Lecture 8

Achromat Design

- Design starts with desired f_D, P_D
- Next choose your glass materials, i.e. ν', ν''
- Find P_D', P_D'' , then get K', K''
- Choose radii (still some freedom left in choice of radii for minimization of monochromatic aberrations). A common, simple choice is to make the crown lens biconvex, and to cement the two lenses together, with no gap. This means:

$$\boxed{r_2' = -r_1'} \text{ biconvex}$$

$$\boxed{r_1'' = r_2' = -r_1'} \text{ cemented}$$

Then r_2'' is set by the constraint of Eq. (7.3).

For crown glass facing parallel light, this gives a good design to minimize spherical and coma. It can be fine tuned by careful choice of ν', ν''

Example: Design 10cm focal length cemented doublet using the crown and flint glasses. $P_D = 10D$

	n_C	n_D	n_F	$n_{G'}$
crown	1.50868	1.511	1.51673	1.52121
flint	1.61611	1.6210	1.63327	1.64369

$\nu' = 63.4783 \quad \nu'' = 36.1888$

$P_D' = 23.2611D \quad P_D'' = -13.2611D$

note $\boxed{P_D = P_D' + P_D'' = 10D}$ (checks!)

Using biconvex for positive element $r_1' = -r_2'$

$$K' = \frac{2}{r_1'} = \frac{P_D'}{n_D - 1} = 45.5207$$

$$r_1' = 0.043961\text{m} = 4.3961\text{cm}$$

with $r_1'' = -r_1'$, $K'' = -\frac{1}{r_1''} - \frac{1}{r_2''} = \frac{P_D''}{n_D'' - 1} = -21.3544$

So, $\boxed{r_2'' = -71.13\text{cm}}$. This completes the design.

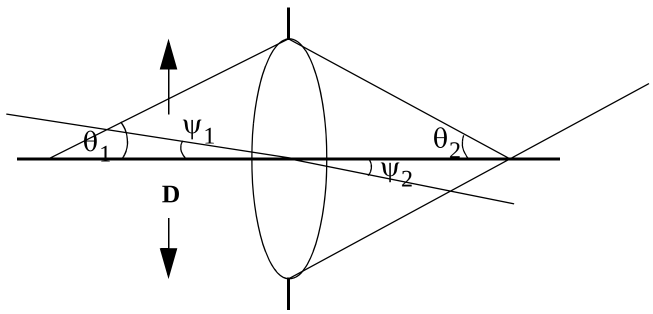
Now check how well it works:

$P_C = (n_C' - 1)K' + (n_C'' - 1)K'' = (0.50868)45.5207 + (0.6161)(-21.3544) = 10.0012D$

$P_F = 9.9988D$

Resolution limit of an optical system

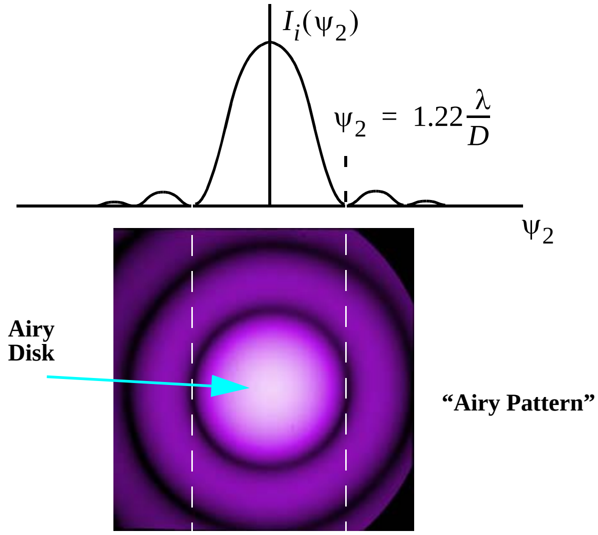
[Reading assignment: Hecht 10.2.6]



Due to diffraction at the aperture stop, the image of a point is slightly blurred. Diffraction theory tells us that the image depends on the shape of the aperture. For a circular aperture:

$$I_i(\psi_2) = I_o\left[\frac{2J_1\left(\pi\frac{D}{\lambda}\sin\psi_2\right)}{\frac{\pi D}{\lambda}\sin\psi_2}\right]^2$$

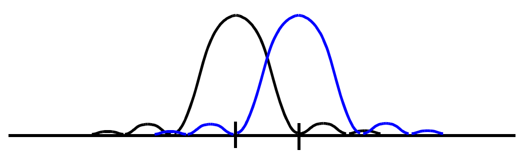
$J_1(x)$ is a special function called the “Bessel Function of the First Kind”.



