1. Parts (e) and (i) of Problem 11.30 on pg. 874 of OWN.

2. Problem 11.43 on pg. 885 of OWN.

3. Problem 11.60 on pp. 907 - 908 of OWN.

4. Let $x[n]$ be a discrete time signal with $z$-transform $X(z)$. Given an integer $M \geq 1$, let $(\downarrow M)$ denote the operation of decimation or downsampling by a factor of $M$, i.e. we write $y[n] = (\downarrow M)x[n]$ if $y[n] = x[nM]$ for all $n$.

Let $(\uparrow M)$ denote the operation of expansion by $M$ (sometimes this is also called upsampling although in this course, following the textbook, we have reserved the word “upsampling” for expansion followed by low pass filtering). Thus, we write $z[n] = (\uparrow M)x[n]$ if

$$z[n] = \begin{cases} 
  x[n/M] & \text{if } n \text{ is an integer multiple of } M \\
  0 & \text{otherwise}
\end{cases}$$

(a) Let $y[n] = (\downarrow M)x[n]$. Verify that

$$Y(z) = \frac{1}{M} \sum_{k=0}^{M-1} X(z \frac{1}{M} e^{j2\pi \frac{k}{M}}).$$

(b) Let $z[n] = (\uparrow M)x[n]$. Verify that

$$Z(z) = X(z^M).$$

(c) Let $u[n] = (\uparrow M)(\downarrow M)x[n]$. Verify that

$$U(z) = \frac{1}{M} \sum_{k=0}^{M-1} X(z e^{j2\pi \frac{k}{M}}).$$

(d) Suppose we first downsample $x[n]$ by a factor of $M$ and then pass the result through a discrete time filter with transfer function $G(z)$. Verify that the output of the filter is the same as the signal we would get if we first pass $x[n]$ through the discrete time filter with transfer function $G(z^M)$ and then downsample the output of this filter by a factor of $M$. We may write this as

$$G(z)(\downarrow M) = (\downarrow M)G(z^M).$$

In the study of multirate signal processing, this is called the first Noble identity.
(e) Suppose we first pass $x[n]$ through a discrete time filter with transfer function $G(z)$ and then expand the output of the filter by a factor of $M$. Verify that the signal we get this way is the same as the one we would get if we were to first expand the signal $x[n]$ by a factor of $M$ and then pass this through the discrete time filter with transfer function $G(z^M)$. We may write this as

$$(\uparrow M)G(z) = G(z^M)(\uparrow M).$$

In the study of multirate signal processing, this is called the second Noble identity.