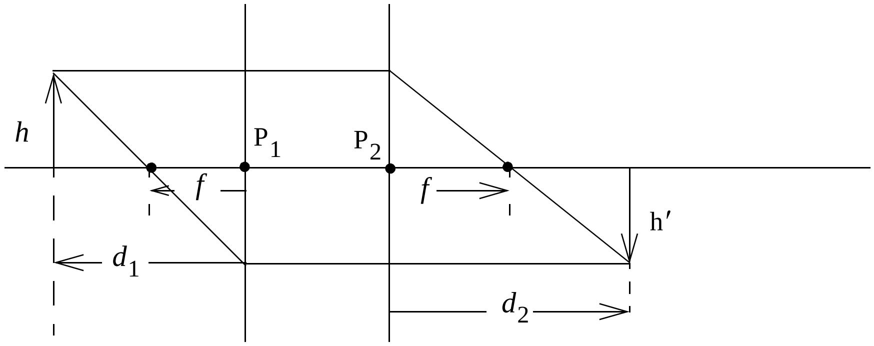


Now, we take the rays entering the system and those emerging from the system and extend them. They intersect on a plane called the "Second principal plane". Similarly, the first focus and "first principal plane" are defined for rays emerging from the system parallel to the axis, which all emanate from a point.



We define f_2 as the distance from the second principal plane to the second focal point. Similarly we define f_1 as the distance from the first principal plane to the first focal point. For a system immersed in air (same index on both sides), $f_1 = f_2$.

With these definitions, the Gaussian lens law applies as follows:

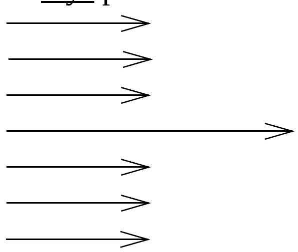


With this geometry, all other relations now apply:

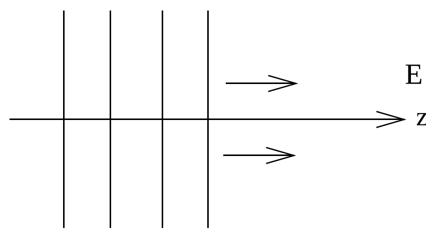
$$\boxed{\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1} \; ; \; m = \frac{d_2}{d_1} \; ; \quad \overline{m} = m^2}$$

Wave optics of lenses

Set of <u>rays</u> parallel to axis

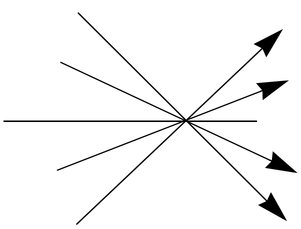


Plane Wave

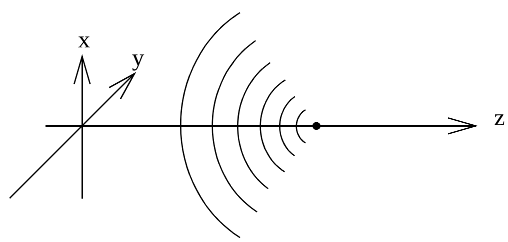


$$E = E_\circ \cos(kz - \omega t)$$
$$k = \frac{2\pi}{\lambda}$$
$$\omega = 2\pi f$$

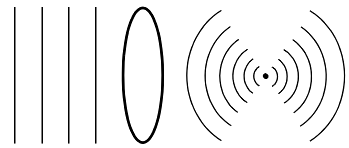
Rays converging to a focus



converging spherical wave



At a given z-plane, the spherical wave has constant phase around circles. The form of the spherical wave is $\cos\left[-\frac{k(x^2+y^2)}{2z_\circ}\right]$ for a spherical wave converting to the point $z_\circ$ on the axis. A lens modifies the wave front, for example from planar to spherical.

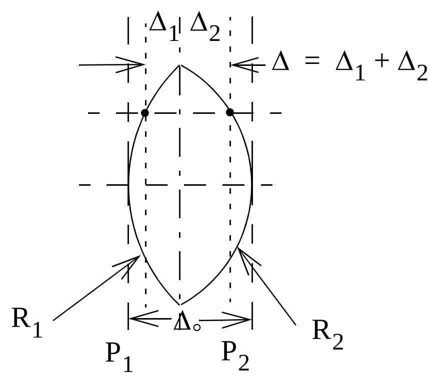


How does this happen?

Optical path length:

Optical waves travel more slowly in the glass since $n > 1$. In glass, the wave is delayed by an amount as if it travelled a distance nl in free space. If $l = l(\mathrm{x,y})$ [or $n = n(\mathrm{x,y})$] then the delay varies with (x,y) so the wavefront gets distorted.

We can analyze the lens in terms of its <u>phase-delay.</u> The light propagates in the glass as $\cos(kn\mathrm{z}) = \cos\phi$, where $\phi = kn\mathrm{z}$ is the <u>phase delay</u>.

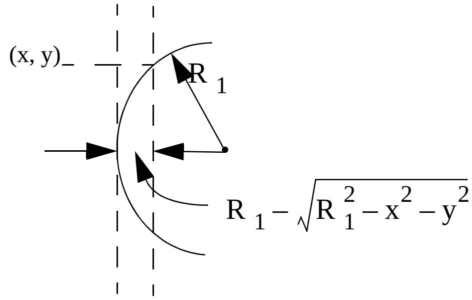


In propagating from plane P_1 to P_2, the light travels a distance $\Delta = \Delta_1 + \Delta_2$ in the glass and a distance $\Delta_\circ - \Delta$ in air, where $\Delta_\circ$ is the thickness at the thickest part of the lens. The phase delay depends on $(\mathrm{x, y})$:

$$\boxed{\phi(\mathrm{x, y}) = kn\Delta(\mathrm{x,y}) + k[\Delta_\circ - \Delta(\mathrm{x, y})]}$$

$$\boxed{= k\Delta_\circ + k(n-1)\Delta(\mathrm{x,y})}$$

We can calculate Δ, assuming spherical surfaces. Recall the sign convention for the surface radii:



From this diagram, we can readily obtain

$$\Delta(x, y) = \Delta_\circ - \left[R_1 - \sqrt{R_1^2 - x^2 - y^2}\right] + \left[R_2 - \sqrt{R_2^2 - x^2 - y^2}\right]$$

$$= \Delta_\circ - R_1\left[1 - \sqrt{1 - \left(\frac{x^2 + y^2}{R_1^2}\right)}\right] + R_2\left[1 - \sqrt{1 - \left(\frac{x^2 + y^2}{R_2^2}\right)}\right]$$

In the <u>paraxial approximation</u> $(x^2 + y^2) \ll R_{1,2}^2$, so

$$\sqrt{1 - \left(\frac{x^2 + y^2}{R_{1,2}^2}\right)} \cong 1 - \left(\frac{x^2 + y^2}{2R_{1,2}^2}\right) \qquad \text{, thus}$$

$$\Delta \cong \Delta_\circ - \left(\frac{x^2 + y^2}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This gives a phase delay:

$$\phi(x, y) = k\Delta_\circ + k(n-1)\left[\Delta_\circ - \left(\frac{x^2 + y^2}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]$$

Apart from the constant delay $kn\Delta_\circ$, the phase delay is:

$$\boxed{\phi(x, y) = -k(n-1)\left(\frac{x^2 + y^2}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

A plane wave incident on the lens has a constant phase. After passing through the lens, the phase is given above. This has the form of a spherical wave, converging to a point at a distance f, where

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)},$$

f is the focal length of the lens. This expression is identical to what we found from the ray optics analysis.