EECS 120 **Fall 2004**

Demo 12 Solutions

1. (6.45) $H_1(j\omega) = \dfrac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$

In general, we know that if we have a term of the form $\dfrac{1}{(1-ae^{-j\omega})}$, then this term will have an inverse fourier transform that exhibits oscillatory behaviour if a < 0.

$H_1(j\omega)$ has no such terms with a < 0, thus it will not exhibit oscillatory behaviour.

$$H_2(j\omega) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$$

$H_2(j\omega)$ has a term, $\dfrac{1}{(1+\frac{1}{2}e^{-j\omega})}$, with a = -1/2 < 0. Thus the impulse response will exhibit oscillatory behaviour.

$$H_3(j\omega) = \frac{1}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{3}{4}e^{-j\omega}+\frac{9}{16}e^{-j2\omega})}$$

In general, we know that if we have a term of the form $\dfrac{1}{(1-2r\cos\theta e^{-j\omega}+r^2e^{-j2\omega})}$, then this term will have an inverse fourier transform that exhibits oscillatory behaviour if $\theta \neq 0$.

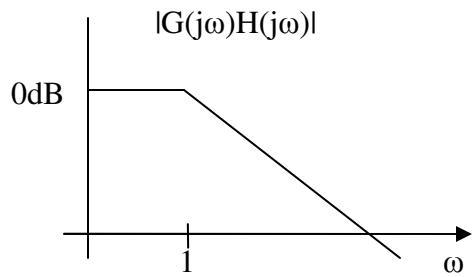
For the $\dfrac{1}{(1-\frac{3}{4}e^{-j\omega}+\frac{9}{16}e^{-j2\omega})}$ term, we have r = ¾, cos(θ) = ½. $\theta = \pi/3 \neq 0$.

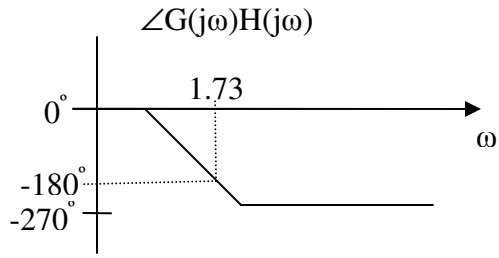
Thus, the impulse response will exhibit oscillatory behaviour.

2. (11.28)

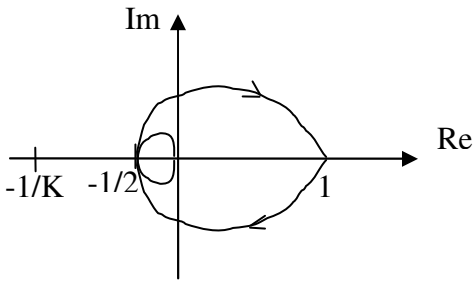
(e) $G(s)H(s) = \dfrac{1-s}{(s+1)^2}$

Sketch of Bode plot:





Sketch of Nyquist plot:



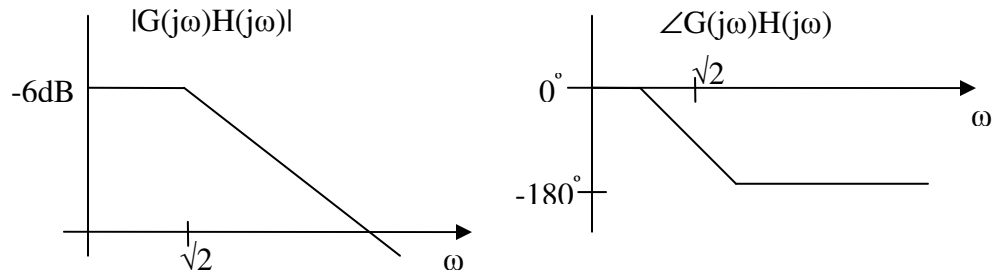
By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point -1/K by the Nyquist plot must equal the number of right-half-plane poles of G(s)H(s). Since G(s)H(s) has 0 right hand pole, the Nyquist plot must not encircle the point -1/K. Thus we get the following condition for the closed loop system to be stable:

$$-\frac{1}{K} < -\frac{1}{2}, \text{ or } -\frac{1}{K} > 1$$

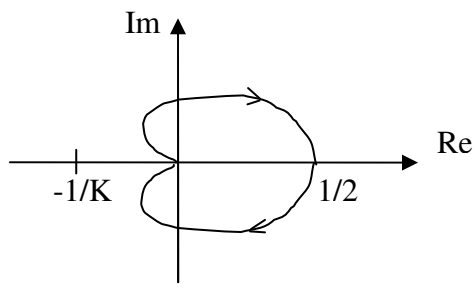
$$K > 1/2 \text{ or } 0 > K > -1$$

(h) $G(s)H(s) = \dfrac{1}{s^2 + 2s + 2}$

Sketch of Bode plot:



