Example of Fourier Series in the Limit of a Long Period

Signal $x(t)$ is a square wave with period $T$ and pulse width $2T_s$.

Fourier series representation: $x(t) = \sum_{k = -\infty}^{\infty} X[k] e^{jk\omega_0 t}$, where $X[k] = \frac{2T_s}{T} \text{sinc}\left(\frac{2T_s}{T}k\right)$ and $\omega_0 = \frac{2\pi}{T}$.

Note that as $T \to \infty$, we obtain an isolated pulse, which is an aperiodic signal. In this limit, $\omega_0 \to 0$, and, although $k$ is a discrete variable, $k\omega_0$ approaches a continuous variable. In this limit, a plot of $2\pi X[k]/\omega_0$ versus $k\omega_0$ approaches a continuous curve.

In the plots below, the Fourier series representations of $x(t)$ include terms for $-64 \leq k \leq 64$. 

![Plots](image-url)
Example of Fourier Series in the Limit of a Long Period

$T_s = 1/2, T = 8, \omega_0 = \pi / 4$

$T_s = 1/2, T = 16, \omega_0 = \pi / 8$