Problem 1 (25 pts.) Consider a DT LTI system having input \( x[n] \), impulse response \( h[n] \) and output \( y[n] \). The system is composed of parallel interconnection of \( N \) DT LTI systems having impulse responses \( h_k[n], k = 0, \ldots, N-1 \).

For any \( k \), \( h_k[n] \) is related to \( h_0[n] \) by \( h_k[n] = e^{j2\pi nk/N} h_0[n] \).

(a) (5 pts.) Let \( H_k(e^{j\Omega}) \) and \( H_0(e^{j\Omega}) \) denote the DTFTs of \( h_k[n] \) and \( h_0[n] \), respectively. Find an expression for \( H_k(e^{j\Omega}) \) in terms of \( H_0(e^{j\Omega}) \).
In parts (b) and (c), let \( h_0[n] \) be an ideal lowpass filter with the frequency response \( H_0(e^{j\Omega}) \) as shown below for the range \(-\pi \leq \Omega < \pi\). The cutoff frequency is \( \Omega_c \), where \( 0 < \Omega_c < \pi \).

\[
H_0(e^{j\Omega})
\]

(b) (10 pts.) Sketch \( H_1(e^{j\Omega}) \) and \( H_{N-1}(e^{j\Omega}) \) for \(-\pi \leq \Omega < \pi\), labeling the vertical and horizontal axes of the plots.

\[
H_1(e^{j\Omega})
\]

\[
H_{N-1}(e^{j\Omega})
\]

(c) (10 pts.) Determine, in terms of \( N \), the value of \( \Omega_c \), \( 0 < \Omega_c < \pi \), such that \( y[n] = x[n] \).
Problem 2  (40 pts.) Consider a system using sampling and ideal bandlimited reconstruction.

Let \( x(t) \) have the FT shown here.

(a) (5 pts.) What is the largest \( T \) such that \( x_r(t) = x(t) \)?

In parts (b), (c) and (c), assume that \( T = 1/7 \).

(b) (10 pts.) Sketch the DTFT of the sampled signal, \( X(e^{j\omega T}) \). Label the vertical and horizontal axes of your plot.

(c) (10 pts.) Sketch the FT of the reconstructed signal, \( X_r(j\omega) \). Label the vertical and horizontal axes of your plot.
(d) (15 pts.) Use Parseval’s identity to calculate the squared error \( \varepsilon = \int_{-\infty}^{\infty} |x(t) - x_r(t)|^2 dt \).

**Problem 3** (35 pts.) A cascade of two LTI systems is shown below.

The first LTI system has the frequency response:

\[
H_1(j\omega) = \begin{cases} 
  e^{-j10\omega} & |\omega| \leq 10\pi \\
  0 & |\omega| > 10\pi 
\end{cases}
\]

(a) (5 pts.) Find an expression for \( H(j\omega) \), the frequency response of the overall system enclosed in the dashed box.
(b) (10 pts.) Sketch $|H(j\omega)|$ and $\arg\{H(j\omega)\}$, labeling the vertical and horizontal axes.

(c) (15 pts.) Find an expression for $h(t)$, the impulse response of the overall system enclosed in the dashed box.

(d) (5 pts.) Let the input be $x(t) = \sin\left(\frac{\pi}{20}t\right)$. Find an expression for the output $y(t)$. 