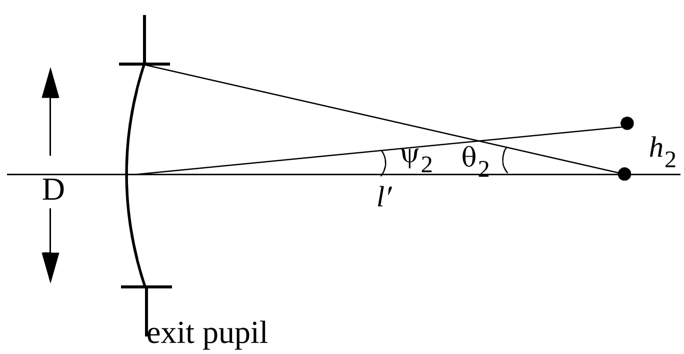
The first zero in the pattern occurs at $\boxed{\psi_2 = 1.22\frac{\lambda}{D}}$.

If 2 points lie close together in the object plane, the Airy patterns will overlap. The criterion for whether the 2 points can be resolved depends on the type of imaging application and it is somewhat arbitrary. A very common criterion is Rayleigh’s criterion.

According to Rayleigh’s criterion, 2 spots are resolved if the maximum of the pattern from one point falls on the first minimum of the other.



We say that the angular resolution in the image plane is $1.22\lambda/D$



with l' the distance to the exit pupil (radius of exit sphere), we have

$$l' = \frac{-D}{2\sin\theta_2}$$

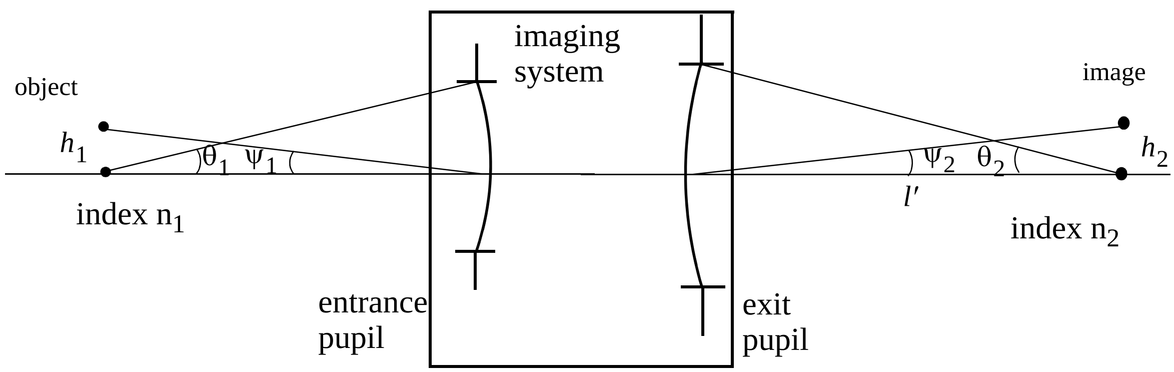
for small ψ_2 (small h_2), $h_2 \cong l'\psi_2$

so $$\boxed{h_2 = \frac{-D\psi_2}{2\sin\theta_2} = \frac{0.61\lambda}{\sin\theta_2} = \frac{0.61\lambda}{NA_2}}$$

where $NA_2 \equiv \sin\theta_2$.

The “Numerical Aperture” or NA is a very important property of an imaging system. It is simply the sine of the half angle subtended by the pupil. Here, NA_2 is the numerical aperture of the exit pupil.