Sketch of Nyquist plot:



By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point -1/K by the nyquist plot must equal the number of right-half-plane poles of G(s)H(s). Since G(s)H(s) has 0 right hand pole, the nyquist plot must not encircle the point -1/K. Thus we get the following condition for the closed loop system to be stable:

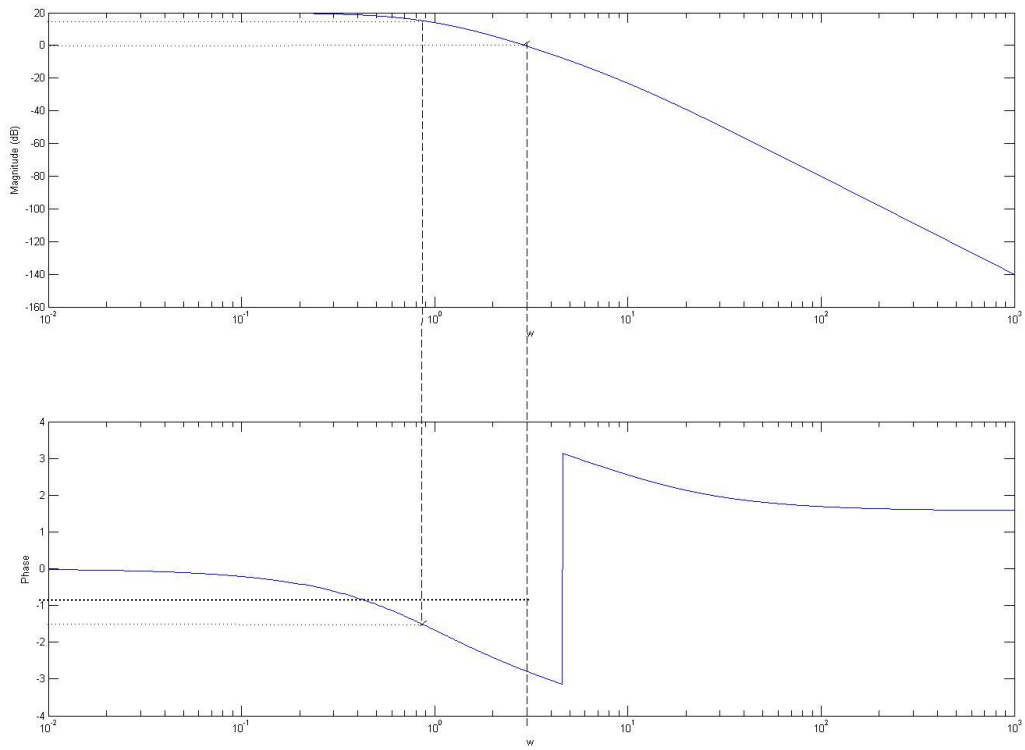
$-\dfrac{1}{K} < 0$, or $-\dfrac{1}{K} > \dfrac{1}{2}$

$K > 0$ or $0 > K > -2$

⇨ K > -2

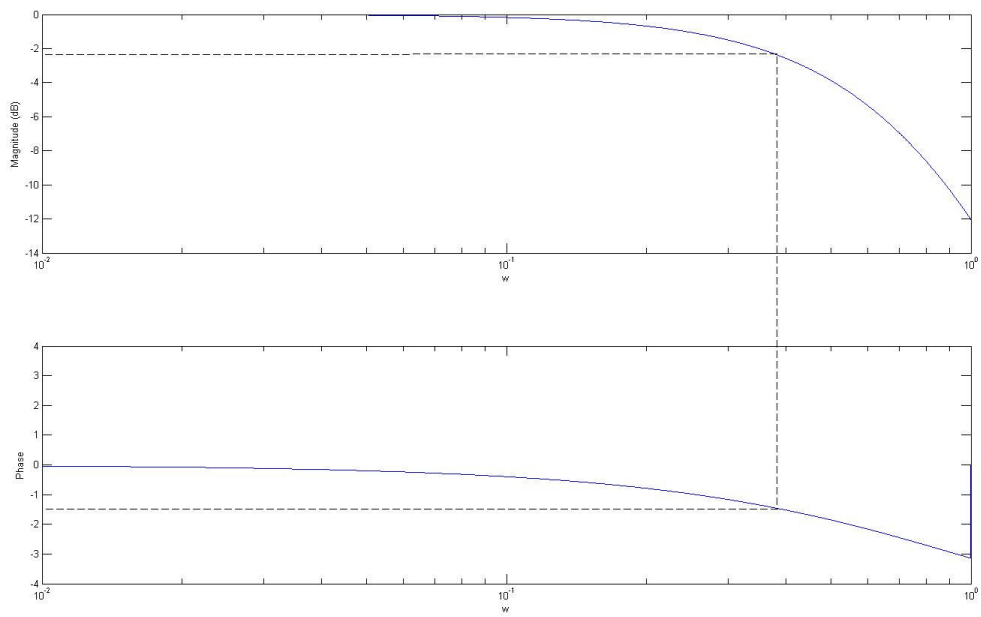
3. (11.29)

(c) $G(s)H(s) = \dfrac{100}{(s+1)^2(s+10)}$



Gain Margin ≈ 15dB
Phase Margin≈3π/4 , found by finding the difference between the phase at the frequency where the magnitude is 0dB and π.

(d) $G(s)H(s) = \dfrac{1}{(s+1)^4}$



Gain Margin ≈ -2dB
Phase Margin≈ π, found by finding the difference between the phase at the frequency where the magnitude is 0dB and π.

4. (11.45)

$$H(s) = \frac{1}{(s+1)(s-2)}$$

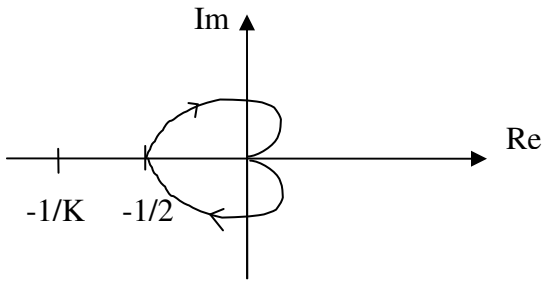
(a) $C(s)H(s) = \dfrac{1}{(s+1)(s+3)}$

This is not considered to be a particularly useful way to attempt to stabilize a system because the pole locations of H(s) will vary due to temperature fluctuations and other external factors. This means that the pole in the RHP may not be perfectly canceled out by the compensator zero and your system will be unstable under some conditions.

(b) Closed loop response: $H_{CL}(s) = \dfrac{kH(s)}{1+kH(s)}$

$$H(j\omega) = \frac{1}{(j\omega+1)(j\omega-2)}$$

Sketch of Nyquist plot:

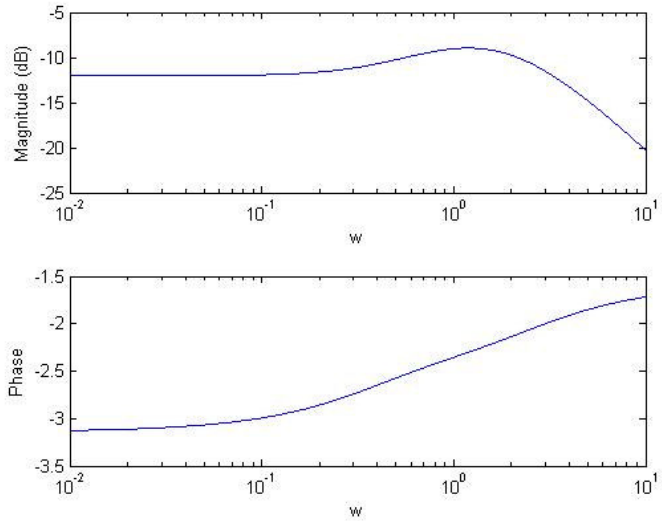


By the Nyquist stability criterion, the number of clockwise encirclements of the point -1/K by the Nyquist plot must equal negative the number of right-half-plane poles of H(s). Since H(s) has 1 right hand pole, the nyquist plot must encircle the point -1/K in the counter-clockwise direction negative one times. This is impossible, so the system cannot be stabilized for any K.

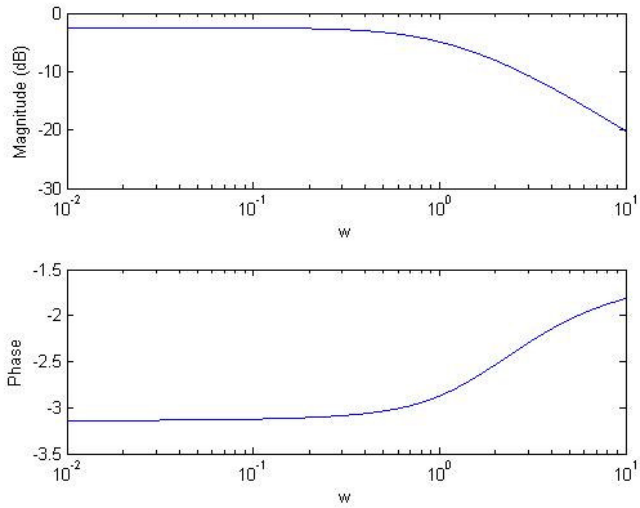
(c) Closed loop response: $H_{CL}(s) = \dfrac{k(s+a)H(s)}{1+k(s+a)H(s)}$

$$(j\omega+a)H(j\omega) = \frac{(j\omega+a)}{(j\omega+1)(j\omega-2)}$$

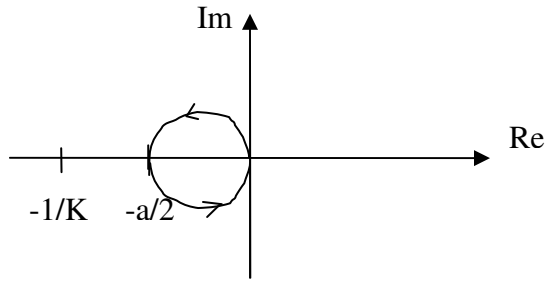
Bode Plot for a = 0.5:



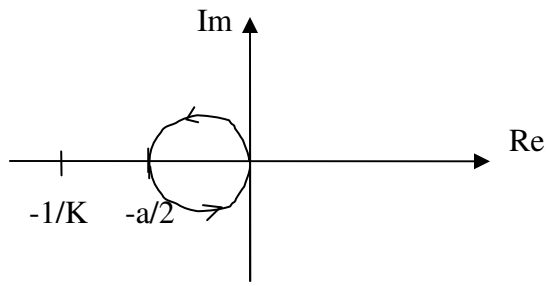
Bode plot for a = 1.5:



Sketch of Nyquist plot for 0 < a < 1:



Sketch of Nyquist plot for 1 < a :



By the Nyquist stability criterion, the number of counter-clockwise encirclements of the point -1/K by the nyquist plot must equal the number of right-half-plane poles of G(s)H(s). Since G(s)H(s) has 1 right hand pole, the Nyquist plot must encircle the point -1/K in the counter-clockwise direction once. Thus we get the following condition for the closed loop system to be stable:

$$-\frac{a}{2} < -\frac{1}{K} < 0$$

=> K > 2/a

(d) Closed loop response: $H_{CL}(s) = \dfrac{k(s+2)H(s)}{1+k(s+2)H(s)} = \dfrac{1}{\dfrac{(s+1)(s-2)}{k(s+2)}+1}$

Solving for the poles we get: $\dfrac{(s+1)(s-2)}{k(s+2)}+1=0$, $(s+1)(s-2)+k(s+2)=0$,

$s^2 + s(k-1) + 2(k-1) = 0$

This is of the form : $s^2 + 2\xi\omega_n s + \omega_n^2$,

Where $\omega_n = \sqrt{2(k-1)}$, $2\xi\omega_n = k-1$, $\xi = \dfrac{k-1}{2\sqrt{2(k-1)}}$

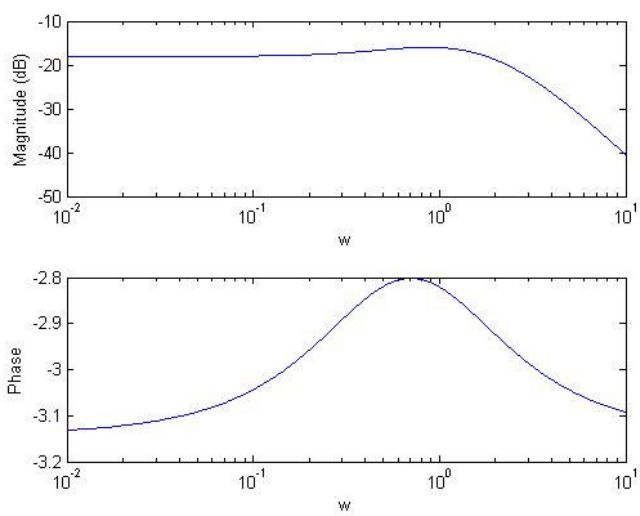
Setting $\xi = 1/2$, we find that we need k = 3.

(e)

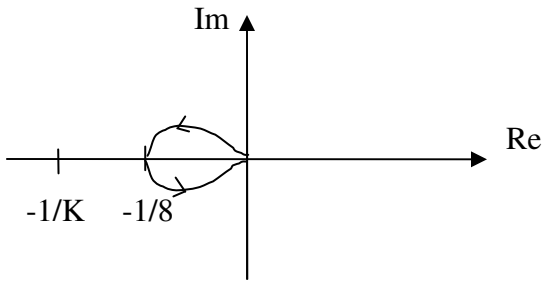
(i) Closed loop response: $H_{CL}(s) = \dfrac{C(s)H(s)}{1+C(s)H(s)}$

$$\frac{(j\omega+1/2)}{(j\omega+2)}H(j\omega) = \frac{(j\omega+1/2)}{(j\omega+1)(j\omega-2)(j\omega+2)}$$

Bode Plot:



Sketch of Nyquist plot:

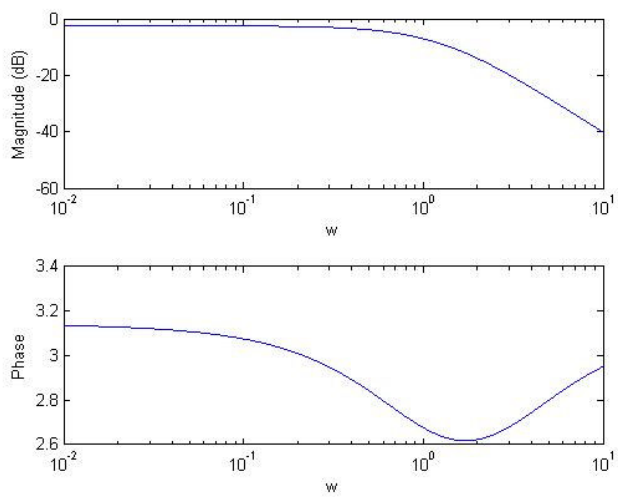


From the Nyquist plot, we see that if K > 8, the closed loop system will be stable.

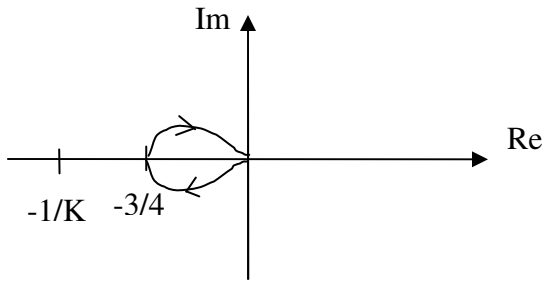
(ii) $H_{CL}(s) = \dfrac{C(s)H(s)}{1+C(s)H(s)}$

$$\frac{(j\omega+3)}{(j\omega+2)}H(j\omega) = \frac{(j\omega+3)}{(j\omega+1)(j\omega-2)(j\omega+2)}$$

Bode Plot:



Sketch of Nyquist plot:



By the Nyquist stability criterion, the number of clockwise encirclements of the point -1/K by the Nyquist plot must equal minus the number of right-half-plane poles of G(S)H(s). Since G(s)H(s) has 1 right hand pole, the Nyquist plot must encircle the point -1/K in the clockwise direction minus one times. This is impossible, so the system will not be stable for any value of K.