Somewhat more generally, consider a complete imaging system. The entrance pupil subtends an angle θ_1 with an object of height h_1,



A general property of imaging systems holds that:

$$h_1 n_1 \sin\theta_1 = h_2 n_2 \sin\theta_2 .$$

The numerical aperture is generalized if the object and image spaces are immersed in different index of refraction.

$$n\sin\theta_1 \equiv NA_1 . \;\; n_2\sin\theta_2 \equiv NA_2$$

Here NA_1 is the “entrance” numerical aperture and NA_2 is the “exit numerical aperture.

Once again we have: $$\boxed{h_2 = \frac{0.61\lambda}{NA_2}}$$

For large $NA \sim 0.6 \quad h_2 = \lambda$, so the resolution ~ wavelength of light.

Also notice that:

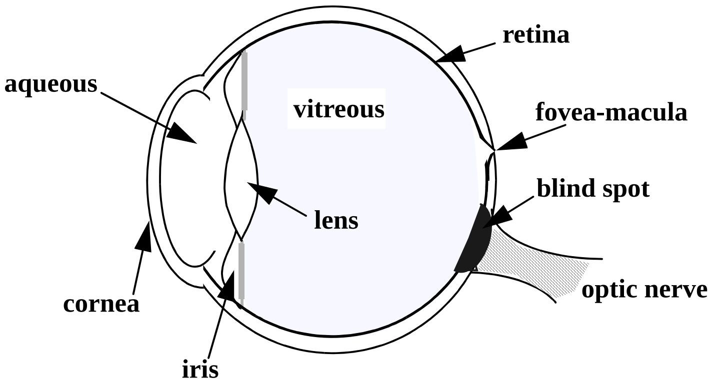
$$\frac{h_2}{h_1} = m = \frac{NA_1}{NA_2}$$

which says that the entrance and exit numerical apertures have a ratio equal to the transverse magnification.

Optical Systems

The Human Eye

[Reading Assignment, Hecht 5.7.1-5.7.3; see also Smith Chapter 5]



	t (mm)	n	R (mm)
cornea f	0	1.376	7.7
b	0.5		6.8
aqueous		1.336	
lens f	4.0	1.386-1.406	10.0 (relaxed), 5 (focused)
b	7.0		–6.0 (relaxed), –5 (focused)
vitreous		1.336	
retina	24.4		

The overall power of the eye is $\sim 58.6\ \mathrm{D}$. The lens surfaces are not spherical, and the lens index is higher at the center (on-axis). Both effects correct spherical aberration. The diameter of the iris ranges from 1.5 $\rightarrow$ 8 mm.

<u>Retina</u>

Rods are most sensitive to light, but do not sense color, motion

Cones are color sensitive in bright light.

You have ~ 6 million cones, ~ 120 million rods, but only 1 million nerve fibers.

Cones are 1 -1.5 μm diameter, 2 –2.5 μm apart in the fovea.

Rods are ~ 2 μm diameter

The macula is $5°$ to the outside of the axis.

The fovea is the central 0.3 mm of the macula. It has **only cones** and is the center of sharp vision.

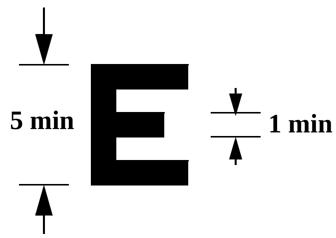
> You can demonstrate to yourself that the fovea only consists of cones, and is less sensitive to light than the surrounding region of your visual field. To see this, look at a faint star in the center of your field of vision. Then look slightly to the side. You see the faint star better when it moves out of the fovea.

Visual Acuity (VA)

The separation between cone cells in the fovea corresponds to about $1'$ (0.3 mrad). At close viewing distance of 25 cm, this gives a resolution of $75\ \mu m$.

This is close to the diffraction limit imposed by NA of the eye.

Visual acuity (VA) is defined relative to a standard of 1 minute of arc. VA = 1/(the angular size of smallest element of a letter that can be distinguished [in min])



VA is usually expressed as $\boxed{\dfrac{\text{dist to target (usually 20 ft)}}{\text{dist at which target element is 1 min}}}$.

For 20/20 vision, the minimum element is 1 min at 20 ft.

The separation of cells increases away from the fovea. This gives a variation of VA with retinal position: